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A New Friedman's Model for Evolutionary Game Problem

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Abstract: The term game in game theory means a problem, where some of the people or groups (players) share a set of rules and regulations that create the conditions and events that make up the beginning of the game. For instance, in the trade market, the buyers and sellers of some commodities interact in a random way. The reputation of sellers effects on increasing of selling. e.g., honest sellers are more attractive than cheaters for the buyers and the buyers can examine the products or unexamined. In this paper, a non-linear discrete dynamical system of Friedman model was considered. Also, we proposed a new model of interaction between these two populations (buyers and sellers). investigate its limit run behavior where we found the limit converges to a fixed point (0,0) i.e., the sellers will always cheat and the sellers will not examine according to Friedman's model which is denoted by the fixed point (0.0).

Keywords: Discrete dynamical system, Evolutionary game, Friedman's godel, Simplex

Introduction

In evolutionary game theory, each individual has the ability to choose among the strategies in such their payoff depend on the others choice. Then, the behavior of players evolves over time in a population where appropriate strategies will be more prevalent (Sandholm, 2020). Also, the changes of these strategies to be more fit or less fit may increase the complexity of the dynamics of the game. It is natural to ask the following question: which of such behaviors or strategies will be dominant and which extinct in the future and may this system converge to a stable steady state? In heredity model, biologists took into account the interaction between random individuals in one population and analyzed according to steady equilibrium shortly (Evolutionary Steady Stable) for a fixed evolutionary strategy. The population state of species probabilities can be represented by *m*-tuple $(x_1, x_2, \dots, x_m)$, where each x_k represents a fraction of the species k in all the population (Haozhen, 2020). The original pairs (parents) i and j in the case of random interbreeding create for a fixed state $x = (x_1, x_2, \dots, x_m)$ with the probability $x_i x_j$. The scale of species was assumed is such that without doubt for the species of parents i, j determines of the probability $p_{ij,k}$ of each types k in the first descent during the direct posterity of the strategy i and strategy j. where $p_{ij,k}$ is called the coefficient of heredity. Hence the overall probability of the species k in the first generation of direct offspring is defined by:

$$\sum_{i,j=1}^{m} p_{ij,k} x_i x_j \quad (k = 1,2, \dots, m) \tag{1}$$

Let

$$S^{m-1} = \left\{ y = (y_1, y_2, \dots, y_m) \in R^m ; y_i \geq 0, \sum_{k=1}^{m} y_k = 1 \right\} \tag{2}$$

be $m-1$-dimensional simplex. A map V of S^{m-1} into itself, which denoted as

$$(Vy)_k = \sum_{i,j=1}^{m} p_{ij,k} y_i y_j \tag{3}$$

is called a quadratic stochastic operator if for any $y \in S^{m-1}$and for all $k = 1,2, \dots, m$:

$$i)\ p_{ij,k} \geq 0, \qquad ii)\ p_{ij,k} = p_{ji,k}, \qquad iii) \sum_{k=1}^{m} p_{ij,k} = 1.$$

For any initial point $y_0 \in S^{m-1}$, assume that $\left\{ y_0^{(n)} \in S^{m-1}; n = 1,2, \dots \right\}$. The trajectory of operator V is denoted by $y^{(n+1)} = V\left(y^{(n)}\right) \forall\, n = 0,1,2, \dots$, where $y^{(0)} = y$.

A fixed point of the quadratic stochastic operators V is a point $y_0 \in S^{m-1}$ satisfies $V(y_0) = y_0$.

If all eigen values at the fixed point α of Jacobian matrix of the dynamical system are real numbers or complex with absolute value less than 1 then the point α is called stable fixed point. The fixed point is unstable if at least one of them has absolute value greater than one.

For any initial point $y_0 \in R^m$, if the limit

$$\lim_{n \to \infty} V^n(y_0), \tag{4}$$

exists. Then, the quadratic stochastic operator V is called regular operator.

Note that for a quadratic stochastic operator V the limit point is a fixed point. Thus, the limit behavior of trajectory for any initial point can be described by the fixed points. The fixed points of quadratic stochastic operators and behavior of their trajectories have a main role in many research problems such as mathematics (Saburov et. al., 2019), computer science (Hofbauer et. al. 1988), and biology (Jamilov et. al., 2020). For example, in mathematical genetic models, the family of Volterra quadratic stochastic operators can describe dynamics of bisexual populations (Friedman, 1991).

Many ideas introduced by biologists was transferred to the field of economic. but the difference is that the biology viewpoint focused on expand and adapt of the species (Sandholm, 2020; Friedman, 1998). Also, the biologists almost treat with the genetic mechanism of natural selection (Zeeman, 1980; Castanos, 2018; Nachbar, 1990). On the other hand, economists consider the mechanisms of genetics are less importance than learning or imitation (Friedman, 1991; Bernstein, 1924).

In (Castanos et. al., 2018; Leibo et. al., 2019), a biological model was considered differ from the economic models which the authors established the bisexual population system with the difference of the gender. Nowadays, there are only a small number of studies on dynamical phenomena on higher dimensional systems that are presently comprehended, even though they are very important. The below model features interaction of many populations which strategically featured to represent the relations between two sides buyers and sellers.

A Model of Interaction Between Two Population

A two-player game can be represented by a bimatrix$[A, B]$, where A and B are $(n+1) \times (m+1)$, with $n + 1$ is the number of strategies of the first player and $m + 1$ is the number of strategies for the second player (9), (11). if player 1 and player 2 play the ith and jth strategies, then the payoff of the player 1 given by the element a_{ij}. Similarly, the payoff for the player 2 given by b_{ij} under same strategies. In (Friedman, 1991) Friedman provided the following model of interaction between two populations. Considered an appropriate mutual influence between two populations sellers and buyers. two possible options are available for each buyer $i = 1$ (examined) and $i = 2$ (unexamined). also, two alternative available options for each seller $i = 1$ (honest) and $i = 2$ (cheat). The strategy state $s = \left((s_1^1, s_2^1), (s_1^2, s_2^2)\right)$ can be described by a point (p, q) in the unit square $[0,1] \times [0,1]$ because the state of strategies is in one-dimensional. The fraction of examining buyers is $(s_1^1 = p)$ and the fraction of seller who are honest is $(s_2^1 = q)$. Then, $s_2^1 = 1 - p$ and $s_2^2 = 1 - q$.

As an example, the game 72 in (Friedman, 1991), consider two-person game that is characterized by a payoff matrix of the form:

$$A = \begin{array}{c c} & \begin{array}{cc} \text{SH} & \text{SC} \end{array} \\ \begin{array}{c} \text{BI} \\ \text{BDI} \end{array} & \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix} \end{array}$$

and

$$B = \begin{array}{c c} & \begin{array}{cc} \text{SH} & \text{SC} \end{array} \\ \begin{array}{c} \text{BI} \\ \text{BDI} \end{array} & \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix} \end{array}$$

The fitness function investigates how close a solution is to the ideal solution of the given problem and determine how Suitable of the solution. According to (6), the buyer fitness is defined as

$$A.(q, 1-q) = \left(3q + 2(1-q), 4q + 1(1-q)\right) = (q + 2, 3q + 1),$$

and the population average fitness among buyers is:

$$(q+2) - p(q+2) - (1-q)(3q+1) = (1-q)(1-2q).$$

Respectively, the seller's fitness is defined as

$$(p, 1-p)\text{B} = \left(2p + 3(1-p), 1p + 4(1-p)\right) = (3-p, 4-3p),$$

and the population average fitness among buyers is:

$$3 - p - [q(3-p) + (1-q)(4-3p)] = (1-q)(2p-1).$$

As for the dynamics of population of p and q, it is assumed to be Malthusian (for Malthusian population dynamics see (Kenneth, 1951) That the strategy's growth rate is proportional to its relative fitness (with a suitable time scale). Hence, the following system of differential equations is obtained:

$$p' = p(1-p)(1-2q), \qquad (5)$$

$$q' = q(1-q)(2p-1).$$

It is easy to verify that the system of differential equations "5" has exactly five fixed points, i.e., four fixed points at the four corners of the unit square $p - q$ and one at the center. It can be shown that all other points are periodic, and their direction is counterclockwise around center. So that, under Malthusian dynamics the four fixed points at the corners are unstable points, while the fixed point at the center is neither asymptotically stable nor unstable (neutral point). Below, a discrete dynamical system for the considered model "5" was produced because the discrete dynamical systems are of partial importance because computer simulations are discrete in essence (Nachbar, 1990). Despite the complexity of the discrete dynamical systems that sometimes happened, many of discussion, theorems, propositions, and its proofs will be for such discrete systems.

Discrete Friedman's Model

Consider a discrete dynamical system for model considered by Friedman. As noted above the discrete systems are of partial importance because computer simulations are discrete in nature. The discrete time dynamical system that corresponding to the system "5" is defined as follows:

$$p' = p(2 - p - 2q + 2pq),$$
$$q' = q(2p + q - 2pq). \tag{6}$$

To find the fixed points of "6", the following system of equations should be solved.

$$p = p(2 - p - 2q + 2pq),$$
$$q = q(2p + q - 2pq). \tag{7}$$

By solving the system of equations "6", it has also five fixed points at the center $M_0 = (\frac{1}{2}, \frac{1}{2})$ and at the four corners $M_1 = (0,0)$, $M_2 = (0,1)$, $M_3 = (1,0)$, $M_4 = (1,1)$ of the $p - q$ unit square. A Jacobian matrix for dynamical system "3" has the form:

$$J(p,q) = \begin{bmatrix} 2 - 2p - 2q + 4pq & -2p + 2p^2 \\ 2q - 2q^2 & 2p + 2q - 4pq \end{bmatrix}.$$

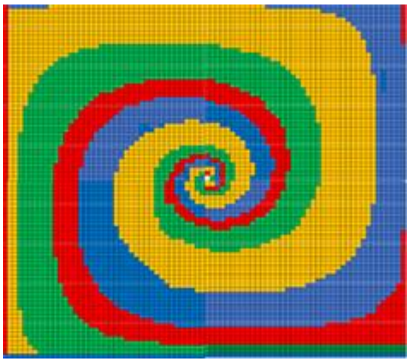


Figure 1 Phase diagram of discrete friedman's model here for arbitrary ınitial blue point: a trajectory converges to point (1,0) and respectively for red point converges to (0,0); for orange point converges to (1,1) and for green point converges to (0,1).

At the corners M_1 and M_4 the Jacobian matrix:

$$J(M_1) = J(M_4) = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix},$$

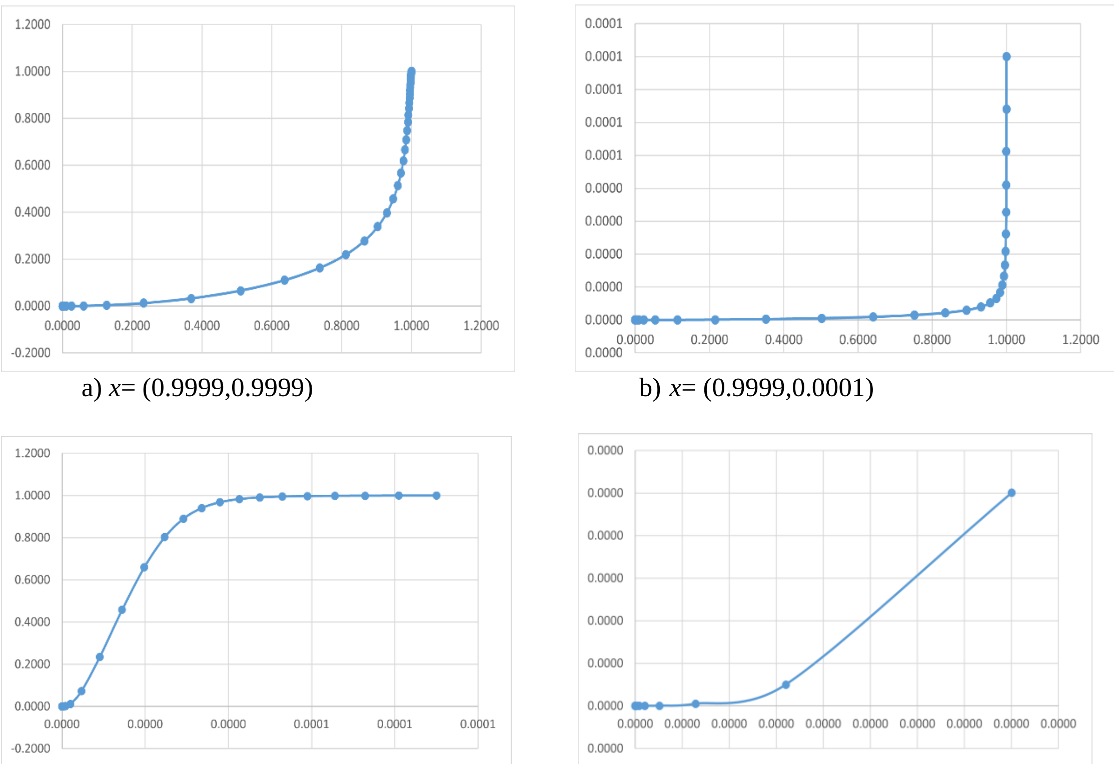
and at the corners M_2 and M_3 Jacobian matrix:

$$J(M_2) = J(M_3) = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}.$$

Also, at the center M_0 Jacobian matrix:

$$J(M_0) = \begin{bmatrix} 1 & -\frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix}.$$

Numerically one can check that the transformation “6” is the regular, that is the trajectory for any initial point converges to one of 5 fixed points “Fig1”. Recall that if the Jacobian matrix at the fixed point of a dynamical system can describe the stability of such system i.e., the fixed point is asymptotically stable if the real part of all the eigenvalues is exactly less than zero. It is easy to verify that all 5 fixed points are non-asymptotically stable. It is easy to verify that all 5 fixed points are non-asymptotically stable.



a) x= (0.9999,0.9999) b) x= (0.9999,0.0001)

c) x= (0.0001,0.9999) d) x= (0.0001,0.0001)

Figure 2. Trajectory of the discrete-dynamical system “9” for a different initial point, x.

New discrete Friedman’s model

We propose a two-person game, as a modification of the game 72 (3), which is characterized by the following payoff matrix of the form:

$$A = \begin{array}{c} \\ \text{BI} \\ \text{BDI} \end{array} \begin{array}{c} \begin{array}{cc} \text{SH} & \text{SC} \end{array} \\ \begin{vmatrix} 3 & 1.4 \\ 3.1 & 2 \end{vmatrix} \end{array},$$

and

SH SC

$$B = \begin{array}{c} \text{BI} \\ \text{BDI} \end{array} \begin{vmatrix} 2.3 & 2.5 \\ 3 & 3.9 \end{vmatrix}.$$

Using the definition of the same fitness function as above, one can produce the following differential equations:

$$f' = f(1-f)(0.5h - 0.6),$$
$$h' = h(1-h)(0.7f - 0.9). \tag{8}$$

The discrete time dynamical system that corresponding to the system "8" is defined as follows:

$$f' = f[1 + (1-f)(0.5h - 0.6)],$$
$$h' = h[1 + (1-h)(0.7f - 0.9)]. \tag{9}$$

It is easy to verify that the system of coupled equations "9" has four fixed points at the four corners $T_1 = (0,0)$, $T_2 = (0,1)$, $T_3 = (1,0)$, $T_4 = (1,1)$ of the *f-h* unit square. and T_1 is attractive fixed point. Thus, for any initial point a trajectory converges to the point T_1 "Fig 2 (a), (b), (c), (d) for some fixed initial points". No matter what the initial population is, eventually all buyers do not examine, and all sellers are cheating (for illustration of the trajectory with different initial points, see "fig 2(a), (b), (c), (d)".

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Scientific Ethics Declaration

The authors declare that the scientific ethical and legal responsibility of this article published in EPSTEM journal belongs to the authors.

References

Bernstein. S. (1924), The solution of a mathematical problem related to the theory of heredity. *Uchn. Zapiski. NI Kaf. Ukr. Otd. Mat. (1)*, 83–115.

Castanos. O, Jamilov. U, & Razikov. U. (2018). On Volterra quadratic stochastic operators of a two-sex population on. *S1×S1 Uzbek. Mat. Zh (3)*, 37-50.

Friedman, D. (1991). Evolutionary games in economics. *Econometrica: Journal of the Econometric Society*, *59*(3) 637-666. Friedman, D., (1998), On economic applications of evolutionary game theory. *J.Evol. Econ. 8*. 15-43.

Haozhen. S., (2020) Evolutionary stable strategies in games with fuzzy Payoffs. *Artificial Intelligence Evolution*, *1*(2): 63-71.

Hofbauer, J. & Sigmund. K., (1988). *The theory of evolution and dynamical systems*. Cambridge Univ. Press.

Jamilov, U. U., & Ladra, M. (2020). On (s, t)-Volterra quadratic stochastic operators of a bisexual population. *Journal of Applied Nonlinear Dynamics*, *9*(4), 575-588.

Kenneth. S, (1951). *The Malthusian controversy economics issues in the 19th century*. Taylor Francis Group.

Leibo. Z, Perolat. J, & Hughes E, (May 2019), Evolving intrinsic motivations for altruistic behavior. *18th International Conference on Autonomous Agents and MultiAgent Systems (AAMAS 2019)*, 13-17, Montreal, Canada.

Nachbar, J. H. (1990). "Evolutionary" selection dynamics in games: Convergence and limit properties. *International Journal of Game Theory*, *19*(1), 59-89.

Saburov, M., & Saburov, K. (2018). Mathematical models of nonlinear uniform consensus II. *Journal of Applied Nonlinear Dynamics*, *7*(1), 95-104.

Sandholm, W. H. (2020). Evolutionary game theory. *Complex Social and Behavioral Systems: Game Theory and Agent-Based Models*, 573-608.

Zeeman, E. C. (1980). Population dynamics from game theory. In *Global theory of dynamical systems* (pp. 471-497). Springer.

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