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Analysis of Temperature Change Effect on Dissipation of Energy in Functionally Graded Beams

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Abstract: In this paper, an analysis of the energy dissipation in functionally graded beam structures of viscoelastic behaviour under bending is presented with considering the effects of temperature change. There is an obvious need of carrying-out of such analyses because very often various load-bearing beam structures made of functionally graded viscoelastic materials are exposed to temperature changes simultaneously with external mechanical loading during their lifetime. Rheological model with one spring and two viscous components is used for treating the beam viscoelastic behaviour. The energy dissipation analysis accounts for the temperature change by correcting the time-dependent modulus of elasticity. A parametric study is performed. The general trend of the results obtained is that the dissipated energy decreases when the temperature increases.

Keywords: Dissipation of energy, Temperature change, Functionally graded beam structure

Introduction

Excellent properties of the functionally graded materials (these materials pertain to the category of continuously inhomogeneous engineering materials) make them potential substitute of the conventional engineering structural materials like metals and classical fiber reinforced composites (Fanani et al., 2021; Gururaja Udupa et al., 2014; Gandra et al., 2011). Therefore, functionally graded materials have been widely used for manufacturing of various load-bearing structural members applied in different spheres of modern engineering in the recent decades (Mahamood & Akinlabi, 2017; Nagaral et al., 2019). Ensuring of safety, stability and durability of functionally graded load-bearing structural applications in the process of their engineering design represents an important condition for guaranteeing of the reliable work of these structures (this necessitates development of methods for analyzing the mechanical behaviour of functionally graded structural members under various external loadings) (Radhika et al., 2020; Rizov, 2020; Toudehdehghan et al., 2017).

The present paper addresses the problem of energy dissipation analysis in functionally graded viscoelastic beam structures subjected to bending simultaneously with temperature change. The necessity of performing of such analysis is aroused by the fact that previous publications devoted to this topic consider mainly the effect of external mechanical loading applied on the beam structures (Narisawa, 1987; Rizov, 2021). However, in many situations the load-bearing engineering structures are under external mechanical loading simultaneously with temperature change. Therefore, the innovation in the present paper is that the strain energy dissipation in a functionally graded beam of viscoelastic behaviour under both external mechanical loading and temperature change is analyzed.

Analysis of Energy Dissipation

The length, thickness and width of the cantilever beam structure displayed in Fig. 1 are marked with l, h and b, respectively. The material is functionally graded along the beam thickness. Besides, the material has

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viscoelastic behaviour. The beam is under bending so that the change of the angle of rotation, φ, of the beam free end with time, t, is given by logarithmic law, i.e.

$$\varphi = \ln\left(1 + \beta_{\varphi} t\right), \tag{1}$$

where β_{φ} is a parameter governing the change of φ.

Figure 1. Functionally graded viscoelastic cantilever beam structure.

The viscoelastic behaviour of the beam structure is analyzed with the help of the rheological model displayed in Fig. 2. The model has two viscous components with coefficients of viscosity, η_{up} and η_{lw}, and a linear-elastic component (a spring) with modulus of elasticity, E.

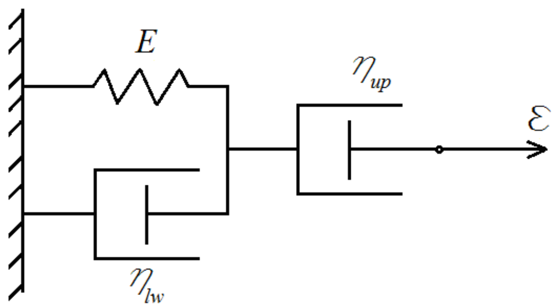

Figure 2. Rheological model.

The model is under strain, ε. The change of the strain with time is written as

$$\varepsilon = \ln\left(1 + \beta_{\varepsilon} t\right), \tag{2}$$

where β_{ε} is a parameter governing the change of ε.

In order to determine the strain, ε_{lw}, in the viscous component with coefficient of viscosity, η_{lw}, the following differential equation is worked-out:

$$\dot{\varepsilon}_{lw} = \frac{\rho_1}{1 + \beta_{\varepsilon} t} + \rho_2 \varepsilon_{lw}, \tag{3}$$

Where

$$\rho_1 = \frac{\eta_{up} \beta_{\varepsilon}}{\eta_{up} + \eta_{lw}}, \ \rho_2 = -\frac{E}{\eta_{up} + \eta_{lw}}. \tag{4}$$

In Eq. (3) $\dot{\varepsilon}_{lw}$ is the first derivative of ε_{lw} with respect to time. By solving Eq. (3), the following expression for ε_{lw} is found:

$$\varepsilon_{lw} = \frac{\rho_1\left(e^{\rho_2 t} - 1\right)}{\rho_2\left(1 + \beta_\varepsilon t\right)}. \tag{5}$$

By applying the Hook's law and Eq. (5), one derives the following expression for the stress, σ_{lw}, in the viscous component with coefficient of viscosity, η_{lw}:

$$\sigma_{lw} = \eta_{lw}\,\frac{e^{\rho_2 t}\rho_1\rho_2\left(1 + \beta_\varepsilon t\right) - \rho_1\beta_\varepsilon\left(e^{\rho_2 t} - 1\right)}{\rho_2\left(1 + \beta_\varepsilon t\right)^2}. \tag{6}$$

By summing up the stresses in the spring and in the viscous component with coefficient of viscosity, η_{lw}, one obtains the following expression for the stress, σ, in the rheological model:

$$\sigma = E\,\frac{\rho_1\left(e^{\rho_2 t} - 1\right)}{\rho_2\left(1 + \beta_\varepsilon t\right)} + \eta_{lw}\,\frac{e^{\rho_2 t}\rho_1\rho_2\left(1 + \beta_\varepsilon t\right) - \rho_1\beta_\varepsilon\left(e^{\rho_2 t} - 1\right)}{\rho_2\left(1 + \beta_\varepsilon t\right)^2}. \tag{7}$$

Equation (7) holds also for the stress, σ_{up}, in viscous component with coefficient of viscosity, η_{lw}. The strain, ε_{up}, in viscous component with coefficient of viscosity, η_{lw}, is determined as

$$\varepsilon_{up} = \varepsilon - \varepsilon_{lw}, \tag{8}$$

where ε and ε_{lw} are obtained by Eq. (2) and Eq. (5), respectively.

Combination of Eq. (2) and Eq. (7) yields the following expression for the time-dependent modulus of elasticity, E_*, of the rheological model:

$$E_* = E\,\frac{\rho_1\left(e^{\rho_2 t} - 1\right)}{\rho_2\left(1 + \beta_\varepsilon t\right)\ln\left(1 + \beta_\varepsilon t\right)} + \eta_{lw}\,\frac{e^{\rho_2 t}\rho_1\rho_2\left(1 + \beta_\varepsilon t\right) - \rho_1\beta_\varepsilon\left(e^{\rho_2 t} - 1\right)}{\rho_2\left(1 + \beta_\varepsilon t\right)^2\,\ln\left(1 + \beta_\varepsilon t\right)}. \tag{9}$$

The change of η_{up}, η_{lw} and E along the beam thickness is given by exponential laws

$$\eta_{up} = \eta_{up0}\,e^{q_1\frac{\frac{h}{2}+z}{h}}, \tag{10}$$

$$\eta_{lw} = \eta_{lw0}\,e^{q_2\frac{\frac{h}{2}+z}{h}}, \tag{11}$$

$$E = E_0\,e^{q_3\frac{\frac{h}{2}+z}{h}}, \tag{12}$$

where η_{up0}, η_{lw0} and E_0 are the values of η_{up}, η_{lw} and E at the upper surface of the beam structure, q_1, q_2 and q_3 are material properties, z is the vertical centric axis of the beam ($-h/2 \le z \le h/2$). Beams of high aspect ratio are under consideration in the present paper. Therefore, the hypothesis for conservation of plane cross-section can be applied. Hence, the change of the strain along the beam thickness is given as

$$\varepsilon = \kappa(z - z_n), \tag{13}$$

where κ is the beam curvature, z_n is the neutral axis coordinate. In order to determine κ the angle of rotation of the beam free end is expressed as a function of κ by applying the integrals of Maxwell-Mohr. The expression obtained is treated as equation with unknown κ. The solution of this equation is found as

$$\kappa = \frac{\ln(1 + \beta_\varphi t)}{l}. \tag{14}$$

The neutral axis coordinate is determined by using the following equation of equilibrium:

$$N = b \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma dz, \tag{15}$$

where σ is given by Eq. (7), N is the axial force (for the beam under consideration $N = 0$). Eq. (15) is solved with respect to z_n by the MatLab computer program at various values of time. The beam structure under consideration is subjected also to change of the temperature, T. This causes variation of the time-dependent modulus of elasticity, E_*. Therefore, E_* is corrected by applying the following relations (Narisawa, 1987):

$$E_{*T_0}\left(\frac{t}{\alpha_T}\right) = \frac{T_0 r_0}{T r_1} E_*, \ \lg \alpha_T = -\frac{C_P(T - T_p)}{C_R + (T - T_p)}, \tag{16}$$

where the value of E_* at room temperature is marked with E_{*T_0}, the ratio, $T_0 r_0 / (T r_1)$, is unit, T_p is a material property, $C_P = 8.86$ and $C_R = 101.6$ (Narisawa, 1987). The unit dissipated energy, u_0, is found as

$$u_0 = \int_0^t (\sigma_{lw}\varepsilon_{lw} + \sigma_{up}\varepsilon_{up}) dt, \tag{17}$$

where ε_{lw}, σ_{lw}, σ_{up} and ε_{up} are given by Eqs. (5), (6), (7) and (8), respectively. The dissipated energy, U, in the beam structure under consideration (Fig. 1) is expressed as

$$U = bl \int_{-\frac{h}{2}}^{\frac{h}{2}} u_0 dz. \tag{18}$$

The integral in Eq. (18) is solved by the MatLab computer program at various values of time.

Numerical Results

Numerical results are presented hereafter. The influence of the temperature change and the material gradient on the dissipated energy in the functionally graded viscoelastic beam structure is studied by performing calculations of U.

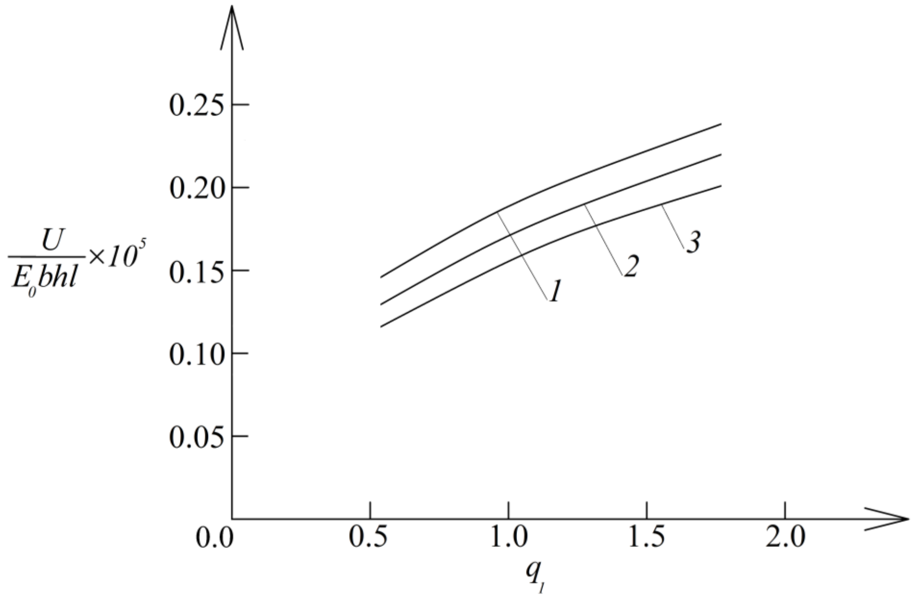

Figure 3. Variation of the dissipated energy with respect to q_1 (curve 1 - at $T / T_p = 1.1$, curve 2 - at $T / T_p = 1.3$ and curve 3 - at $T / T_p = 1.5$).

The following data are used: $l = 0.300$ m, $b = 0.015$ m, $h = 0.020$ m and $\beta_\varphi = 0.3 \times 10^{-7}$ 1/s.

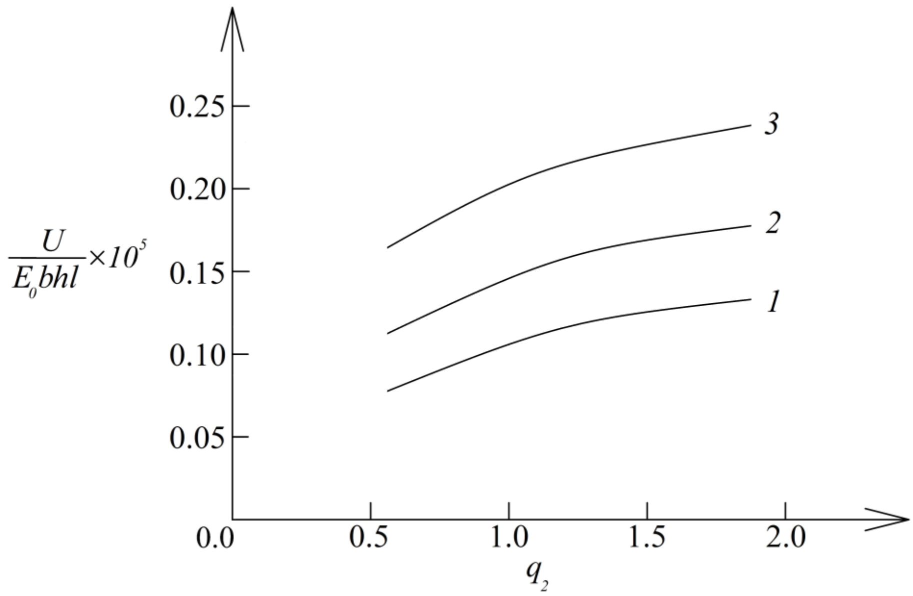

Figure 4. Variation of the dissipated energy with respect to q_2 (curve 1 – at $q_3 = 0.5$, curve 2 - at $q_3 = 1.0$ and curve 3 – at $q_3 = 2.0$).

The dissipated energy is calculated for a range of values of q_1 for three T / T_p ratios. The curves obtained are plotted on the graph presented in Fig. 3. One can observe in Fig. 3 that the shape of each curve is similar for every T / T_p ratio. One can notice also the increase of dissipated energy with increase of q_1 (Fig. 3). Since η_{up} depends on q_1 through the exponential relationship (10), one can draw a general conclusion from the graph in Fig. 3 that a higher coefficient of viscosity, η_{up}, results in higher values of the dissipated energy in the beam structure under consideration. Concerning the influence of the temperature on U, one can see in Fig. 3

that as the T/T_p ratio increases, the dissipated energy decreases (this observation agrees with results of previous studies dealing with homogeneous engineering structures (Kishkilov & Apostolov, 1994).

In order to investigate the effects of the change of the coefficient of viscosity, η_{lw}, and the modulus of elasticity, E, along the beam thickness, the dissipated energy is evaluated for various values of q_2 and q_3. A graph of the dissipated energy in non-dimensional form against q_2 for three values of q_3 is given in Fig. 4. It can be seen from Fig. 4 that the higher the values of q_2 and q_3, the higher the value of the dissipated energy (these observations agree with results of previous papers (Narisawa, 1987; Rizov, 2021).

Conclusion

A theoretical study of the effects of temperature change on the energy dissipation in functionally graded beam structures which exhibit viscoelastic behaviour under bending is conducted. A rheological model constructed by one spring and two viscous components is applied in the analysis. The unit dissipation energy is obtained and then integrated in order to derive the dissipated energy in the beam. The time-dependent modulus of elasticity is corrected to take into account the effect of the temperature change on the dissipated energy. The study leads to several main observations. Firstly, the dissipated energy decreases when the temperature increases. In other words, the energy dissipation capacity of the two viscous components of the rheological model is reduced with increasing the temperature. Also, it is observed that increase of q_1, q_2 and q_3 generates an increase of the dissipated energy. Having in mind that q_1, q_2 and q_3 are exponents in the functions describing the change of η_{up}, η_{lw} and E along the beam thickness, one can draw the conclusion that when the coefficients of viscosity and the modulus of elasticity increase, the dissipated energy increases too.

Recommendations

Analyzing energy dissipation in functionally graded beam structures exhibiting non-linear viscoelastic behavior can be recommended as a future task.

Scientific Ethics Declaration

The author declares that the scientific ethical and legal responsibility of this article published in EPSTEM journal belongs to the author.

Acknowledgements or Notes

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