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Comparing Performance of OFDM based V2V System in Rayleigh and Weibull Fading Channels

Didem Kivanc TURELI
Istanbul Okan University

Zainab QASSAB
Istanbul Okan University

Abstract: The performance of communication systems across wireless channels is limited by fading. This is particularly true for vehicular networks, which require highly reliable links. Orthogonal Frequency Division Multiplexing (OFDM) is used today in a wide range of communication systems. It may also be part of the standard for vehicular networks. This paper compares the performance of a vehicular communication system using Quadrature Phase Shift Keying (QPSK) communication over Orthogonal Frequency Division Multiplexing (OFDM) in two channel models found to fit the vehicular channel: Rayleigh and Weibull fading models by comparing the bit error rate under constant signal to noise ratio (SNR). The exact BER has been derived based on the derived of the effective noise sample distribution at the OFDM demodulator output over the Weibull Fading and the log-normal fading channel in AWGN. The Monte Carlo simulation results have verified the validity of the derived analytical BER expressions with an exact match to the simulation results.

Keywords: Fading, Multipath channels, Raleigh fading model, Weibull fading model

Introduction

In multipath channels copies of the signal arrive from different directions due to reflection and refraction off different surfaces. Frequency selective fading, the amplification of certain frequencies and the attenuation of others occurs due to the phase delay induced by the reflections. In OFDM the cyclic prefix allows the gain to remain relatively constant over the bandwidth of a given carrier, reducing the effect of frequency selective fading to multiplication by a scalar for each individual subcarrier. However, the gain of each subcarrier is not identical. This means that certain carriers are more suitable for communication than others. The effect of fading can be split into two: fast fading which changes on the time scale of a single symbol and slow fading which refers to signal changes on the order of several symbols (Bingham, 1990).

While systems can usually adapt to large scale fading through power control or adaptive modulation and coding (AMC), small scale fading changes quickly so that the process of channel estimation, feedback and adaptation of system parameters may take too long to have the desired effect. For this reason, it is important to study the effect of small scale fading on communication. In this paper we study the effects of Rayleigh and Weibull fading models, which model small scale fading. These models have been found to accurately model a vehicular communication channel scenario (Sen & Matolak, 2008).

With the advent of intelligent transportation systems, it becomes necessary to characterize the channel used for communication between vehicles. The wireless channel environment has a significant impact on the performance of wireless communication systems in these cases. Some studies have found that Rayleigh and Weibull distributions accurately model the small scale fading on vehicular links (Sen & Matolak, 2008). These links are time varying and need to be modeled accurately (Matolak & Frolik, 2011). There has been prior work

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on modeling these links at the packet scale (Wang *et al.*, 2019; Jameel *et al.*, 2017). In this paper we calculate analytically and by simulation the probability of bit error in these scenarios and compare the results.

Orthogonal Frequency Division Multiplexing (OFDM) was first introduced in the late 1960s, for military applications but the technique required may RF chains and was not practical for commercial use (Bingham, 1990). To eliminate the need for analog subcarrier oscillators, Weinstein proposed employing a Discrete Fourier Transform (DFT) to generate and receive OFDM signals. Using the Fast Fourier Transform (FFT), an efficient implementation of DFT, allowed for an easy implementation of OFDM. 4G and 5G cellular systems and WiFi also use OFDM (Prasad, 2004). This paper compares theoretically and through simulations the average bit error rate (BER) experienced under Rayleigh and Weibull fading channels.

OFDM System Model

Orthogonal Frequency Division Multiplexing (OFDM) divides the available spectrum into many narrow subcarriers (Goldsmith, 2005). The subcarrier frequencies are selected so that they are orthogonal, packing the signal into a tight spectrum. OFDM systems have high spectral efficiency and are resistant to multipath. In terrestrial environments, OFDM is better for transmitting a lot of data than single-carrier transmissions, because OFDM can handle a lot of different types of interference. This is due to the frequency diversity introduced by multiple channels. OFDM does have some drawbacks, for instance frequency and timing estimation are critical for accurate symbol estimation and the high peak to average power ratio (PAPR) of the OFDM waveform needs to be compensated for at the receiver (Ladaycia *et al.*, 2017). Figure 1 shows the architecture of the OFDM system.

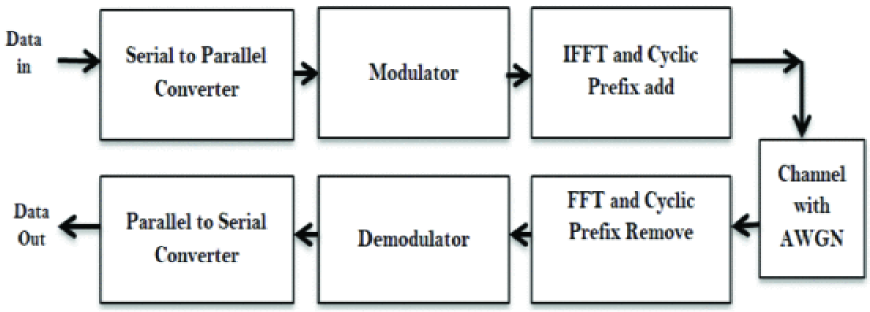

Figure 1. OFDM architecture (Ladaycia *et al.*, 2017).

Consider a system with N information symbols $x_0, x_2, \ldots, x_{N-1}$ to be transmitted. The *N*-symbol OFDM block

$X = (X_k)$ where $k = 0,1,\ldots,N-1$ is calculated as:

$$X_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x_n \, e^{j2\pi kn/N} \quad k = 0,1,\ldots,N-1 \tag{1}$$

To make this system easier to analyze, now assume that the message symbols which are the input of the OFDM system are statistically independent and identically distributed. Considering the central limit theorem, where *N* is large, the real and imaginary parts of the OFDM signal are the linear combination of Gaussian random variables, which also makes these symbols independent and identically distributed Gaussian random variables with mean zero and variance (Mishra and Sood, 2011).

$$\sigma^2 = E\left\{ \left(\mathrm{Re}\{X_k\}\right)^2 + \left(\mathrm{Im}\{X_k\}\right)^2 \right\} \tag{2}$$

where $E\{\cdot\}$ denotes the expected value of the expression.

Then the probability density function (pdf) of the OFDM symbol is

$$f\left(X_k\right)=\frac{1}{\sqrt{2\pi\sigma}}e^{-X_k{}^2/2\sigma^2} \quad k=0,1,...,N-1 \tag{3}$$

Assuming the pulse shape is purely rectangular and each symbol duration is exactly T_s, the baseband representation of the transmitted analog OFDM waveform is

$$s\left(t\right)=\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}x_n\, e^{jn\Delta f t} \qquad 0\le t\le T_s,\, \Delta f=\frac{1}{T_s} \tag{4}$$

The output of the OFDM channel is:

$$r\left(t\right)=s\left(t\right)*h\left(t\right)+n\left(t\right) \tag{5}$$

where $h\left(t\right)$ is the impulse response of the channel and $n\left(t\right)$ is white Gaussian noise with $N_0/2$ power spectral density, and * is the convolution operator.

The impulse response of the multipath channel is:

$$h\left(t\right)=\sum_{i=1}^{L}h_i\delta\left(t-\tau_i\right) \tag{6}$$

In a multipath environment with time-invariant channel, we assume that the transmitted subcarrier bandwidth $\Delta f=1/T_s$ is narrow relative to the channel coherence bandwidth. In this case where inter-symbol interference (ISI) is neglected, further neglecting the pulse shape the effect of the channel is to multiply the message on each subcarrier by a constant channel gain and phase offset. In the baseband representation:

$$\begin{aligned} r\left(t\right)&=\frac{1}{\sqrt{N}}\sum_{i=1}^{L}h_i\sum_{n=0}^{N-1}x_n\, e^{jn\Delta f\left(t-\tau_i\right)}+n\left(t\right)=\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}\sum_{i=1}^{L}h_i x_n\, e^{jn\Delta f\left(t-\tau_i\right)}+n\left(t\right) \\ &=\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}\sum_{i=1}^{L}h_i e^{-jk\Delta f n\tau_i}x_n\, e^{j\Delta f nt}+n\left(t\right)=\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}\underbrace{\left(\sum_{i=1}^{L}h_i e^{-jk\Delta f n\tau_i}\right)}_{H_n}x_n\, e^{j\Delta f nt}+n\left(t\right)\end{aligned} \tag{7}$$

where H_n is the complex valued baseband gain for the nth subcarrier channel. The output of the multipath channel is then:

$$r\left(t\right)=\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}H_n x_n\, e^{jn\Delta f t}+n\left(t\right) \qquad 0\le t\le T_s \tag{8}$$

The receiver will get N samples from this, making the OFDM symbol

$$r_k=r\left(kT_s\right)=\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}H_n x_n\, e^{jn\Delta f\left(kT_s\right)}+n\left(kT_s\right)=\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}H_n x_n\, e^{j\frac{nk}{N}}+n_k \qquad 0\le k\le N-1 \tag{9}$$

At the output of the OFDM receiver (FFT) we will see

$$\begin{aligned} y_m&=\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}r_k\, e^{-j\frac{km}{N}}=\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}\left(\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}H_n x_n\, e^{j\frac{nk}{N}}+n_k\right)e^{-j\frac{km}{N}} \qquad 0\le k\le N-1 \\ &=\frac{1}{N}\sum_{k=0}^{N-1}\sum_{n=0}^{N-1}H_n x_n\, e^{j\frac{(n-m)k}{N}}+\frac{1}{N}\sum_{k=0}^{N-1}n_k\, e^{-j\frac{km}{N}}=\frac{1}{N}\sum_{n=0}^{N-1}H_n x_n\sum_{k=0}^{N-1}e^{j\frac{(n-m)k}{N}}+\tilde{n}_k=\frac{1}{N}\sum_{n=0}^{N-1}H_n x_n\, N\delta\left(n-m\right)+\tilde{n}_k \\ y_m&=H_m x_m+\tilde{n}_k\end{aligned} \tag{10}$$

where $\tilde{n}_k$ has the same probability distribution as n_k since it is the linear combination of independent Gaussian random variables, and the coefficients add to 1.

Multipath Channel

Communication in multipath channels is impaired due to their frequency selectivity. This section derives the probability of error when data is transmitted across a frequency selective channel using OFDM and QPSK modulation. The effect of phase is not considered since phase variation is considered to be slow enough for the system to adapt to. Only the effect of magnitude on the SNR is considered. An average BER is calculated for transmission across the channel. Standard Rayleigh fading is considered as well as Weibull fading in the channel. Weibull channels are interesting because they have been used to model vehicular channels.

Rayleigh Fading

It can be seen from Equation (8) that the subcarrier gain is the sum of many smaller components. When the number of multipath components L is large, the sum approaches a Gaussian random variable. In this assumption, the real and imaginary parts of the baseband channel gain H_n are both Gaussian distributed. Since the transmitted symbols x_n are independent and identically distributed, the sum of these terms makes two identically distributed Gaussian random variables.

In Equation (10) we see that the symbol is multiplied by the subcarrier gain H_n which is the Discrete Fourier Transform (DFT) of the channel impulse function:

$$H_n = \sum_{i=1}^{L} h_i e^{-jk\Delta f n \tau_i} \tag{11}$$

For the Rayleigh fading channel since the channel coefficients h_i are circular complex Gaussian random variables with mean 0 and standard deviation 1, the linear combination of those random variables H_n will also be a Gaussian random variable, thus the received OFDM signal power will be Rayleigh distributed with variance L.

The magnitude $r = |H_n|$ is a Rayleigh random variable with probability density function

$$f(r) = 2re^{-r^2} \tag{12}$$

For these types of channel, the probability of error for Quadrature Phase Shift Keying (QPSK) modulated signals is (Goldsmith, 2005).

$$P_b = \frac{1}{2}\left[1 - \sqrt{\frac{\gamma_0/2}{1+\gamma_0/2}}\right] \quad \text{where} \quad \gamma_0 = \frac{E_b}{\sigma_n^{\,2}} \tag{13}$$

Weibull Fading

Weibull fading is generally considered to be a worse type of fading channel than Rayleigh fading, in that the channel gains change more and cause a higher probability of error. However, similarly to Rayleigh fading, the received signal is given by Equation 10 with channel gain again given by Equation 11. The difference in this case is that the multipath coefficients h_i are Weibull distributed. Even when this is true, as long as the number of multipath components L is large, on the order of 10 or more, the real and imaginary parts of the subcarrier gain H_n will be well approximated with the Gaussian distribution. Then the channel experienced will also be a Rayleigh fading channel, but with a higher variance equal to:

$$\sigma^2 = L\beta^2\left[\Gamma\left(1+\frac{2}{k}\right)-\left(\Gamma\left(1+\frac{1}{k}\right)\right)^2\right] \tag{14}$$

The probability of error is then given by:

$$P_b = \frac{1}{2}\left[-\sqrt{\frac{\beta^2\left[\Gamma\left(1+\frac{2}{k}\right)-\left(r\left(1+\frac{1}{k}\right)\right)^2\right]\frac{E_b}{2\sigma_n^2}}{1+\beta^2\left[\Gamma\left(1+\frac{2}{k}\right)-\left(\Gamma\left(1+\frac{1}{k}\right)\right)^2\right]\frac{E_b}{2{\sigma_n}^2}}}\right] \tag{15}$$

Log-normal

The log-normal distribution is recognized to statistically define fading in various propagation contexts, including ultra wide-band indoor channels, radio channels influenced by body worn devices , and the shadowing phenomenon in outdoor scenarios . The calculation of the average bit error rate or average symbol error rate is thought to be a crucial factor in determining how well digital modulation schemes perform in fading channels. The statistical expectation of error rate in an additive white gaussian noise (AWGN) over a probability density function describing a fading environment determines the average error rate for coherent detection of various digitally modulated signals (Khandelwal, 2013).

A continuous distribution in which the logarithm of a variable has a normal distribution. A log normal distribution results if the variable is the product of a large number of independent, identically-distributed variables in the same way that a normal distribution results if the variable is the sum of a large number of independent, identically distributed variables (Singh and Singh, 2013).
log-normal distribution can be expressed as:

$$p_h(h_k^r) = \frac{1}{\sqrt{2\pi\sigma^2}h_k^r}e^{\left(-(\ln h_k^r-\mu)^2/(2\sigma^2)\right)} \tag{16}$$

where $h_k^r > 0$, Real and imaginary parts of the Gaussian random variable are r = {ℜ, ℑ}, σ 2 and μ the variance and the mean of the random variable (Al-Rubaye *et al.*, 2019).

Simulation Results

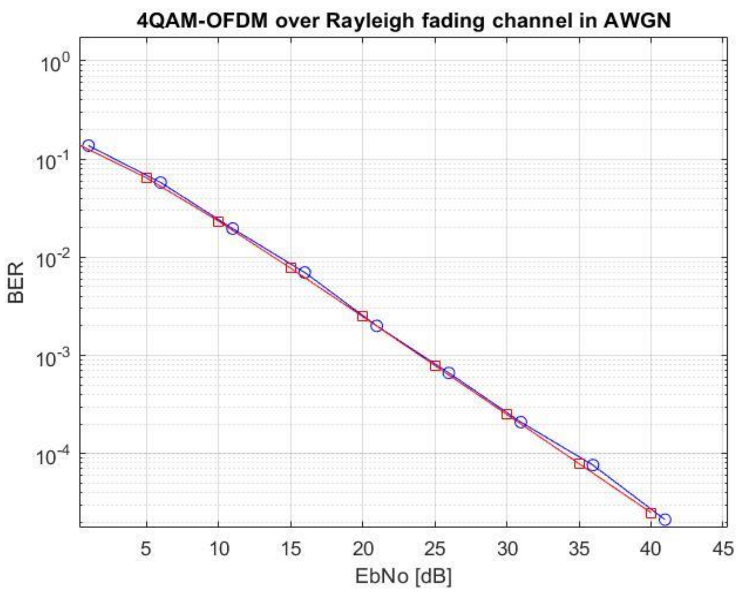

Figure 2. SNR vs BER for OFDM-QPSK over Rayleigh fading channel in AWGN.

Simulations using Matlab were performed to verify the probability of error derived in the previous section. In this work, we consider an OFDM system where the data symbols x_k are drawn from QPSK symbols $\{\pm 1 \pm 1j\}$. The performance of OFDM is presented under the Rayleigh fading channel considering Bit Error Rate and Signal to Noise Ratio. The simulation results for QPSK-OFDM in AWGN channel are shown Rayleigh fading channel in Figure 2.

As can be seen the BER of Rayleigh and the difference between Weibull. Thus, it can be concluded that using QPSK modulation in AWGN channel to see difference when we chose three values equal to Rayleigh and over, less than Rayleigh as shown in Figure 3. And for log normal channel, the BER of Rayleigh and the difference between Log-Normal. Thus, it can be concluded that using QPSK modulation in AWGN channel to see difference when we chose three values equal to Rayleigh and over, less than Rayleigh as shown in Figure 4.

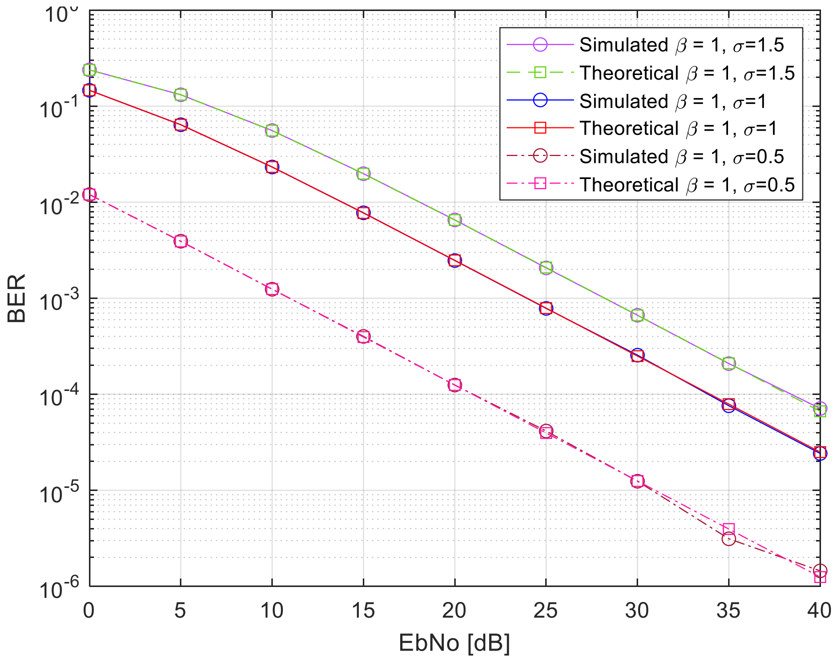

Figure 3. SNR vs BER for OFDM-QPSK over Weibull fading channel in AWGN

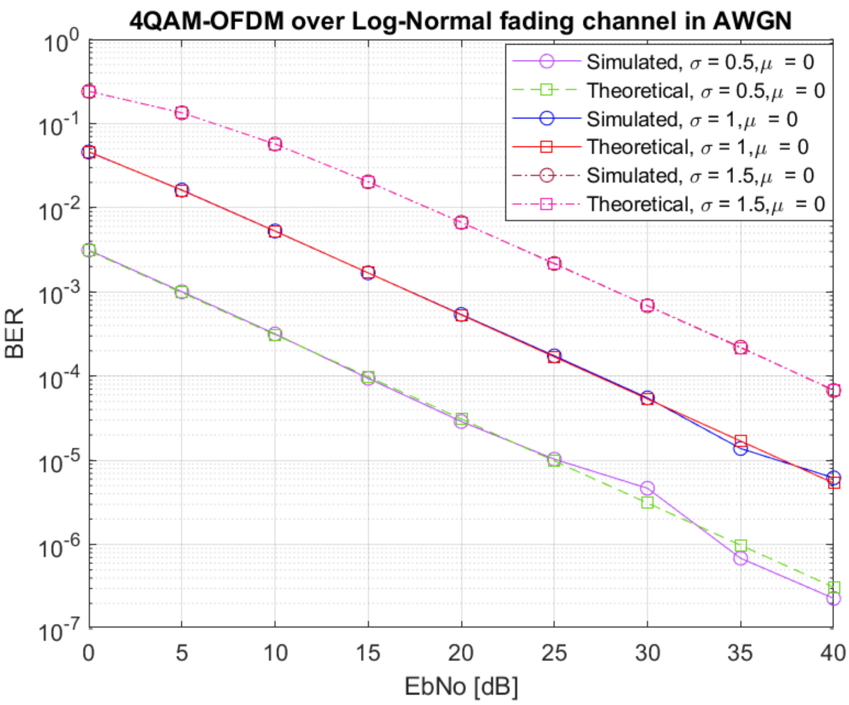

Figure 4. SNR vs BER for OFDM-QPSK over log-normal fading channel in AWGN

Conclusion

In this paper we have compared the performance of QPSK modulated OFDM systems in Rayleigh, Log-Normal and Weibull fading channels. The bit error rate results are compared using both simulations and analytical results. We find that Weibull and Log Normal fading is more challenging than Rayleigh fading.

Appendix

Derivation of BER of QPSK in Rayleigh fading in Equation 13

This section outlines the calculation of BER for QPSK-OFDM in Rayleigh fading and derives Equation 13 following Alouini and Goldsmith (1999). First it is observed that the magnitude of the subcarrier gain in Equation 11 is Rayleigh distributed. Since H_n is the sum of complex Gaussian random variables, it is also complex Gaussian random variables and its magnitude is Rayleigh distributed. In calculating the probability of error for QPSK we consider the in phase and quadrature components separately, since phase and frequency offsets are set to zero. In this case for the Rayleigh fading channel the gain of the in phase component is a Gaussian distributed random variable with mean 0 and variance $0.5\sigma_h^2$. Define $\gamma_0 = E_s/N_0$. The SNR $\gamma_s = |h|^2 \gamma_0$ can be shown to be an exponential distributed random variable (Goldsmith, 2005):

$$f(\gamma) = \frac{1}{\gamma_0} e^{-\gamma/\gamma_0} u(\gamma) \tag{17}$$

The instantaneous BER for QPSK in a channel with instantaneous SNR $\gamma_s = |h|^2 \gamma_0$ is

$$P_b(h) = Q\left(\sqrt{\gamma_s}\right) = Q\left(\sqrt{|h|^2 \gamma_0}\right) \tag{18}$$

Averaging the BER over the Rayleigh distribution of the random channel gain h gives

$$P_b(\gamma_0) = E\left\{P_b(h\gamma_0)\right\} = \int_0^\infty Q\left(\sqrt{\gamma_s}\right) \frac{1}{\gamma_0} e^{-\gamma_s/\gamma_0} d\gamma \tag{19}$$

By definition the above function is the moment generating function (MGF) for the Gaussian distribution with some scalar adjustments. From this observation and the definition of the MGF leads to Equation 13.

Derivation of BER of QPSK in Weibull fading in Equation 15

The probability of error for OFDM-QPSK in Weibull fading is derived similarly to that in Rayleigh fading. The most important observation is that for larger numbers of multipath components, that is, larger values of *L*, the sum of independent identically distributed Weibull random variables with uniformly distributed phase will be a complex Gaussian random variable. Thus, the magnitude of H_n will be Rayleigh distributed. Define $\gamma_0 = E_s/N_0$. The SNR $\gamma_s = |h|^2 \gamma_0$ can be shown to be an exponential distributed random variable (Goldsmith, 2005), with probability density function (pdf) as given by Equation 16. The derivation then proceeds the same as the derivation of Equation 13 but with the variance given by Equation 14.

Scientific Ethics Declaration

The authors declare that the scientific ethical and legal responsibility of this article published in EPSTEM journal belongs to the authors.

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Author Information

Zainab Qassab	**Didem Kivanc Tureli**
Istanbul Okan University	Istanbul Okan
Istanbul, Turkey	University Istanbul,
Contact e-mail: *zainabusagi143@gmail.com*	Turkey

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