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Dynamic Control of Non-Linearly Tapering FGM Beams

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Abstract: This work presents the dynamics and active vibration control of a non-uniform functionally graded beam. Thanks to the strong use of beams and specifically with non-uniform section, in the different industrial applications, such as helicopter rotor blades, wind turbines, space and marine structures, we present in this article, a comparison of linearly and non-linearly tapering beams. The FGM beam is equipped with four layers of piezoelectric materials as sensors and actuators, bonded on the upper and lower surfaces of the main structure, on different finite elements to see the influence of its location on the dynamics and active control. In this study, the Timoshenko beam's theory combined with FEM is applied to a beam divided into a finite number of elements. Hamilton's principle is applied to generate the equation of motion. The structure is modeled analytically and numerically and the simulation results are presented at the end. The optimal LQG control with Kalman filtering is applied.

Keywords: Vibration, LQG-kalman control, FGM, Piezoelectric materials, Timoshenko's beam theory

Introduction

The mathematical finite element model of Euler Bernoulli has shown its limitations because it does not take into account shear effects, rotational inertia, axial displacements etc... In order to remedy this, we model our smart structure by the Timoshenko shear deformation theory. Timoshenko's beam model corrects the classical beam model by considering the effects of first order transverse shear deformation and axial displacements. Thus, the Timoshenko's beam theory overcomes the disadvantages of the EB's beam theory and the resulting mathematical model is closer to an exact model. For isotropic beams, Timoshenko's theory takes into account six fundamental global deformations (bending and transverse shearing in two directions, extension and torsion). In this model, a plane cross-section perpendicular to the beam axis before deformation no longer remains normal to the beam axis after deformation due to shear. The derivation of normality is produced by a transverse shear which is assumed to be constant over the cross section. Thus, the Timoshenko beam model performs better than the EB's model for the accurate prediction of the response of a beam and is of additional importance in the dynamics of the structure and the control of its vibrations.

In the mid-1980s, a new class of composites, called functionally graded materials (FGMs), emerged thanks to a group of Japanese scientists (Koizumi, 1997). That year, Japanese researchers faced a problem that required a specific type of composite material that could withstand a very large temperature difference in a space project. These materials exhibit a gradual change in their composition as a function of volume (Bharti et al., 2013). These composites are widely used in various engineering applications due to their high temperature resistance.

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Mechanical structures are often subject to vibrations from various sources. These vibrations are generally sources of problems affecting the proper functioning of many systems and processes in different industrial sectors and many engineering fields such as automotive, aeronautics, naval... Thanks to the strong use of beams and specifically with non-uniform section, in the different industrial applications, such as helicopter rotor blades, wind turbines, space and marine structures. These systems increasingly integrate more composite materials in the manufacture of structures. The amplitude of vibrations can cause a significant number of problems such as damage or fatigue of structures. One solution to such a problem is active vibration control (AVC) (Beards 1996).

Examples of systems where AVC can be applied are numerous, such as fans, vehicle interiors, precision equipment, engines, electric and hydraulic drives, noise barriers and enclosures… (Bendine et al., 2016, El Harti et al., 2017, Panda et al., 2016). There have been extensive theoretical and experimental studies on FGM structures that have been published, especially beams and are still of interest to researchers because of their applications. (El Harti et al., 2017) studied the AVC of an FGM beam with a piezo-materials with a new geometry. They also addressed the active control of the porous Euler-Bernoulli FGM beam in a thermal environment with symmetrically bonded piezo materials (El Harti et al., 2020), as well as the dynamic analysis and AVC of the distributed piezo-thermo-elastic porous FGM beam modeled by the FEM (El Harti et al., 2021). (Sahabni & Cunedioglu, 2020) studied the free vibration analysis of cracked FG non-uniform beams. (Ebrahimi & Hashemi, 2017) investigated the vibration analysis of non-uniform FG porous beams under thermal loading. (Kumar et al., 2015) analyzed the geometrically nonlinear free vibration of axially FG taper beams. (Chen, 2021) studied the vibration analysis of axially FG Timoshenko beams with non-uniform section-cross. (Huang et al., 2013) examined the free vibration of axially functionally graded Timoshenko beam with non-uniform cross section. (Ozdemir et al., 2010) presented a study of the vibration analysis of a rotating tapered beam using DTM. (Eberle & Oberguggenberger, 2022) established a new method for estimating the bending stiffness curve of non-uniform Euler-Bernoulli beams using static deflection data.

Mathematical Formulation

A non-uniform FG beam with dimension $(L \times b(x) \times h)$, the width $b(x)$ is considered variable in this study. The general equations for the cross-sectional area, width and moment of inertia of the beam are given by (Ebrahimi & Hashemi, 2017):

$$\begin{aligned} b(x) &= b_0 \left(1 - C_b \frac{x}{L}\right)^m \\ A(x) &= A_0 \left(1 - C_b \frac{x}{L}\right)^m \\ I_y(x) &= b_{y0} \left(1 - C_b \frac{x}{L}\right)^m \end{aligned} \quad (1)$$

The width taper ratio (C_b) must satisfy $0 \leq C_b < 1$ for $A(x)$ and $I_y(x)$ to be positive. Knowing the width quantities at the embedding and free end of the structure, the width taper ratio can be calculated as follows:

$$C_b = 1 - \frac{b}{b_0} \quad (2)$$

In this study, the beam shrink linearly (m=1). The quantities of cross sectional area, moment of inertia at the beam embedment can be written as follows:

$$\begin{aligned} A_0 &= b_0 h \\ I_{y0} &= \frac{b_0 h^3}{12} \end{aligned} \quad (3)$$

The properties of the materials vary from pure metal on the bottom side to pure ceramic on the top side using power law, presented as follows: (El Harti et al., 2020).

$$V_c = \left(\frac{z}{h}\right)^k = 1 - V_m \quad (4)$$

According to the theory of Timoshenko, the longitudinal and transverse displacements are written, respectively (Friedman & Kosmataka, 1993):

$$u(x, y, z, t) = z\theta(x, t) \qquad w(x, y, z, t) = w(x, t) \tag{5}$$

The nonzero components of strain can be written by (Eisenberger & Abramovich, 1997):

$$\varepsilon_{xx} = z\frac{\partial\theta}{\partial x} \qquad \gamma_{xz} = \frac{\partial w}{\partial x} + \theta \tag{6}$$

The equations of motion are derived via the Hamilton's principle:

$$\delta\prod = \int_{t_1}^{t_2}(\delta U - \delta T - \delta W_e)dt = 0 \tag{7}$$

with δU, δT , δW_e represent the variations of strain energy, kinetic energy and the work of external forces, respectively.

The strain energy U of the element can be written as:

$$U = \frac{1}{2}\int_0^l \begin{bmatrix} \frac{\partial\theta}{\partial x} \\ \frac{\partial w}{\partial x} + \theta \end{bmatrix}^T \begin{bmatrix} EI & 0 \\ 0 & KGA \end{bmatrix} \begin{bmatrix} \frac{\partial\theta}{\partial x} \\ \frac{\partial w}{\partial x} + \theta \end{bmatrix} dx \tag{8}$$

where E, G, I and ρ are respectively, Young modulus, shear modulus, the area moment of inertia of the cross-section and the mass density, and $K = 10(1 + \nu)/(12 + 11\nu)$ being the shear coefficient (Cowper, 1996).

The kinetic energy of the element is given as:

$$T = \frac{1}{2}\int_0^l \begin{bmatrix} \frac{\partial w}{\partial t} \\ \frac{\partial\theta}{\partial t} \end{bmatrix}^T \begin{bmatrix} \rho A & 0 \\ 0 & \rho I \end{bmatrix} \begin{bmatrix} \frac{\partial w}{\partial t} \\ \frac{\partial\theta}{\partial t} \end{bmatrix} dx \tag{9}$$

The total work W_e due to the external forces in the beam is given by:

$$W_e = \int_0^l \begin{bmatrix} w \\ \theta \end{bmatrix}^T \begin{bmatrix} q_d \\ m \end{bmatrix} dx \tag{10}$$

Finite Element Formulation

The equations governing the beam based on the Timoshenko theory could be satisfied if their polynomial order in w is far greater by one order of θ (Friedman & Kosmataka, 1993). Using the boundary conditions for the embedded-free beam, the transversal displacement $w(x, t)$ and its first, second spatial derivatives are given in matrix form as:

$$w(x, t)[N_w][q] \qquad \dot{w}(x, t)[N_\theta][q] \qquad \ddot{w}(x, t)[N_a][q] \tag{11}$$

With

$$q = [w_1 \quad \theta_1 \quad w_2 \quad \theta_2]^T \tag{12}$$

The equation of motion's element is developed by substituting the shape functions in the Hamilton principle, then integrating over the entire length of the element:

$$M\ddot{q} + Kq = \quad (13)$$

where the elementary mass matrices of the piezoelectric and FGM elements are expressed respectively as:

$$[M^P] = \frac{1}{2}\int_0^{l_p} \begin{bmatrix} N_w \\ N_\theta \end{bmatrix}^T \begin{bmatrix} \rho_p A_p & 0 \\ 0 & \rho_p I_p \end{bmatrix} \begin{bmatrix} N_w \\ N_\theta \end{bmatrix} dx \quad (14)$$

$$[M^{FGM}] = \frac{1}{2}\int_0^{l_b} \begin{bmatrix} N_w \\ N_\theta \end{bmatrix}^T \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} \begin{bmatrix} N_w \\ N_\theta \end{bmatrix} dx \quad (15)$$

and the elementary stiffness matrices of the piezoelectric and FGM elements are also expressed respectively as:

$$[K^P] = \frac{1}{2}\int_0^{l_p} \begin{bmatrix} \frac{\partial N_\theta}{\partial x} \\ \frac{\partial N_w}{\partial x} + N_\theta \end{bmatrix}^T \begin{bmatrix} E_p I_p & 0 \\ 0 & KG_p A_p \end{bmatrix} \begin{bmatrix} \frac{\partial N_\theta}{\partial x} \\ \frac{\partial N_w}{\partial x} + N_\theta \end{bmatrix} dx \quad (16)$$

$$[K^{FGM}] = \frac{1}{2}\int_0^{l_b} \begin{bmatrix} \frac{\partial N_\theta}{\partial x} \\ \frac{\partial N_w}{\partial x} + N_\theta \end{bmatrix}^T \begin{bmatrix} C_3 & 0 \\ 0 & C_4 \end{bmatrix} \begin{bmatrix} \frac{\partial N_\theta}{\partial x} \\ \frac{\partial N_w}{\partial x} + N_\theta \end{bmatrix} dx \quad (17)$$

with C_1, C_2, C_3, and C_4 are constants which depend on the characteristics of the FGM material, given by (El Harti et al., 2019).

Piezoelectric Equations

The linear piezoelectric coupling between the elastic field and the electric field can be expressed by the direct and inverse piezoelectric constitutive equation (Aldraihem et al., 1997, Anjanappa & BI, 1994) as:

$$D_z = d_{31}\sigma + e^\sigma E \qquad D_z = d_{31}E_f + S^c c \quad (18)$$

The voltage will be applied as an input to the actuator with an accurate gain depending on the degree of damping desired.

$$V^c(t) = G_c e_{31} z b \int_0^{l_p} n_1^T \dot{q} \, dx \quad (19)$$

where $z = \left(\frac{h}{2} + t_s\right)$, and e_{31} being the piezoelectric stress/load constant.

$$V_1^c(t) = S_c[0 \quad -1 \quad 0 \quad 1] \quad (20)$$

$$V_2^c(t) = -S_c[0 \quad -1 \quad 0 \quad 1 \quad (21)$$

where $S_s = G_C e_{31} z_1 b$ is called the sensor constant.

Dynamic Equation

The dynamic equation of the structure and the control equation are finally given by:

$$M^{*}\ddot{g} + C^{*}\dot{g} + K^{*}g = f^{*}_{ext} + f^{*}_{ctr1} + f^{*}_{ctr2} \qquad (22)$$

The state space model in MIMO mode is given by:

$$\dot{x} = Ax(t) + Bu(t) + Er(t) \qquad (23)$$

$$y(t) = C^{T}x(t) + Du(t) \qquad (24)$$

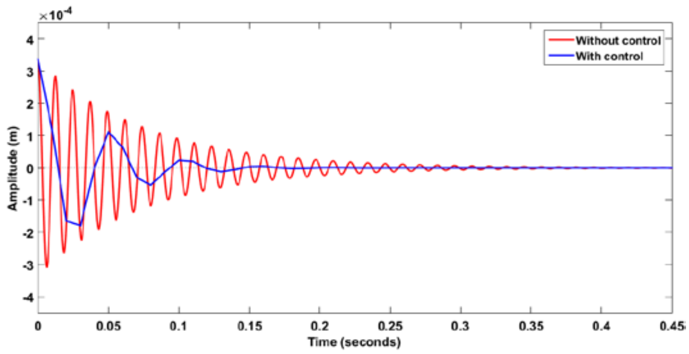
Results and Discussion

In the aim to validate the control's procedure, we consider an embedded-free FGM beam with a non-uniform section, partially covered by four layers of piezoelectric materials. A pulse of amplitude (1N) is applied as external force. The figures below show the response of the beam to the pulse. In all results, we presents the comparison of the responses, with and without control, varying the location of piezoelectric actuators as well as the taper ratio, and the power index. The geometrical and physical characteristics of the materials are shown in (Table 1).

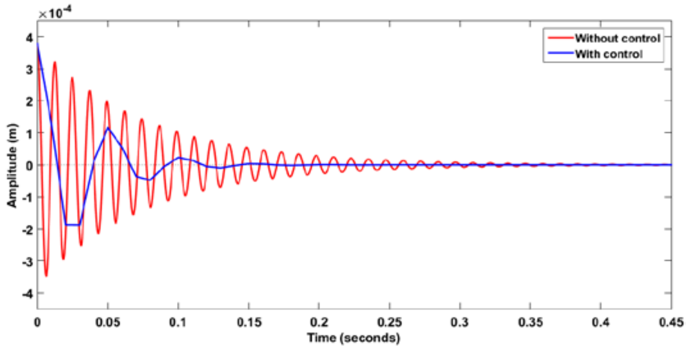
Table 1. Geometric and physical characteristics of the structure

Physical properties	FG material	Material (PZT) Sensor/Actuator
Length (m)	$L = 5 \times l_b = 0.$	$l_p = 0.05$
Width (m)	$b = 0.03$ $b_0 = 0.02$	$b = 0.03$ $b_0 = 0.02$
Thickness (m)	$h = 0.002$	$t_s = t_a = 0.00001$
Density (Kg/m^3)	$\rho_m = 2780$ $\rho_c = 3800$	$\rho_p = 7700$
Young's Modulus (G Pa)	$E_m = 70$ $E_c = 380$	$E_p = 68.1$
Poisson's ratio	$\nu_m = 0.3$ $\nu_c = 0.3$	$\nu_p = 0.3$
PZT Strain constants (m/V)		$d_{31} = 125 \times 10^{-12}$
PZT Stress constant (Vm/N)		$g_{31} = 10.5 \times 10^{-3}$

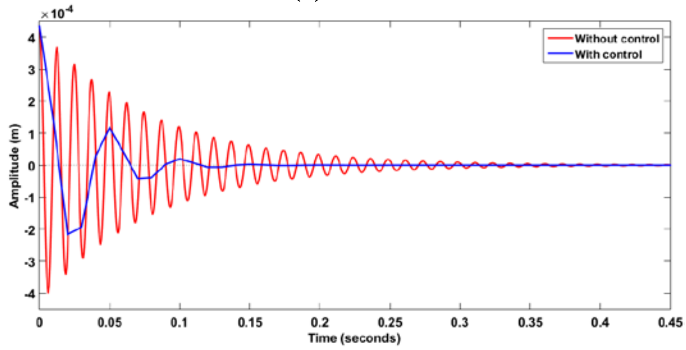
Figure 1 shows the variation of the width from b=3 to b=2, keeping the power index k=1. Figure 2 presents the variation of the power index for k=(0.5; 1; 5) and b=2. in the first two figures, the actuator pair is in the FE2. Figure 3 shows the variation of the actuator pair between FE2 and FE3 for k=1 and b=2.



(a) b=3



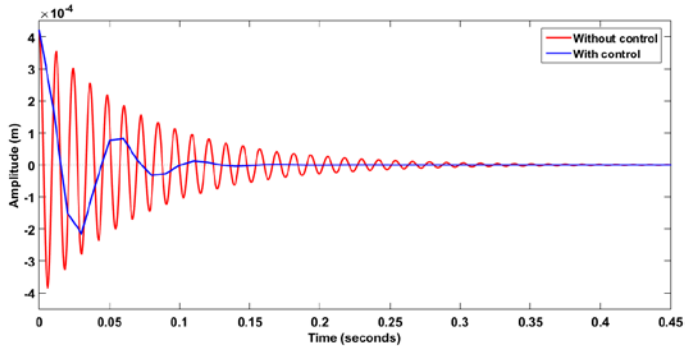
(b) b=2.5



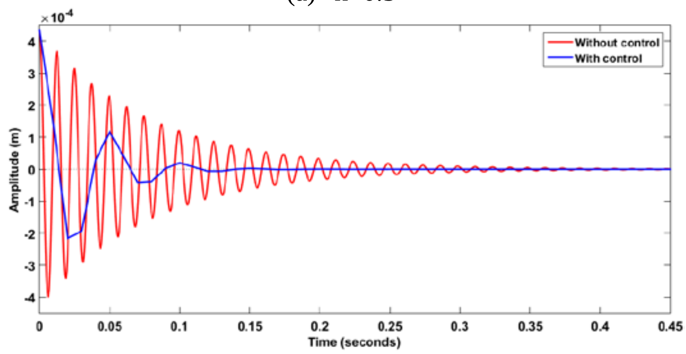
(c) b=2

Figure 1. Impulse response of the smart beam for k=1 (Actuators in FE2)

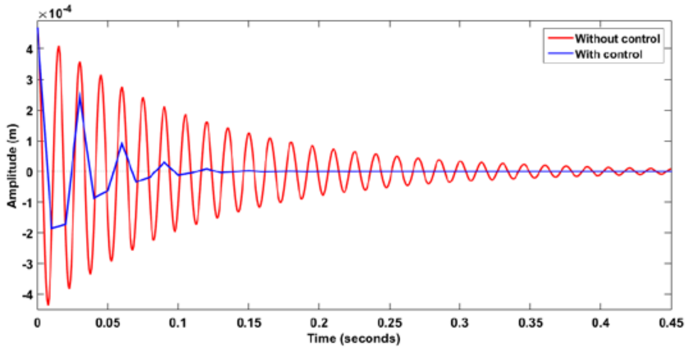
From (figure 1), we find that the increase in the width taper ratio (decrease in the width) between b=3 and b=2, implies increases in the amplitudes of the vibrations. The decrease in width implies a decrease in the mass matrix, it means that the structure becomes less resistive.



(a) k=0.5



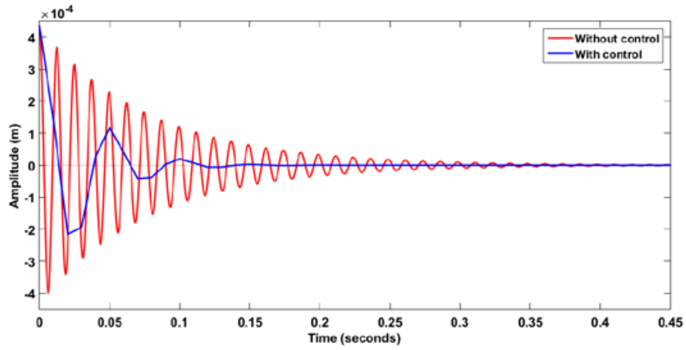
(b) k=1



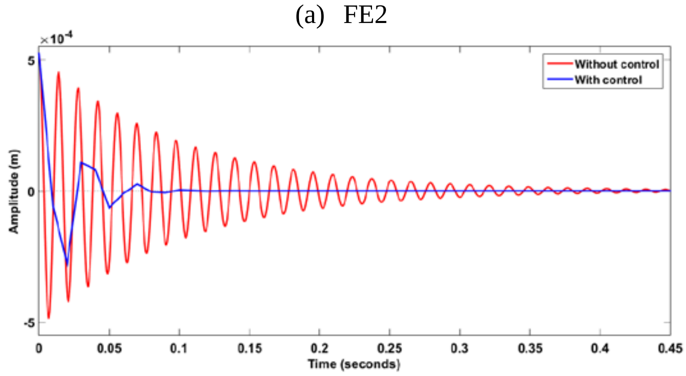
(c) k=5

Figure 2. Impulse response of the smart beam for b=2 (Actuators in FE2)

Figure 2 shows the variation of the power index. From the figure, we find that the increase in the power index between k=0.5 and k=5, implies an increase in the vibration amplitudes. This increase is due to the fact that the increase in the power index, the beam contains more metal than ceramic, the thing which explains the weakness of the matrix of matrix and rigidity.



(a) FE2



(b) FE3

Figure 3. Impulse response of the smart beam for k=1 and b=2

From Figure 3, the displacement of the actuator pair from the embedded end to the free end implies an increase in the vibration amplitudes. This increase is explained, that the displacement of a quantity of material towards the free end is equivalent to an increase in the excitation force.

Conclusion

In order to study the dynamics and active control of a non-uniform section FGM beam, based on Timoshenko theory and FEM, we consider a free-embedded FGM beam containing four layers of piezoelectric materials. The analysis of results shows that the increase in the power index, the increase in the width taper ratio as well as the displacement of the pair of actuators towards the free end, imply increases in the amplitudes of the vibrations. also the comparison of the responses in open and closed loop shows the success of the LQG-Kalman active control procedure.

Scientific Ethics Declaration

The authors declare that the scientific ethical and legal responsibility of this article published in EPSTEM journal belongs to the authors.

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