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On Tzitzeica Curves According to Q-Frame in Euclidean 3-Space

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Abstract: *Tzitzeica curve* is a special spatial curve, which introduced Gheorghe Tzitzeica in 1911. Gheorghe Tzitzeica defined this curve as follows; for a spatial curve ϕ, "if the ratio of its torsion τ and the square of the distance d from the origin to the osculating plane at an arbitrary point of the curve is constant, then this spatial curve is a *Tzitzeica curve* in Euclidean space." Moreover Gheorghe Tzitzeica defined a special surface which is named *Tzitzeica surface* in 1907. In this surface, the asymptotic lines of *Tzitzeica surfaces* with negative Gaussian curvature are *Tzitzeica curves*. Also, the ratio of its Gaussian curvature K and the distance d from the origin to the tangent plane at any arbitrary point of the surface is constant. In this paper, we study the *Tzitzeica curve* via q-frame in $\mathbb{E}^3$. Firstly, we redefine the *Tzitzeica curve* q-frame in Euclidean 3- space. Then, it is obtained some conditions for the *Tzitzeica curve* as a spherical curve. Finally, we investigate harmonic vector cases of the *Tzitzeica curve* according to q- frame in Euclidean space.

Keywords: Tzitzeica curve, q-frame, Spherical curve

Introduction

Tzitzeica curve satisfies the following condition

$$\frac{\tau}{d_{osc}^2} = a,$$

where $a \neq 0 \in \mathbb{R}$, and $d_{osc} = \langle \mathbf{B}, \phi \rangle$ and Tzitzeica surface satisfies the following condition

$$\frac{K}{d_{osc}^4} = a,$$

where $a \neq 0 \in \mathbb{R}$, (Bayram et al., 2018, Bila, 2012 and Crâsmareanu, 2002). Then, these curves and surfaces are often used in scientific research from the past to the present with geometric properties. For example, Crâsmareanu studied the cylindrical form of Tzitzeica curves in (Crâsmareanu, 2002), Karacan gave the elliptic cylindrical form of Tzitzeica curves in (Karacan & Bukcu, 2009), Agnev computed the effect of a general centro-affine transformation on *Tzitzeica surface* in (Agnew et al.,2010) and finally, Bayram characterized *Tzitzeica curves* in $\mathbb{E}^3$ in terms of their curvatures, (Bayram et al., 2018). See more details (Bobe et al., 2012, Yazici et al., 2022)

Although there are many studies on these curves and surfaces in Euclidean space and Minkowski space, it is almost never studied according to q- frame in Euclidean space. We redefine the *Tzitzeica curve* q-frame in Euclidean 3- space. Then, it is obtained some conditions for the *Tzitzeica curve* as a spherical curve. Finally, we give harmonic vector cases of the *Tzitzeica curve* according to q- frame in Euclidean space.

Preliminaries

Let $\phi: I \subseteq \mathbb{R} \to \mathbb{E}^3$ be an unit speed curve. The q-frame of the curve ϕ is obtained as follows

$$\mathbf{t} = \phi', \ \mathbf{n}_q = \frac{\mathbf{t}\times z}{\|\mathbf{t}\times z\|}, \ \mathbf{b}_q = \mathbf{t}\times \mathbf{n}_q, \tag{1}$$

where $\mathbf{n}_q$ is the quasi-normal vector, $\mathbf{b}_q$ is the quasi-binormal vector, and z is the projection vector. The q formulas are expressed as

$$\begin{bmatrix}\nabla_{\phi'}\mathbf{t}\\ \nabla_{\phi'}\mathbf{n}_q\\ \nabla_{\phi'}\mathbf{b}_q\end{bmatrix} = \begin{bmatrix}0 & k_1 & k_2\\ -k_1 & 0 & k_3\\ -k_2 & -k_3 & 0\end{bmatrix}\begin{bmatrix}\mathbf{t}\\ \mathbf{n}_q\\ \mathbf{b}_q\end{bmatrix}, \tag{2}$$

where k_i are called q-curvatures $(1 \le i \le 3)$,

$$\begin{aligned} k_1 &= \kappa\cos\gamma = \langle \mathbf{t}', \mathbf{n}_q\rangle,\\ k_2 &= -\kappa\sin\gamma = \langle \mathrm{t}', \mathrm{b}_q\rangle,\\ k_3 &= d\gamma + \tau = -\langle \mathbf{n}_q, \mathbf{b}_q'\rangle, \end{aligned} \tag{3}$$

and γ is the angle between the principal normal vector and the quasi normal vector. In this paper, it will be assumed that the projection vector $z = (0,0,1)$. If ϕ and z are parallel, the vector z can be chosen as $z = (0,1,0)$, (Dede & Ekici, 2018).

On the other hand, let ϕ be a space curve with the arclength parameter s, and $\nabla_{\phi'} = \frac{d}{ds}$. The Laplacian operator Δ of ϕ can write as, (Kocayigit et al., 2016),

$$\triangle = -\nabla_{\phi'}\nabla_{\phi'}.$$

Definition 2.1 *Let $\phi: I \subset \mathbb{R} \to \mathbb{E}^3$ be a unit speed curve. The curve ϕ is called a harmonic tangent vector if*

$$\triangle\, \mathbf{t} = 0.$$

Also, *The curve ϕ is called a harmonic 1-type tangent vector if*

$$\triangle\, \mathbf{t} = \lambda \mathbf{t},$$

where $\lambda \in \mathbb{R}$, (Kocayigit et al., 2016).

Definition 2.2 *Let $\phi: I \subset \mathbb{R} \to \mathbb{E}^3$ be a unit speed curve. The curve ϕ is called a harmonic principal vector if*

$$\triangle\, \mathbf{n} = 0.$$

Also, *The curve ϕ is called a harmonic 1-type principal normal vector if*

$$\triangle\, \mathbf{n} = \lambda \mathbf{n},$$

where $\lambda \in \mathbb{R}$, (Kocayigit et al., 2016).

Definition 2.3 *Let $\phi: I \subset \mathbb{R} \to \mathbb{E}^3$ be a unit speed curve. The curve ϕ is called a harmonic binormal vector if*

$$\triangle\, \mathbf{b} = 0.$$

Also, *The curve ϕ is called a harmonic 1-type binormal vector if*

$$\triangle\, \mathbf{b} = \lambda \mathbf{b},$$

where $\lambda \in \mathbb{R}$, (Kocayigit et al., 2016).

Some Characterizations of Tzitzeica Curves

In this chapter, we will define *Tzitzeica curves* according to q-frame in $\mathbb{E}^3$. Then, we will do some characterizations of *Tzitzeica curves* with q-curvatures.

Definition 3.1 *Let $\phi: I \subset \mathbb{R} \to \mathbb{E}^3$ be a unit speed curve, $\{\mathbf{t}, \mathbf{n}_q, \mathbf{b}_q\}$ be its q-frame, and k_1, k_2, k_3 be q-curvatures. If the inner product of the two vectors α and β in Euclidean space is denoted by $g(\alpha, \beta)$, the curve ϕ that provides the following condition is called the Tzitzeica curve:*

$$k_3 = \ell g(\mathbf{b}_q, \phi), \tag{4}$$

where $k_3 \neq 0$, $k_1 > 0$,and $k_2 > 0$.

Proposition 3.2 *Let $\phi: I \subset \mathbb{R} \to \mathbb{E}^3$ be a unit speed Tzitzeica curve, $\{\mathbf{t}, \mathbf{n}_q, \mathbf{b}_q\}$ be its q-frame. Then, the Tzitzeica curve ϕ provides the following equality*

$$k_2 k_3 g(\phi, \mathbf{t}) + k_3^2 g(\phi, \mathbf{n}_q) + k_3' g(\phi, \mathbf{b}_q) = 0. \tag{5}$$

Proof. Because the *Tzitzeica curve ϕ* provides eq. (4), $\ell \neq 0$. In this case, the proof is clear.

Theorem 3.3 *Let $\phi: I \subset \mathbb{R} \to \mathbb{E}^3$ be a unit speed Tzitzeica curve, $\{\mathbf{t}, \mathbf{n}_q, \mathbf{b}_q\}$ be its q-frame, and k_1, k_2, k_3 be q-curvatures. The Tzitzeica curve ϕ is a spherical curve if and only if*

$$(-k_1^2 - k_2^2) g(\phi, \mathbf{t}) + (k_1' - k_2 k_3) g(\phi, \mathbf{n}_q) + (k_1 k_3 + k_2') g(\phi, \mathbf{b}_q) = 0. \tag{6}$$

Proof. If the *Tzitzeica curve ϕ* is a spherical curve, we can write

$$\|\phi\| = r.$$

Take the derivative last equation, it is obtained as

$$g(\phi, \mathbf{t}) = 0.$$

If the last equation is taken again derivative, we can write as

$$(-k_1^2 - k_2^2) g(\phi, \mathbf{t}) + (k_1' - k_2 k_3) g(\phi, \mathbf{n}_q) + (k_1 k_3 + k_2') g(\phi, \mathbf{b}_q) = 0.$$

In this case, the proof is completed.

Proposition 3.4 *Let $\phi: I \subset \mathbb{R} \to \mathbb{E}^3$ be a unit speed spherical Tzitzeica curve, $\{\mathbf{t}, \mathbf{n}_q, \mathbf{b}_q\}$ be its q-frame, and k_1, k_2, k_3 be q- curvatures. The following equations can be written for the curvatures of the spherical Tzitzeica curve;*

$$\begin{aligned} k_1 &= \frac{k_3' - k_2'}{k_3}, \\ k_2 &= \frac{k_1' - k_3^2}{k_3}, \\ k_3 &= -\frac{k_1^2 + k_2^2}{k_2}. \end{aligned} \tag{7}$$

Proof. The proof is completed considering the Proposition 3.2 and the Theorem 3.3 together.

Theorem 3.5 *Let $\phi: I \subset \mathbb{R} \to \mathbb{E}^3$ be a unit speed Tzitzeica curve, $\{\mathbf{t}, \mathbf{n}_q, \mathbf{b}_q\}$ be its q-frame, and k_1, k_2, k_3 be q-curvatures. If the position vector of the Tzitzeica curve is*

$$\phi(s) = \pi_1(s)\mathbf{t}(s) + \pi_2(s)\mathbf{n}_q(s) + \pi_3(s)\mathbf{b}_q(s),$$

the following equations can be written

$$\begin{aligned}\pi_1' - \pi_2 k_1 - \pi_3 k_2 &= 1,\\ \pi_1 k_1 + \pi_2' - \pi_3 k_3 &= 0,\\ \pi_1 k_2 + \pi_2 k_3 + \pi_3' &= 0.\end{aligned}$$

Theorem 3.6 *Let $\phi: I \subset \mathbb{R} \to \mathbb{E}^3$ be a unit speed Tzitzeica curve, $\{\mathbf{t}, \mathbf{n}_q, \mathbf{b}_q\}$ be its q-frame, and k_1, k_2, k_3 be q-curvatures. The differential equation characterizing Tzitzeica curve according to its tangent vector is obtained*

$$\begin{aligned}&\mu_1 \nabla^3_{\phi'}\mathbf{t} + (k_1\eta_2 - k_2\eta_1)\nabla^2_{\phi'}\mathbf{t} + (\lambda_3\eta_1 + \lambda_2\eta_2)\nabla_{\phi'}\mathbf{t}\\ &+(\mu(\lambda_3 k_2 + \lambda_2 k_1 - \lambda_1') + \eta_1 k_2 \lambda_1 - \eta_2 k_1 \lambda_1)\mathbf{t} = 0,\end{aligned} \tag{8}$$

where

$$\begin{aligned}\lambda_1 &= -k_1^2 - k_2^2,\\ \lambda_2 &= k_1' - k_2 k_3,\\ \lambda_3 &= k_1 k_3 + k_2',\\ \mu_1 &= k_2\lambda_2 - k_1\lambda_3\\ \eta_1 &= \lambda_1 k_1 + \lambda_2' - \lambda_3 k_3,\\ \eta_2 &= \lambda_1 k_2 + \lambda_3' + \lambda_2 k_3.\end{aligned}$$

Proof. Assume that ϕ is the *Tzitzeica curve* and $\{\mathbf{t}, \mathbf{n}_q, \mathbf{b}_q\}$ is its q-frame. Then, the following equations can be written

$$\nabla_{\phi'}\mathbf{t} = k_1\mathbf{n}_q + k_2\mathbf{b}_q, \tag{9}$$

$$\nabla^2_{\phi'}\mathbf{t} = (-k_1^2 - k_2^2)\mathbf{t} + (k_1' - k_2 k_3)\mathbf{n}_q + (k_1 k_3 + k_2')\mathbf{b}_q, \tag{10}$$

And

$$\begin{aligned}\nabla^3_{\phi'}\mathrm{t} = &\left(-2k_1k_1' - 3k_2k_2' - k_1'k_2 + k_2^2k_3 - k_1k_2k_3\right)\mathrm{t}\\ &+(-k_1^3 - k_1k_2^2 + k_1'' - 2k_2'k_3 - k_2k_3' - k_1k_3^2)\mathbf{n}_q\\ &+(-k_1^2k_2 - k_2^3 + 2k_1'k_3 - k_2k_3^2 + k_1k_3' + k_2')\mathbf{b}_q\end{aligned} \tag{11}$$

Considering eq. (9) and eq. (10) together, q-normal vector and q-binormal vector of the *Tzitzeica curve* obtained as

$$\mathbf{n}_q = \frac{1}{k_2\lambda_2 - k_1\lambda_3}\left(k_2\nabla^2_{\phi'}\mathbf{t} - \lambda_3\nabla_{\phi'}\mathbf{t} - k_2\lambda_1\mathbf{t}\right),$$

and

$$\mathbf{b}_q = \frac{1}{k_2\lambda_2 - k_1\lambda_3}\left(-k_1\nabla^2_{\phi'}\mathbf{t} + \lambda_2\nabla_{\phi'}\mathbf{t} + k_1\lambda_1\mathbf{t}\right).$$

Then, we can write

$$L_1\nabla^3_{\phi'}\mathbf{t} + L_2\nabla^2_{\phi'}\mathbf{t} + L_3\nabla_{\phi'}\mathbf{t} + L_4\mathbf{t} = 0,$$

where

$$\begin{aligned}L_1 &= 2k_1k_2\lambda_2\lambda_3 - k_1^2\lambda_3^2 - k_2^2\lambda_3^2,\\ L_2 &= -(k_2\eta_1 + k_1\eta_2),\\ L_3 &= \lambda_3\eta_1 + \lambda_2\eta_2,\\ L_4 &= (2k_1k_2\lambda_2\lambda_3 - k_1^2\lambda_3^2 - k_2^2\lambda_3^2)(\lambda_3 k_2 + \lambda_2 k_1 - \lambda_1')\\ &\quad+\eta_1 k_2\lambda_1 + \eta_2 k_1\lambda_1.\end{aligned}$$

Corollary 3.7 *Let $\phi: I \subset \mathbb{R} \to \mathbb{E}^3$ be a unit speed Tzitzeica curve, $\{\mathbf{t}, \mathbf{n}_q, \mathbf{b}_q\}$ be its q-frame, and k_1, k_2, k_3 be q-curvatures. The Tzitzeica curve ϕ is not both harmonic tangent vector and harmonic 1-type tangent vector.*

Proof. Assume that ϕ is a the *Tzitzeica curve*. Considering the Definition 2.1 and the Corollary 3.7 together, If the curve ϕ is a harmonic 1-type tangent vector, The following equations can be written

$$\begin{aligned} k_1^2 + k_2^2 &= 0, \\ k_1' - k_2 k_3 &= 0, \\ k_1 k_3 + k_2' &= 0. \end{aligned}$$

In this case, we can write $k_1 = k_2 = 0$, but the last equations conflict the Definition 3.1. That is, the *Tzitzeica curve ϕ i*s not harmonic 1-type tangent vector according to q-frame in Euclidean space.
The other case can be shown similarly. That is, the *Tzitzeica curve ϕ i*s not harmonic tangent vector according to q-frame in Euclidean space.

Theorem 3.8 *Let $\phi: I \subset \mathbb{R} \to \mathbb{E}^3$ be an unit speed Tzitzeica curve, $\{\mathbf{t}, \mathbf{n}_q, \mathbf{b}_q\}$ be its q-frame, and k_1, k_2, k_3 be q-curvatures. The differential equation characterizing Tzitzeica curve according to its q-normal vector is obtained*

$$\begin{aligned} &\mu_2 \nabla^3_{\phi'} \mathbf{n}_q - (\eta_3 k_3 + \eta_4 k_1) \nabla^2_{\phi'} \mathbf{n}_q + (\eta_3 \lambda_3 - \eta_4 \lambda_1) \nabla_{\phi'} \mathbf{n}_q \\ &+ (\eta_3 k_3 \lambda_2 - \mu_2 (\lambda_1 k_1 + \lambda_2' - \lambda_3 k_3) + \eta_4 k_1 \lambda_2) \mathbf{n}_q = 0, \end{aligned} \tag{12}$$

where

$$\begin{aligned} \lambda_4 &= -k_1' - k_2 k_3, \\ \lambda_5 &= -k_1^2 - k_3^2, \\ \lambda_6 &= -k_1 k_2 + k_3', \\ \mu_2 &= k_1 \lambda_3 + \lambda_1 k_3 \\ \eta_3 &= \lambda_1' + \lambda_2 k_1 - \lambda_3 k_2, \\ \eta_4 &= \lambda_1 k_2 + \lambda_2 k_3 + \lambda_3'. \end{aligned}$$

Proof. Assume that ϕ is the *Tzitzeica curve* and $\{\mathbf{t}, \mathbf{n}_q, \mathbf{b}_q\}$ is its q-frame. Then, the following equations can be written

$$\nabla_{\phi'} \mathbf{n}_q = -k_1 \mathbf{t} + k_3 \mathbf{b}_q, \tag{13}$$

$$\nabla^2_{\phi'} \mathbf{n}_q = (-k_1' - k_2 k_3) \mathbf{t} + (-k_1^2 - k_3^2) \mathbf{n}_q + (-k_1 k_2 + k_3') \mathbf{b}_q, \tag{14}$$

and

$$\begin{aligned} \nabla^3_{\phi'} \mathrm{n}_q = &\left(-\mathrm{k}_1'' - \mathrm{k}_2' \mathrm{k}_3 - 2\mathrm{k}_2 \mathrm{k}_3' - \mathrm{k}_1^3 - \mathrm{k}_1 \mathrm{k}_3^2 + \mathrm{k}_1 \mathrm{k}_2^2\right) \mathrm{t} \\ &+ (-3k_1 k_1' - k_1 k_2 k_3 + k_1 k_2 k_3 - 3k_3 k_3') \mathbf{n}_q \\ &+ (-2k_1' k_2 - k_2^2 k_3 - k_1^2 k_3 - k_3^3 - k_1 k_2' + k_3'') \mathbf{b}_q. \end{aligned} \tag{15}$$

The proof is completed.

Corollary 3.9 *Let $\phi: I \subset \mathbb{R} \to \mathbb{E}^3$ be a unit speed Tzitzeica curve, $\{\mathbf{t}, \mathbf{n}_q, \mathbf{b}_q\}$ be its q-frame, and k_1, k_2, k_3 be q-curvatures. The Tzitzeica curve ϕ is not both harmonic q-normal vector and harmonic 1-type q-normal vector.*

Proof. The proof is clear from the proof of the Corollary 3.7.

Theorem 3.10 *Let $\phi: I \subset \mathbb{R} \to \mathbb{E}^3$ be an unit speed Tzitzeica curve, $\{\mathbf{t}, \mathbf{n}_q, \mathbf{b}_q\}$ be its q-frame, and k_1, k_2, k_3 be q- curvatures. The differential equation characterizing Tzitzeica curve according to its q-binormal vector is obtained*

$$\begin{aligned} &\mu_3 \nabla^3_{\phi'} \mathbf{b}_q - (\eta_5 k_3 + \eta_6 k_2) \nabla^2_{\phi'} \mathbf{b}_q - (\eta_5 \lambda_8 + \eta_6 \lambda_7) \nabla_{\phi'} \mathbf{b}_q \\ &+ \left(\eta_5 k_3 \lambda_9 + \eta_6 k_2 \lambda_9 - \mu_3 (\lambda_7 k_2 - \lambda_8 k_3 + \lambda_9')\right) \mathbf{b}_q, \end{aligned} \tag{16}$$

where

$$\lambda_7 = -k_2' + k_1 k_3,$$
$$\lambda_8 = -k_1 k_2 - k_3',$$
$$\lambda_9 = -k_2^2 - k_3^2,$$
$$\mu_3 = \lambda_8 k_2 - \lambda_7 k_3,$$
$$\eta_5 = \lambda_7' - \lambda_8 k_1 - \lambda_9 k_2,$$
$$\eta_6 = \lambda_7 k_1 + \lambda_8' - \lambda_9 k_3.$$

Proof. Assume that ϕ is the *Tzitzeica curve* and $\{\mathbf{t}, \mathbf{n}_q, \mathbf{b}_q\}$ is its q-frame. Then, the following equations can be written

$$\nabla_{\phi'} \mathbf{b}_q = -k_2 \mathbf{t} - k_3 \mathbf{n}_q, \tag{17}$$

$$\nabla_{\phi'}^2 \mathbf{b}_q = (-k_2' + k_1 k_3)\mathbf{t} + (-k_1 k_2 - k_3')\mathbf{n}_q + (-k_2^2 - k_3^2)\mathbf{b}_q, \tag{18}$$

and

$$\nabla_{\phi'}^3 \mathbf{b}_q = (\lambda_7' - \lambda_8 k_1 - \lambda_9 k_2)\mathbf{t} + (\lambda_7 k_1 + \lambda_8' - \lambda_9 k_3)\mathbf{n}_q + (\lambda_7 k_2 - \lambda_8 k_3 + \lambda_9')\mathbf{b}_q.$$

Corollary 3.11 *Let* $\phi: I \subset \mathbb{R} \to \mathbb{E}^3$ *be a unit speed Tzitzeica curve,* $\{\mathbf{t}, \mathbf{n}_q, \mathbf{b}_q\}$ *be its q-frame, and* k_1, k_2, k_3 *be q-curvatures. The Tzitzeica curve* ϕ *is not both harmonic* q-binormal *vector and harmonic 1-type* q-binormal *vector.*

Proof**.** The proof is clear from the proof of the Corollary 3.7.

Conclusion

Tzitzeica curve is a special spatial curve which is introduced by Gheorghe Tzitzeica in 1911. We redefine the *Tzitzeica curve* according to q-frame in Euclidean 3-space. We obtain the necessary and sufficient condition for a *Tzitzeica curve* to be a spherical curve. Then, we give some curvature properties satisfied by these curves. Finally, we obtain differential equations that characterize *Tzitzeica curves* according to q-frame and we determine if the *Tzitzeica curve* can both be harmonic and harmonic 1-type vector for three cases according to q-frame in Euclidean 3-space.

Scientific Ethics Declaration

The authors declare that the scientific ethical and legal responsibility of this article published in EPSTEM journal belongs to the authors.

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