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Determination of the Shear Force in RC Interior Beam-Column Connections

Albena Doicheva
University of Architecture, Civil Engineering and Geodesy (UACEG)

Abstract: The calculation of frame structures requires special attention when modeling the beam-column connection. Often the joint is assumed to be rigid, but this does not correspond to the real behavior of the beam-column connection, as well as the real response of frame structures. The leading countries in seismic research (USA, New Zealand, Japan) have uniform procedures introduced in their seismic codes for studying the moment resisting frame. However, regarding the determination of the shear force in the beam-column connection, there is still a discrepancy in how it is determined. In the present work, a mathematical model is proposed for the analytical determination of the shear force. The emerging large deformations in the beam, which could be realized during earthquake, have been taken into account. The material is elastic. The obtained values are compared with results determined by mathematical procedures proposed in other literature sources.

Keywords: Beam-column connection, Shear force, Reinforced concrete, Elastic material, Large deformations

Introduction

The beam-column connection is a basic element of frame structures. Its task is to transfer the loads between the connected elements. The preservation of the integrity of the joint in moment resisting reinforced concrete frames is guaranteed in static load calculations by the recommendations in the current codes. However, failures in many frame structures during cyclic loading (such as earthquakes) indicate abrupt destruction due to joint shear. Detailed studies of the beam-column connection date back to the last 5 to 6 decades.

The first quantitative definition of the shear force was given by Hanson and Connor (1967). In their report of the test results of RC interior beam-column connections they define joint shear as a horizontal force transferred at the midheight of a horizontal section of a beam-column connection. They suggested that joint shear failure may be precluded by limiting the joint shear stress to the level at which joint shear failure occurs. This definition has been adopted worldwide and subsequent studies lead to the adoption of design provisions providing a limiting value of joint shear stress. The distribution of forces and the response of beam-column connection has occupied scientists for the past few decades (Park & Paulay, 1975; Park & Keong, 1979; Paulay, 1989; Paulay & Priestley, 1992; Park, 2002; Lowes & Altoontash, 2003; Altoontash, 2004; Celik & Ellingwood, 2008; Sharma et al., 2009; Shafaei et al., 2014). However, research in different countries has led to different proposals for the modeling and detailing of frame joints in terms of shear force. Detailed review of interior and exterior joints of special moment resisting reinforced concrete frames, with reference to three codes of practices: American Concrete Institute (ACI 318M-02), New Zealand Standards (NZS 3101, 1995) and Eurocode 8 (EN 1998-1, 2003) was performed by Uma & Jain (2006). Tran et al. (2014) propose a new empirical model to estimate the joint shear strength of both exterior and interior beam-column. A parametric study was carried out to evaluate the dependence of the predicted to tested joint shear strength ratio on the four influence parameters.

Contrary to the general acceptance, Shiohara (2001) proposed a new model for the calculation and detailing of the beam-column connection. The study shows an irrationality in the joint shear failure model, which is adopted in the most current design codes of reinforced concrete beam-to-column joint. It is based on the data of tests of twenty reinforced concrete interior beam-to-column-joint failed in joint shear. The analysis indicated that joint shear stress had increased in the most specimens, even after apparent joint shear failure starts.

Problem

Hanson and Connor (1967) defined the joint shear V_j in an interior beam-column connection from Figure 1 as given in (1). The joint shear V_j in (1) is an internal force acting on the free body along the horizontal plane at the midheight of the beam-column connection.

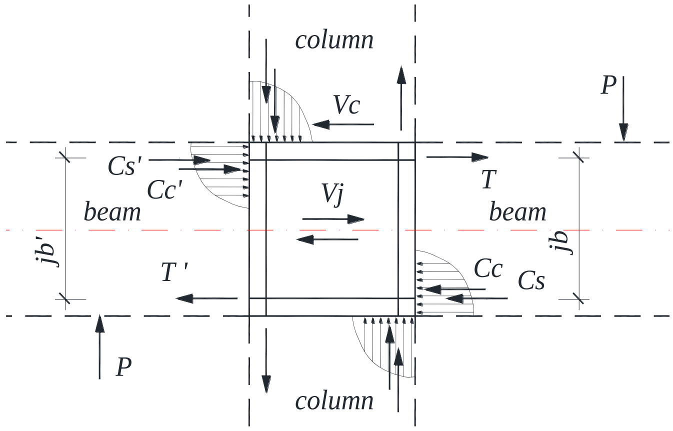


Figure 1. Definition of joint shear in interior RC beam-column connection

$$V_j = T + C_S' + C_C' - V_C = T + T' - V_C \quad (1)$$

where: C_S and C_S' - compressive force in bottom and top longitudinal reinforcing bars in beam passing through the connection;

C_C and C_C' - compressive force in concrete on the bottom and top edge of beam;

T and T' - tensile forces in top and bottom reinforcing bars in beam passing through the connection;

V_C - column shear force

This definition is clear and has been used in the design of beam-column connections. The contribution of steel and concrete is taken into account separately. The difficulty encountered in determining the forces from (1) leads to the adoption of another way of writing the expression for the shear force in the literature. Usually T and T' are defined by (2).

$$T = \frac{M_b}{j_b} \text{ and } T' = \frac{M_b'}{j_b} \quad (2)$$

where: M_b and M_b' - moment at column face;

j_b and j_b' - the length of bending moment arm at the column face. It is assumed to be constant and unchanging in the process of deformation.

Then (1) is rewritten from moment in the beam section at column faces into (3).

$$V_j = \frac{M_b}{j_b} + \frac{M_b'}{j_b'} - V_C \quad (3)$$

The assumption (2) obliges us to assume equal forces in the bottom and top reinforcement of the beam at the face of the column. In the author's previous publications, these values were shown to differ substantially. In this article, the following tasks are set: 1. to determine expressions for the forces from Figure 1, at the column face, 2. to perform comparisons of the obtained results with the results of (2) and (3).

Method

Mathematical Model of Beams

Case I - Cantilever Beam

A cantilever beam is considered. It is supported on one side by column as is the case in the specimens in beam-to-column joint subassemblages for tests. The beam is statically indeterminate, prismatic and symmetric. The beam is under the conditions of special bending with tension/compression and Bernoulli-Euler hypothesis is considered.

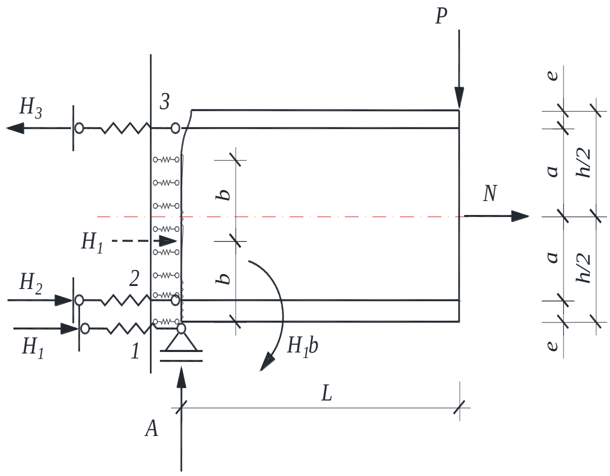


Figure 2. Supports of cantilever beam to column

The beam is loaded with a vertical force $P\left[kN\right]$. The support takes place in vertical support 1, where a vertical support reaction $A\left[kN\right]$ occurs. At the level of the reinforcing bars, elastic supports 2 and 3, with linear spring coefficients k_2 and k_3, are introduced. They are set as the reduced tension/compression stiffness of the reinforcing bar.

$$k_2 = \frac{E_2 A_2}{L} \quad \text{and} \quad k_3 = \frac{E_3 A_3}{L} \tag{4}$$

where: $L\left[cm\right]$ - the length of the beam;

$A_2\left[cm^2\right]$ and $A_3\left[cm^2\right]$ - the area of the cross-section of bottom and top longitudinal reinforcing bars in beam passing through the connection;

$E_2\left[kN/cm^2\right]$ and $E_3\left[kN/cm^2\right]$ - the modulus of elasticity of the bottom and top longitudinal reinforcing bars in beam passing through the connection

The supporting reactions that occur here are $H_2\left[kN\right]$ and $H_3\left[kN\right]$.

Linear spring supports act along the vertical wall of the beam. They account for the connection of the concrete of the beam to that of the column. The forces in all the springs are reduced to one force $H_1\left[kN\right]$. In case of large deformations, part of the vertical edge is destroyed. The unbroken edge has length $2b\left[cm\right]$. The reaction $H_1\left[kN\right]$, which is symmetrically located with respect to the intact lateral edge, moves along the height of the beam as the crack length increases. For convenience, it has been transferred $H_1\left[kN\right]$ to the support along the lower edge (support one), after applying Poinsot's theorem concerning the transfer of forces in parallel to their directrix. This necessitated the introduction of compensating moments $H_1 b\left[kN.cm\right]$. The coefficient of the linear spring is k_1. It is set as the reduced tensile/compressive stiffness of the concrete section.

$$k_1 = \frac{E_1 A_1}{L} \tag{5}$$

where: $L\left[cm\right]$ - the length of the beam;

$A_1\left[cm^2\right]$ - the area of the cross-section of the concrete

$E_1\left[kN/cm^2\right]$ - the modulus of elasticity of the concrete

As a consequence of the linear deformations in the cantilever beam, a normal axial force occurs $N\left[kN\right]$, which is introduced at the free end of the beam.

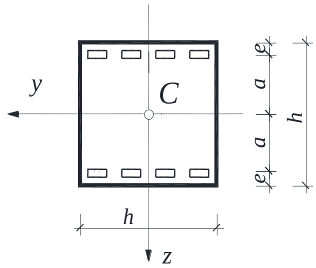


Figure 3. Cross-section of the beam

The following notations have also been introduced:

$h\left[cm\right]$ - the length of the beam;

$e\left[cm\right]$ and $a\left[cm\right]$ - offset of the reinforcing bars from the bottom and top edges of the beam and from the axis of the beam, respectively;

E_2A_2 and E_3A_3 - tensile (compressive) stiffness of the reinforcing bars;

E_1A_1 - tensile (compressive) stiffness of the concrete cross-section;

E_2I_2 and E_3I_3 - bending stiffness of the reinforcing bars;

E_1I_1 - bending stiffness of the concrete cross-section;

$I_1\left(I_{y1}\right)$, $I_2\left(I_{y2}\right)$ and $I_3\left(I_{y3}\right)$ are the moment of inertia of the concrete cross section and the moment of inertia of the bottom and top reinforcing bars relative to the principal axis of inertia *y*, respectively;

$EA = E_1A_1 + E_2A_2 + E_3A_3$ - tensile (compressive) stiffness of the composite section;

$EI = E_1I_1 + E_2I_2 + E_3I_3$ - bending stiffness of the composite section.

Support Reactions

The solution is based on Menabria's theorem about statically indeterminate systems in first-order theory. The potential energy of deformation in special bending, combined with tension (compression) and with the effect of linear springs taken into account, will be:

$$\Pi = \frac{1}{2}\int_0^L \frac{M^2\left(x\right)}{EI}dx + \frac{1}{2}\int_0^L \frac{N^2\left(x\right)}{EA}dx + \frac{H_1^2}{2k_1} + \frac{H_2^2}{2k_2} + \frac{H_3^2}{2k_3}\,. \tag{6}$$

It is a well-known fact that, according to Menabria's theorem, the desired hyperstatic unknown is determined by the minimum potential energy condition with respect to it or will be:

$$\frac{\partial \Pi}{\partial H_1} = 0; \quad \frac{\partial \Pi}{\partial H_2} = 0; \quad \frac{\partial \Pi}{\partial H_3} = 0 \; . \tag{7}$$

The three equilibrium conditions of statics give us respectively:

$$1. \sum V = 0 \quad \rightarrow \quad A_v = P \tag{8}$$

$$2. \sum H = 0 \quad \rightarrow \quad N = H_3 - H_1 - H_2 \tag{9}$$

$$3. \sum M_1 = 0 \quad \rightarrow \quad -PL - H_1 b + H_3 \left(h - e\right) - H_2 e - N\frac{h}{2} = 0 \tag{10}$$

Substitute (9) in (10) and after simplifying for H_2 we get:

$$H_2 = \frac{PL - H_1\left(\dfrac{h}{2} - b\right)}{a} - H_3 \tag{11}$$

The bending moment for the beam is:

$$M\left(x\right) = Ax + H_1 b - H_1\frac{h}{2} - H_2 a - H_3 a \; , \tag{12}$$

Substitute (8) in (12) and substitute the resulting expression in (6) together with (9). Expressions (7) apply. A system of three linear equations with respect to the three unknowns is obtained. The solutions give the formulas for the horizontal support reactions shown below:

$$H_1 = \frac{-PLk_1\left\{EAh_1N_1 + 2EIL\left[k_2n_1 - k_3n_2\right] + 2L^2a^2K_{23}n_2\right\}}{EI\left\{EAD_1 + D_2\right\}} \tag{13}$$

$$H_2 = \frac{PLk_2\left\{EAaN_2 + 4EIL\left[k_1n_1 + k_3 4a\right] + L^2aK_{13}n_2^2\right\}}{2EI\left\{EAD_1 + D_2\right\}} \tag{14}$$

$$H_3 = \frac{PLk_3\left\{4EAaN_1 + 4EIL\left[k_1n_2 + k_2 4a\right] - L^2aK_{12}n_1n_2\right\}}{2EI\left\{EAD_1 + D_2\right\}} \tag{15}$$

where $h_1 = 2b + h$; $\quad n_1 = 2a + h_1$; $\quad n_2 = 2a - h_1$

$K = k_1 + k_2 + k_3$; $\quad K_{12} = k_1k_2$; $\quad K_{13} = k_1k_3$; $\quad K_{23} = k_2k_3$.

$N_1 = 2EI - k_2La^2$; $\quad N_2 = 8EI + L\left(k_1h_1^2 + k_3 4a^2\right)$

$D_1 = \left(k_2 + k_3\right)4a^2 + k_1h_1^2$

$$D_2 = L\left[k_1k_2\left(2a + h_1\right)^2 + k_1k_3\left(2a - h_1\right)^2 + k_2k_3 16a^2\right] \tag{16}$$

The solution was performed in the symbolic environment of the MATLAB R2017b program.

Case II - Simple Beam

A beam from a frame structure is considered. The supporting is analogous to that of a cantilever beam.

Due to the symmetry of the beam, with respect to the mid-section, the horizontal forces on the left side are equal to those on the right side. The beam is three times statically indeterminate. All geometric and material characteristics introduced up to this point are preserved.

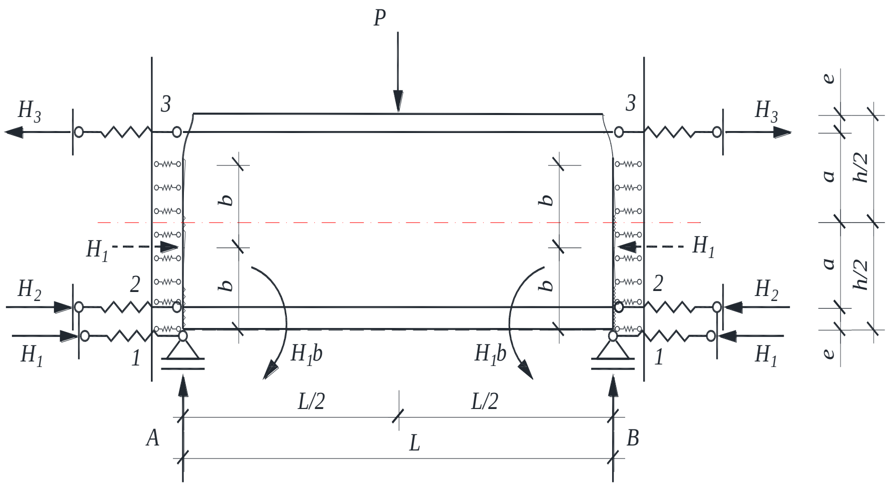


Figure 4. Supports of simple beam to columns

The vertical support reactions are:

$$A = \frac{P}{2} \quad \text{and} \quad B = \frac{P}{2}\,. \tag{17}$$

The bending moments for both part of the beam will be:

$$M_1 = \frac{P}{2}x - H_3 a - H_2 a - H_1\left(\frac{h}{2} - b\right); \tag{18}$$

$$M_2 = \frac{P}{2}\left(\frac{L}{2} - x\right) - H_3 a - H_2 a - H_1\left(\frac{h}{2} - b\right); \tag{19}$$

and the normal force respectively:

$$N = H_3 - H_1 - H_2\,. \tag{20}$$

Substitute (18), (19) and (20) in (21)

$$\Pi = \frac{1}{2}\int_0^{L/2}\frac{M_1^2(x)}{EI}dx + \frac{1}{2}\int_0^{L/2}\frac{M_2^2(x)}{EI}dx + \frac{1}{2}\int_0^{L}\frac{N^2(x)}{EA}dx + \frac{H_1^2}{k_1} + \frac{H_2^2}{k_2} + \frac{H_3^2}{k_3} \tag{21}$$

and it is made a solution proceeds in a similar way like a Case I. Derived the formulas of the horizontal support reactions are:

$$H_1 = \frac{-PL^2 k_1\left\{2EAh_1 + Lk_2 n_1 - Lk_3 n_2\right\}}{4\left\{2EA\left[8EI + LD_1\right] + 8EILK + LD_2\right\}} \tag{22}$$

$$H_2 = \frac{PL^2 k_2\left\{4EAa + Lk_1 n_1 + Lk_3 4a\right\}}{4\left\{2EA\left[8EI + LD_1\right] + 8EILK + LD_2\right\}} \tag{23}$$

$$H_3 = \frac{PL^2 k_3\left\{4EAa + Lk_1 n_2 + Lk_2 4a\right\}}{4\left\{2EA\left[8EI + LD_1\right] + 8EILK + LD_2\right\}} \tag{24}$$

The expressions show good agreement with the expressions reported in Doicheva (2021) and Doicheva (2022), taking into account the relevant geometrical and force conditions embedded in them.

Results and Discussion

For the numerical results, a beam with a cross-section of $25/25\,cm$ was introduced. For all examples considered $P = const$, the distances $e = 3[cm]$ and $a = 9,5[cm]$. And more $A_2 = A_3 = 12,5[cm^2]$ and $E_2 = E_3 = 21000[kN/cm^2]$. The distance $b[cm]$ varies in the interval $[12,5;0)$ and is monitored by the ratio h/b.

Case I - Cantilever Beam

The beam is with a length of $L = 125\,cm$. Two examples with a difference only in the modulus of elasticity of concrete are considered. The modules used are $E_1 = 1700\left[kN/cm^2\right]$ for normal concrete and $E_1 = 3700\left[kN/cm^2\right]$ for High-strength concrete.

Example I- the modulus of elasticity of the concrete is $E_1 = 1700\left[kN/cm^2\right]$

Example II- the modulus of elasticity of the concrete is $E_1 = 3700\left[kN/cm^2\right]$

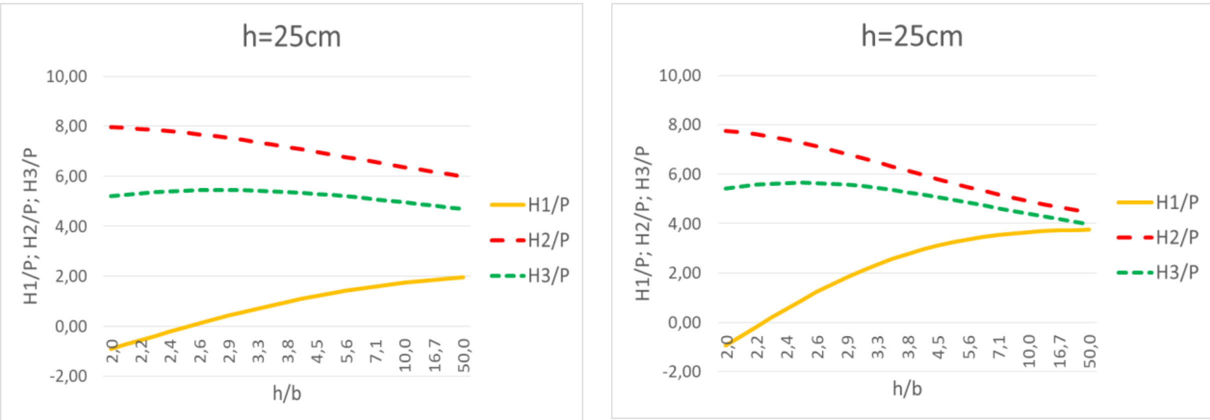


Figure 5. The parameters of the three support reactions - cantilever beam

Case II - Simple Beam

The beam is with a length of $L = 1000\,cm$.

Example I- the modulus of elasticity of the concrete is $E_1 = 1700\left[kN/cm^2\right]$

Example II- the modulus of elasticity of the concrete is $E_1 = 3700\left[kN/cm^2\right]$

Figure 6. The parameters of the three support reactions - simple beam

The graphs in Figure 5 and Figure 6 clearly show the discrepancy between the magnitudes of the forces in the top and bottom reinforcement, i.e. $H_2 \neq H_3$, as well as the significant increase in the force absorbed by the concrete in its intact part (H_1). Also is visible the significant quantitative change of the forces with the change of only one of the material characteristics - the modulus of elasticity of the concrete $E_1\left[kN/cm^2\right]$. These inferences call into question the acceptance in (2) and (3). A new method for the analytical determination of the shear force is proposed.

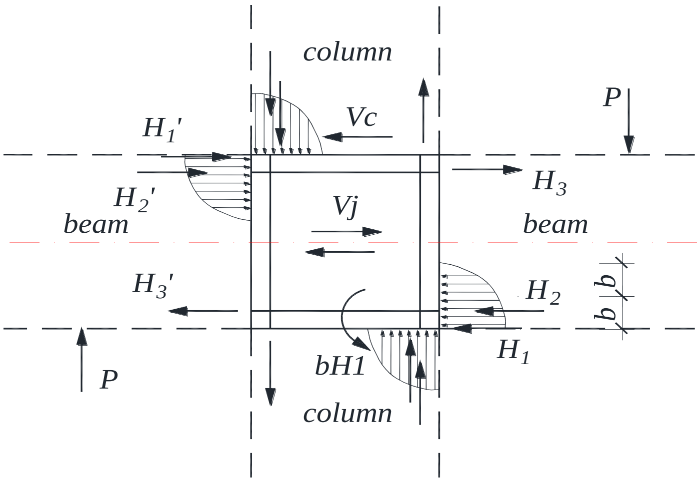


Figure 7. New definition of joint shear in interior RC beam-column connection

A new model to Determination of the Shear Force in RC Interior Beam-Column Connections is proposed.

$$V_j = H_3 + H_2' + H_1' - V_c \quad (25)$$

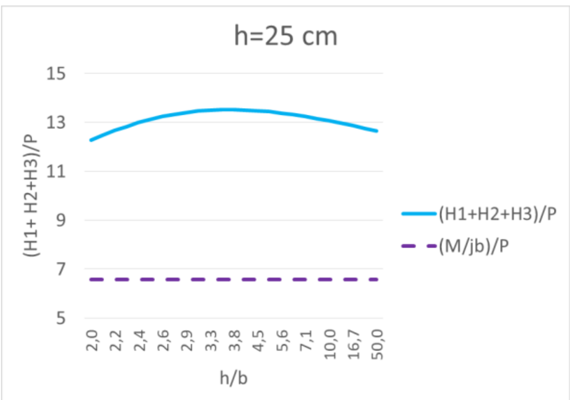
If the frame is symmetric and other conditions being equal, we will have the equality of $H_1 = H_1'$, $H_2 = H_2'$ and $H_3 = H_3'$. Then (25) becomes

$$V_j = H_3 + H_2 + H_1 - V_c \,. \quad (26)$$

Comparison of the Results of (3) and (26)

Case I - Cantilever Beam

Example I- the modulus of elasticity of the concrete is $E_1 = 1700\left[kN/cm^2\right]$



Example II- the modulus of elasticity of the concrete is $E_1 = 3700\left[kN/cm^2\right]$

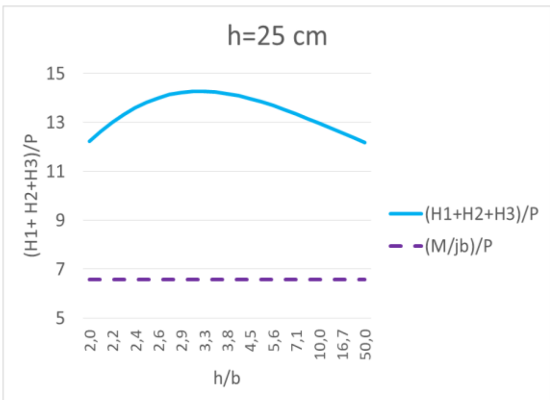


Figure 8. Comparison of the results of (3) and (26) - cantilever beam

The result comparison of the new model and the one known from the literature show the following:

A large discrepancy is observed in the results of (3) and (26), with certainty in favor of (26). The difference determined for the extreme values of (26) with these in (3) is respectively:

- *Example I* – 105% at $h/b = 3{,}8$
- *Example II* – 112% at $h/b = 4{,}5$

There is a serious underestimation of the contribution of the beam forces to the value of the joint shear force, from the expressions known in the literature. The new model shows, that the contribution of the beam forces is greater.

Case II - Simple Beam

Example I- the modulus of elasticity of the concrete is $E_1 = 1700\left[kN/cm^2\right]$

Example II- the modulus of elasticity of the concrete is $E_1 = 3700\left[kN/cm^2\right]$

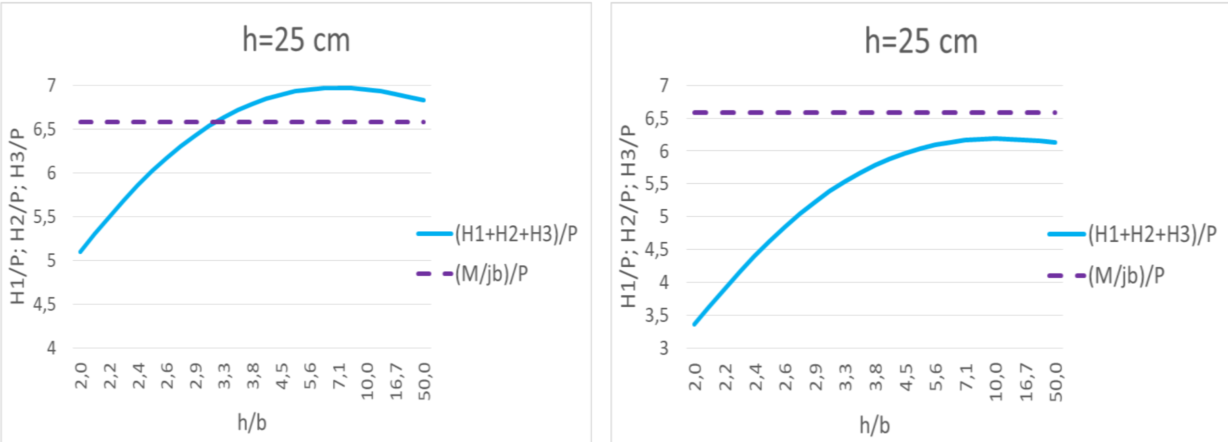


Figure 9. Comparison of the results of (3) and (26) - simple beam

For the simple beam, the results of the old and the new solution are almost similar.

The difference determined for the extreme values of (26) with these in (3) is respectively:

- *Example I* – 6% at $h/b = 7{,}1$, the certainty is in the direction of (26)
- *Example II* – 6% at $h/b = 10$, the certainty is in the direction of (3)

Conclusion

A solution of a cantilever beam with a special arrangement of the support devices was carried out. The real height of the beam was take into account.

The derived expressions for the horizontal supports reactions, although not very short, give results which clearly show the distribution of the forces along the height of the beam, into coresponding support.

The derived expressions for the support reactions take into account the influence of both the geometry of the beam and the material properties of its components.

A comparison is made for the contribution to the value of the shear force from the forces in the beam determined with the obtained expressions and the formulas known from the literature.

A new model is proposed for determining the contribution of the beam forces to the value of the Shear Force in RC Interior Beam-Column Connections.

The obtained results can be of interest to both researchers and practicing engineers. The research from this article can help in the interpretation of the results obtained from structural analyzes and experimental tests.

Recommendations

This article will focus attention on how forces are distributed along the height of the beam and the subsequent load from the beam, on the beam-column connection, with an emphasis on determining the shear force at the joint.

Scientific Ethics Declaration

The author declares that the scientific ethical and legal responsibility of this article published in EPSTEM journal belongs to the author.

Acknowledgements or Notes

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Author Information

Albena Doicheva
University of Architecture, Civil Engineering and Geodesy (UACEG)
Sofia, Bulgaria
Contact e-mail: *doicheva_fhe@uacg.bg*

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