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Soft G*β-Separation Axioms in Soft Topological Spaces

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Abstract: Soft set theory is a newly emerging tool to deal with uncertain problems and has been studied by researchers in theory and practice. The concept of soft topological space is a very recently developed area having many research scopes. Soft sets have been studied in proximity spaces, multicriteria decision-making problems, medical problems, mobile cloud computing networks, defense learning system, approximate reasoning etc. In 2020 Punitha Tharani and H. Sujitha introduced a new class of soft generalized star β-closed $(S_{ft}g*\beta-\text{closed})$ sets and $S_{ft}g*\beta-\text{open}$ sets in soft topological spaces. They investigated some basic properties of $S_{ft}g*\beta-\text{closed}$ sets and $S_{ft}g*\beta-\text{open}$ sets. They also studied the relationship between this type of closed sets and other existing closed sets in soft topological spaces. The aim of this paper is to introduce some soft separation axioms called $S_{ft}g*\beta-R_0$ space, $S_{ft}g*\beta-R_1$ space, $S_{ft}g*\beta-T_0$ space, $S_{ft}g*\beta-T_1$ space, $S_{ft}g*\beta-T_2$ space, $S_{ft}g*\beta-\text{regular}$ space and $S_{ft}g*\beta-\text{normal}$ space in soft topological spaces. We investigate several properties and characterizations of this spaces in soft topological spaces.

Keywords: $S_{ft}-\text{Top}-\text{Space}$, $S_{ft}g*\beta-R_i$ space, $S_{ft}g*\beta-T_i$ space, $S_{ft}g*\beta-\text{normal}$ space

1. Introduction

During recent years, soft set theory emerged as a best mathematical tool to deal with uncertainties, imprecision and vagueness. Many engineering, medical science, economics, environment problems have various uncertainties, and the soft set theory came up with the reasonable solutions to these problems. A soft set is a collection of approximate descriptions of objects. Some researchers have presented a systematic survey of the literature and the developments of Topological Spaces in soft set theory. They have also provided some applications of soft set theory in software engineering, innovation, medical diagnosis, data analysis, decision making etc. All these tools require the specification of some parameter to start with. The theory of soft sets gives a vital mathematical tool for handling uncertainties and vague concepts. Recently several researchers introduced the notion of soft topology and established that every soft topology induces a collection of topologies called the parametrized family of topologies induced by the soft topology. They discussed soft set-theoretical operations and gave an application of soft set theory to a decision-making problems. Several mathematicians published papers on applications of soft sets and soft topology. Soft sets and soft topology have applications in data mining, image processing, decision-making problems, spatial modeling, and neural patterns. Research works on soft set theory and its applications in various fields are progressing rapidly. Decision-making and topology have a long joint tradition since the modern statement of the classical Weierstrass extreme value theorem. It combines two topological concepts called continuity of a real-valued function and compactness of the domain (both with respect to a given topology). They represent a necessary and sufficient condition to guarantee the existence of the maximum and minimum values of the function. The success of Mathematical Problems in Engineering technique was amplified by its adoption in fields like engineering sciences, computer sciences, and mathematical economics. This matter can be adopted on the version of soft setting by replacing the classical notions (compactness, function, and real numbers) by their soft counterparts (soft compactness, soft function, and soft real numbers). Some practical experiments in the civil engineering require classification of the

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materials according to their characteristics (attribute set or parameter set E) which can be expressed using the concept of soft sets. We study the separation of them with respect to the group of soft sets which are constructed from the practical experiments. In this group of soft sets, we add the absolute and null soft sets to initiate a soft weak structure. The researchers in the communication engineering endeavor to select the best protocol to solve the noisy problems in wireless networks. They evaluate the performance of these protocols according to the proposed scenarios. The researchers in Soft Theory may plan with some engineers to propose some protocols using the appropriate soft structure to select the optimal protocol to solve the interference problems in wireless networks. The main objective of this paper is to introduce some soft separation axioms called $S_{ft}g{*}\beta-R_0$ space, $S_{ft}g{*}\beta-R_1$ space, $S_{ft}g{*}\beta-T_0$ space, $S_{ft}g{*}\beta-T_1$ space, $S_{ft}g{*}\beta-T_2$ space, $S_{ft}g{*}\beta-regular$ space and $S_{ft}g{*}\beta-normal$ space in soft topological spaces. We investigate several properties and characterizations of these new notions in soft topological spaces.

2. Preliminaries

Definition 2.1. Let X be an initial universe set and E be a collection of all possible parameters with respect to X, where parameters are the characteristics or properties of objects in X Let $P(X)$ denote the power set of X, and let A be a non-empty subset of E . A pair (F,A) is called a soft set over X, where F is a mapping given by $F:A\rightarrow P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X . For $e\in A$, $F(e)$ may be considered as the set e–approximate elements of the soft set (F,A). Clearly, a soft set is not a set. For two soft sets (F,A) and (G,B) over the common universe X, we say that (F,A) is a soft subset of (G,B) if (i) $A\subseteq B$ and (ii) for all $e\in A$, $F(e)$ and $G(e)$ are identical approximations. We write $(F,A)\subseteq(G,B)$. (G,B) is said to be a soft superset of (F,A), if (F,A) is a soft subset of (G,B). Two soft sets (F,A) and (G,B) over a common universe X are said to be soft equal if (F,A) is a soft subset of (G,B) and (G,B) is a soft subset of (F,A).

Definition 2.2. The union of two soft sets of (F,A) and (G,B) over the common universe X is soft set $(H,C)=(F,A)\bigcup(G,B)$, where $C=A\bigcup B$, and $H(e)=F(e)$ if $e\in A-B$, $H(e)=G(e)$ if $e\in B-A$ and $H(e)=F(e)\bigcup G(e)$ if $e\in A\bigcap B$.

Definition 2.3. The Intersection (H,C) of two soft sets (F,A) and (G,B) over a common universe X denoted $(F,A)\bigcap(G,B)$ is defined as $C=A\bigcap B$ and $H(e)=F(e)\bigcap G(e)$ for all $e\in C$.

Definition 2.4. For a soft set (F,A) over the universe U, the relative complement of (F,A) is denoted by $(F,A)^C$ and is defined by $(F,A)^C=(F^C,A)$, where $F^C:A\rightarrow P(X)$ is a mapping defined by $F^C(e)=X-F(e)$ for all $e\in A$.

Definition 2.5. A soft set (F,E) over X is said to be (i) A null soft set, denoted by $\tilde{\phi}$, if $\forall$ $e\in E$, $F(e)=\phi$. (ii) An absolute soft set, denoted by $\tilde{X}$, if $\forall$ $e\in E$, $F(e)=X$.

Definition 2.6. Let $\tilde{\tau}$ be the collection of soft sets over X. Then $\tilde{\tau}$ is said to be a soft Topology on X if

(i) $\tilde{\phi}$, $\tilde{X}$ belong to $\tilde{\tau}$

(ii) The union of any number of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$

(iii) The intersection of any two soft sets belongs to $\tilde{\tau}$.

The triplet $(\tilde{X},\tilde{\tau},E)$ is called a soft topological space over X. The complement of a soft open set is called soft closed set over X.

Definition 2.7. Let $(\tilde{X},\tilde{\tau},E)$ be a soft topological space over X and (F,E) be a soft set over X. Then

(i) Soft interior of a soft set (F,E) denoted by $S_{ft}Int(F,E)$ is defined as the union of all soft open sets over X contained in (F,E).

(ii) Soft closure of a soft set (F,E) denoted by $S_{ft}Cl(F,E)$ is defined as the intersection of all soft closed super sets over X containing (F,E).

Definition 2.8. A $S_{ft}-$subset (F,E) of a $S_{ft}-Top-Space$ $(\tilde{X},\tilde{\tau},E)$ is called a $S_{ft}\alpha-open$ set if $(F,E)\subseteq S_{ft}Int\left[S_{ft}Cl\left(S_{ft}Int(F,E)\right)\right]$ and a $S_{ft}\alpha-closed$ set if $S_{ft}Cl\left[S_{ft}Int\left(S_{ft}Cl(F,E)\right)\right]\subseteq(F,E)$. The complement of a $S_{ft}\alpha-closed$ set is called $S_{ft}\alpha-open$ set.

Definition 2.9. A $S_{ft}-$subset (F,E) of a $S_{ft}-Top-Space$ $(\tilde{X},\tilde{\tau},E)$ is called a $S_{ft}\beta-open$ set if $(F,E)\subseteq S_{ft}Cl\left[S_{ft}Int\left(S_{ft}Cl(F,E)\right)\right]$ and $S_{ft}\beta-closed$ set if $S_{ft}Int\left[S_{ft}Cl\left(S_{ft}Int(F,E)\right)\right]\subseteq(F,E)$. The complement of a $S_{ft}\beta-closed$ set is called $S_{ft}\beta-open$ set.

Definition 2.10. A $S_{ft}-$subset (F,E) of a $S_{ft}-Top-Space$ $(\tilde{X},\tilde{\tau},E)$ is called a $S_{ft}g-closed$ set if $S_{ft}Cl(F,E)\subseteq(U,E)$ whenever $(F,E)\subseteq(U,E)$ and (U,E) is $S_{ft}-open$ in $(\tilde{X},\tilde{\tau},E)$. The complement of a $S_{ft}g-closed$ set is called a $S_{ft}g-open$ set.

Definition 2.11. A $S_{ft}-$subset (F,E) of a $S_{ft}-Top-Space$ $(\tilde{X},\tilde{\tau},E)$ is called a soft generalized β closed $(S_{ft}g\beta-closed)$ set if $S_{ft}\beta-Cl(F,E)\subseteq(U,E)$ whenever $(F,E)\subseteq(U,E)$ and (U,E) is $S_{ft}-open$ in $(\tilde{X},\tilde{\tau},E)$. The complement of a $S_{ft}g\beta-closed$ set is called a $S_{ft}g\beta-open$ set.

Definition 2.12. A $S_{ft}-$subset (F,E) of a $S_{ft}-Top-Space$ $(\tilde{X},\tilde{\tau},E)$ is called a $S_{ft}S_{emi}-open$ set if $(F,E)\subseteq S_{ft}Cl\left[S_{ft}Int(F,E)\right]$ and $S_{ft}S_{emi}-closed$ if $S_{ft}Int\left[S_{ft}Cl(F,E)\right]\subseteq(F,E)$. The complement of a $S_{ft}S_{emi}-closed$ set is called a $S_{ft}S_{emi}-open$ set.

Definition 2.13. A $S_{ft}-$subset (F,E) of a $S_{ft}-Top-Space$ $(\tilde{X},\tilde{\tau},E)$ is called a $S_{ft}g*-closed$ set if $S_{ft}Cl(F,E)\subseteq(U,E)$ whenever $(F,E)\subseteq(U,E)$ and (U,E) is $S_{ft}g-open$ set in X. The complement of a $S_{ft}g*-closed$ set is called a $S_{ft}g*-open$ set.

Definition 2.14. A $S_{ft}-$subset (F,E) of a $S_{ft}-Top-Space$ $(\tilde{X},\tilde{\tau},E)$ is called a soft generalized star β closed $(\text{briefly } S_{ft}g*\beta-closed)$ set if $S_{ft}\beta Cl(F,E)\subseteq(U,E)$, whenever $(F,E)\subseteq(U,E)$ and (U,E) is $S_{ft}g*-open$ in $(\tilde{X},\tilde{\tau},E)$. The complement of a $S_{ft}g*\beta-closed$ set is called a $S_{ft}g*\beta-open$ set. The family of all $S_{ft}g*\beta$-open (respectively $S_{ft}g*\beta$-closed) in $(\tilde{X},\tilde{\tau},E)$ is denoted by $S_{ft}g*\beta\text{-}O(\tilde{X},\tilde{\tau},E)=S_{ft}g*\beta\text{-}O(\tilde{X})$ (respectively $S_{ft}g*\beta\text{-}C(\tilde{X},\tilde{\tau},E)=S_{ft}g*\beta\text{-}C(\tilde{X})$).

Definition 2.15. Let $(\tilde{X},\tilde{\tau},E)$ be a S_{ft}-Top-Space over X and (F,A) be a soft set over X. Then

(i) $S_{ft}g*\beta$-interior of a soft set (F,A) denoted by $S_{ft}g*\beta\text{-}Int(F,A)$ is defined as the union of all soft $S_{ft}g*\beta$-open sets over X contained in (F,A).

(ii) $S_{ft}g*\beta$-closure of a soft set (F,A) denoted by $S_{ft}g*\beta\text{-}Cl(F,A)$ is defined as the intersection of all $S_{ft}g*\beta$-closed sets over X containing (F,A).

Theorem 2.16. (i) Every S_{ft}-closed set in a soft topological space is $S_{ft}g*\beta$-closed set.

(ii) Every S_{ft}-open set in a S_{ft}-Top-Space is $S_{ft}g*\beta$-open set.

Theorem 2.16. Let (F,A) and (G,B) be two S_{ft}-subsets of a S_{ft}-Top-Space $(\tilde{X},\tilde{\tau},E)$ Then the following statements are true.

(i) (F,A) is $S_{ft}g*\beta$-open if and only if $S_{ft}g*\beta\text{-}Int(F,A)=(F,A)$.

(ii) $S_{ft}g*\beta\text{-}Int(F,A)$ is $S_{ft}g*\beta$-open.

(iii) (F,A) is $S_{ft}g*\beta$-closed if and only if $S_{ft}g*\beta\text{-}Cl(F,A)=(F,A)$.

(iv) $S_{ft}g*\beta\text{-}Cl(F,A)$ is $S_{ft}g*\beta$-closed.

(v) $S_{ft}g*\beta\text{-}Cl[(X,E)-(F,A)]=(X,E)-[S_{ft}g*\beta\text{-}Int(F,A)]$.

(vi) $S_{ft}g*\beta\text{-}Int[(X,E)-(F,A)]=(X,E)-[S_{ft}g*\beta\text{-}Cl(F,A)]$.

(vii) If (F,A) is $S_{ft}g*\beta$-open in $(\tilde{X},\tilde{\tau},E)$ and (G,B) is S_{ft}-open in $(\tilde{X},\tilde{\tau},E)$ then $(F,A)\cap(G,B)$ is $S_{ft}g*\beta$-open in $(\tilde{X},\tilde{\tau},E)$.

(viii) A point $x_\alpha\in S_{ft}g*\beta\text{-}Cl(F,A)$ if and only if every $S_{ft}g*\beta$-open set in $(\tilde{X},\tilde{\tau},E)$ containing x intersects (F,A).

(ix) Finite intersection of $S_{ft}g*\beta$-closed sets in $(\tilde{X},\tilde{\tau},E)$ is $S_{ft}g*\beta$-closed in $(\tilde{X},\tilde{\tau},E)$.

(x) Finite union of $S_{ft}g*\beta$-open sets in $(\tilde{X},\tilde{\tau},E)$ is $S_{ft}g*\beta$-open in $(X,\tilde{\tau},E)$.

3. $S_{ft}g*\beta\text{-}R_0$ Space and $S_{ft}g*\beta\text{-}R_1$ Space

In this section we define two types of spaces called $S_{ft}g*\beta\text{-}R_i$ spaces for $i=0,1$.

Definition 3.1. A S_{ft}-Top-Space $(\tilde{X},\tilde{\tau},E)$ is called $S_{ft}g*\beta\text{-}R_0$ if for every $S_{ft}g*\beta$-open set (F,E), $S_{ft}g*\beta\text{-}Cl(\{x_\alpha\})\subseteq(F,E)$ for every $x_\alpha\in(F,E)$.

Definition 3.2. Let $(\tilde{X},\tilde{\tau},E)$ be a S_{ft}-Top-Space and $(F,E)\subseteq\tilde{X}$, Then $S_{ft}g*\beta$-Kernel of (F,E) is defined to be the intersection of all $S_{ft}g*\beta$-open sets containing (F,E) and denoted by $S_{ft}g*\beta\text{-}Ker(F,E)$ that is $S_{ft}g*\beta\text{-}Ker(F,E)=\cap\{(G,E)\in S_{ft}g*\beta\text{-}O(X):(F,E)\subseteq(G,E)\}$.

Lemma 3.3. Let $(\tilde{X},\tilde{\tau},E)$ be a $S_{ft}-Top-Space$ and $x_\alpha \in \tilde{X}$. Then $y_\beta \in S_{ft}g*\beta-Ker(\{x_\alpha\})$ if and only if $x_\alpha \in S_{ft}g*\beta-Cl(\{y_\beta\})$.

Proof. Suppose that $x_\alpha \notin S_{ft}g*\beta-Ker(F,E)$. Then there exists a $S_{ft}g*\beta-open$ set (F,E) containing x_α such that $y_\beta \notin (F,E)$. Therefore, we have $x_\alpha \notin S_{ft}g*\beta-Cl(\{y_\beta\})$. The proof of converse can be done similarly.

Theorem 3.4. Let $(\tilde{X},\tilde{\tau},E)$ be a $S_{ft}-Top-Space$. Then the following statements are equivalent:

(1). (X,τ,E) is $S_{ft}g*\beta-R_{0}$.

(2). For any $(K,E)\in S_{ft}g*\beta-C(X)$ and $x_\alpha \notin (K,E)$ there exists $(F,E)\in S_{ft}g*\beta-O(X)$ such that $(K,E)\subseteq(F,E)$ and $x_\alpha \notin (F,E)$.

(3). For any $(K,E)\in S_{ft}g*\beta-C(X)$ and $x_\alpha \in (K,E)$ implies that $(K,E)\cap S_{ft}g*\beta-Cl(\{x_\alpha\}) = \tilde{\phi}$.

(4). For any two distinct soft points $x_\alpha, y_\beta \in \tilde{X}$ either $S_{ft}g*\beta-Cl(\{x_\alpha\}) = S_{ft}g*\beta-Cl(\{y_\beta\}) = \tilde{\phi}$ or $\left[S_{ft}g*\beta-Cl(\{x_\alpha\})\right]\cap\left[S_{ft}g*\beta-Cl(\{y_\beta\})\right]=\tilde{\phi}$.

Proof $(1)\Rightarrow(2)$: Let $(K,E)\in S_{ft}g*\beta-C(\tilde{X})$ and $x_\alpha \notin (K,E)$. Then by (1) $S_{ft}g*\beta-Cl(\{x_\alpha\})\subseteq \tilde{X}-(K,E)$. Let $(F,E)=\tilde{X}-(K,E)$. Then $(F,E)\in S_{ft}g*\beta-O(\tilde{X})$, $(K,E)\subseteq(F,E)$ and $x_\alpha \notin (F,E)$.

$(2)\Rightarrow(3)$: Let $(K,E)\in S_{ft}g*\beta-C(\tilde{X})$ and $x_\alpha \notin (K,E)$. Then there exists $(F,E)\in S_{ft}g*\beta-O(\tilde{X})$ such that $(K,E)\subseteq(F,E)$ and $x_\alpha \notin (F,E)$. Since $(K,E)\subseteq(F,E)$, so by (2) $(F,E)\cap S_{ft}g*\beta-Cl(\{x_\alpha\})=\tilde{\phi}$. This implies that $(K,E)\cap S_{ft}g*\beta-Cl(\{x_\alpha\})=\tilde{\phi}$.

$(3)\Rightarrow(4)$: Let x_α and y_β be two distinct soft points of $\tilde{X}$. Suppose that $S_{ft}g*\beta-Cl(\{x_\alpha\})\neq S_{ft}g*\beta-Cl(\{y_\beta\})$. Then there exists a soft point z_γ such that $z_\gamma \in S_{ft}g*\beta-Cl(\{x_\alpha\})$ and $z_\gamma \notin S_{ft}g*\beta-Cl(\{y_\beta\})$ or $z_\gamma \in S_{ft}g*\beta-Cl(\{y_\beta\})$ such that $z_\gamma \notin S_{ft}g*\beta-Cl(\{x_\alpha\})$ and there exists $(F,E)\in S_{ft}g*\beta-O(\tilde{X})$ such that $y_\beta \notin (F,E)$ and $z_\gamma \in (F,E)$, hence $x_\alpha \in (F,E)$, therefore, we have $x_\gamma \notin S_{ft}g*\beta-Cl(\{y_\beta\})$ by (3) we obtain $\left[S_{ft}g*\beta-Cl(\{x_\alpha\})\right]\cap\left[S_{ft}g*\beta-Cl(\{y_\beta\})\right]=\tilde{\phi}$.

$(4)\Rightarrow(1)$: Let $(F,E)\in S_{ft}g*\beta-O(\tilde{X})$ and $x_\alpha \in (F,E)$, for each $y_\beta \notin (F,E)$. Then $x_\alpha \neq y_\beta$ and $x_\alpha \notin S_{ft}g*\beta-Cl(\{y_\beta\})$, this shows that $S_{ft}g*\beta-Cl(\{x_\alpha\})\neq S_{ft}g*\beta-Cl(\{y_\beta\})$, then by (4) $\left[S_{ft}g*\beta-Cl(\{x_\alpha\})\right]\cap\left[S_{ft}g*\beta-Cl(\{y_\beta\})\right]=\tilde{\phi}$, for each $y_\beta \in \tilde{X}-(F,E)$. On the other hand, since $(F,E)\in S_{ft}g*\beta-O(\tilde{X})$ and $y_\beta \in \tilde{X}-(F,E)$, we have $S_{ft}g*\beta-Cl(\{y_\beta\})\subseteq \tilde{X}-(F,E)$. Hence $\tilde{X}-(F,E)=\bigcup\left\{S_{ft}g*\beta-Cl(\{y_\beta\}) : y_\beta \in \tilde{X}-(F,E)\right\}$. Therefore, we obtain that $\left[\tilde{X}-(F,E)\right]\cap S_{ft}g*\beta-Cl(\{x_\alpha\})=\tilde{\phi}$ and $S_{ft}g*\beta-Cl(\{x_\alpha\})\subseteq(F,E)$. This shows that $(\tilde{X},\tilde{\tau},E)$ is $S_{ft}g*\beta-R_{0}$.

Theorem 3.5. A $S_{ft}-\text{Top}-\text{Space}\ (\tilde{X},\tilde{\tau},E)$ is a $S_{ft}g*\beta-R_0$ space if and only if for any $x_\alpha, y_\beta \in \tilde{X}$, $S_{ft}g*\beta-Cl(\{x_\alpha\}) \neq S_{ft}g*\beta-Cl(\{y_\beta\}) \Rightarrow [S_{ft}g*\beta-Cl(\{x_\alpha\})] \cap [S_{ft}g*\beta-Cl(\{y_\beta\})] = \tilde{\phi}$.

Proof. This is an immediate consequence of Theorem 3.4.

Theorem 3.6. Let x_α and y_β be any distinct soft points in a $S_{ft}-\text{Top}-\text{Space}\ (\tilde{X},\tilde{\tau},E)$. Then $S_{ft}g*\beta-Ker(\{x_\alpha\}) \neq S_{ft}g*\beta-Ker(\{y_\beta\})$ if and only if $S_{ft}g*\beta-Cl(\{x_\alpha\}) \neq S_{ft}g*\beta-Cl(\{y_\beta\})$.

Proof. __Necessity__. Suppose that $S_{ft}g*\beta-Ker(\{x_\alpha\}) \neq S_{ft}g*\beta-Ker(\{y_\beta\})$. Then there exists a soft point $z_\gamma \in X$ such that $z_\gamma \in S_{ft}g*\beta-Ker(\{x_\alpha\})$ and $z_\gamma \notin S_{ft}g*\beta-Ker(\{y_\beta\})$. Since $z_\gamma \in S_{ft}g*\beta-Ker(\{x_\alpha\})$. So $\{x_\alpha\} \cap [S_{ft}g*\beta-Cl(\{z_\gamma\})] \neq \tilde{\phi}$. This implies that $x_\alpha \in S_{ft}g*\beta-Cl(\{z_\gamma\})$ and since $z_\gamma \notin S_{ft}g*\beta-Ker(\{y_\beta\})$. We have $\{y_\beta\} \cap [S_{ft}g*\beta-Cl(\{z_\gamma\})] = \tilde{\phi}$. Since $x_\alpha \in S_{ft}g*\beta-Cl(\{z_\gamma\})$, so $S_{ft}g*\beta-Cl(\{x_\alpha\}) \subseteq S_{ft}g*\beta-Cl(\{z_\gamma\})$ and hence $\{y_\beta\} \cap S_{ft}g*\beta-Cl(\{x_\alpha\}) = \tilde{\phi}$. Therefore, $S_{ft}g*\beta-Cl(\{x_\alpha\}) \neq S_{ft}g*\beta-Cl(\{y_\beta\})$.

__**Sufficiency.**__ Suppose that $S_{ft}g*\beta-Cl(\{x_\alpha\}) \neq S_{ft}g*\beta-Cl(\{y_\beta\})$. Then there exists a soft point z_γ in $\tilde{X}$ such that $z_\gamma \in S_{ft}g*\beta-Cl(\{x_\gamma\})$ and $z_\gamma \notin S_{ft}g*\beta-Cl(\{y_\beta\})$. Thus there exists a $S_{ft}g*\beta-$*open* set (F,E) containing z_γ (and hence x_α) but not y_β, that is $y_\beta \notin S_{ft}g*\beta-Ker(\{x_\alpha\})$. Therefore, $S_{ft}g*\beta-Ker(\{x_\alpha\}) \neq S_{ft}g*\beta-Ker(\{y_\beta\})$.

Theorem 3.7. A $S_{ft}-\text{Top}-\text{Space}\ (\tilde{X},\tilde{\tau},E)$ is a $S_{ft}g*\beta-R_0$ space if and only if for any two distinct soft points $x_\alpha, y_\beta \in \tilde{X}$, $S_{ft}g*\beta-Ker(\{x_\alpha\}) \neq S_{ft}g*\beta-Ker(\{y_\beta\})$ implies $[S_{ft}g*\beta-Ker(\{x_\alpha\})] \cap [S_{ft}g*\beta-Ker(\{y_\beta\})] = \tilde{\phi}$.

Proof. Necessity. Suppose that $(\tilde{X},\tilde{\tau},E)$ is $S_{ft}g*\beta-R_0$. Then by Theorem 3.6, for any distinct soft points x_α and y_β in $\tilde{X}$, if $S_{ft}g*\beta-Ker(\{x_\alpha\}) \neq S_{ft}g*\beta-Ker(\{y_\beta\})$, then $S_{ft}g*\beta-Cl(\{x_\alpha\}) \neq S_{ft}g*\beta-Cl(\{y_\beta\})$. Assume that $S_{ft}g*\beta-Ker(\{x_\alpha\}) \cap S_{ft}g*\beta-Ker(\{y_\beta\})$. Since $x_\alpha \in S_{ft}g*\beta-Ker(\{x_\alpha\})$, and by Theorem 3.4 it follows that $x_\alpha \in S_{ft}g*\beta-Cl(\{z_\alpha\})$. Since $x_\alpha \in S_{ft}g*\beta-Cl(\{x_\alpha\})$, by Theorem 3.4 it follows that $S_{ft}g*\beta-Cl(\{x_\alpha\}) = S_{ft}g*\beta-Cl(\{z_\gamma\})$. Similarly we have $S_{ft}g*\beta-Cl(\{x_\alpha\}) = S_{ft}g*\beta-Cl(\{z_\gamma\}) = S_{ft}g*\beta-Cl(\{y_\beta\})$, which is contradiction. Thus, $[S_{ft}g*\beta-Ker(\{x_\alpha\})] \cap [S_{ft}g*\beta-Ker(\{y_\beta\})] = \tilde{\phi}$.

Sufficiency. Let $(\tilde{X},\tilde{\tau},E)$ be a $S_{ft}-\text{Top}-\text{Space}$ such that for any distinct soft points x_α and y_β in $\tilde{X}$, $[S_{ft}g*\beta-Ker(\{x_\alpha\})] \neq [S_{ft}g*\beta-Ker(\{y_\beta\})]$ implies that $[S_{ft}g*\beta-Ker(\{x_\alpha\})] \cap [S_{ft}g*\beta-Ker(\{y_\beta\})] = \tilde{\phi}$. If $[S_{ft}g*\beta-Cl(\{x_\alpha\})] \neq [S_{ft}g*\beta-Cl(\{y_\beta\})]$, hence by Theorem 3.6, $\left[S_{ft}g*\beta-Ker\left(\{x_\alpha\}\right)\right]\neq\left[SS_{ft}g*\beta-Ker\left(\{y_\beta\}\right)\right]$. Therefore, $\left[S_{ft}g*\beta-Ker\left(\{x_\alpha\}\right)\right]\cap\left[S_{ft}g*\beta-Ker\left(\{y_\beta\}\right)\right]=\tilde{\phi}$. Which implies that $\left[S_{ft}g*\beta-Cl\left(\{x_\alpha\}\right)\right]\cap\left[S_{ft}g*\beta-Cl\left(\{y_\beta\}\right)\right]=\tilde{\phi}$, because $z_\gamma\in S_{ft}g*\beta-Cl\left(\{x_\alpha\}\right)$ implies that $x_\alpha\in S_{ft}g*\beta-Ker\left(\{z_\gamma\}\right)$. Therefore, $\left[S_{ft}g*\beta-Ker\left(\{x_\alpha\}\right)\right]\cap\left[S_{ft}g*\beta-Ker\left(\{y_\beta\}\right)\right]\neq\tilde{\phi}$. By hypothesis we have, $\left[S_{ft}g*\beta-Ker\left(\{x_\alpha\}\right)\right]=\left[S_{ft}g*\beta-Ker\left(\{z_\gamma\}\right)\right]$, then $z_\gamma\in\left[S_{ft}g*\beta-Cl\left(\{x_\alpha\}\right)\right]\cap\left[S_{ft}g*\beta-Cl\left(\{y_\beta\}\right)\right]$ implies that $S_{ft}g*\beta-Ker\left(\{x_\alpha\}\right)=S_{ft}g*\beta-Ker\left(\{z_\gamma\}\right)=S_{ft}g*\beta-Ker\left(\{y_\beta\}\right)$, this is a contradiction. Therefore, $\left[S_{ft}g*\beta-Cl\left(\{x_\alpha\}\right)\right]\cap\left[S_{ft}g*\beta-Cl\left(\{y_\beta\}\right)\right]=\tilde{\phi}$. Hence by Theorem 3.4, (X,τ,E) is $S_{ft}g*\beta-R_{\mathbf{0}}$.

Theorem 3.8. Let $(\tilde{X},\tilde{\tau},E)$ be a $S_{ft}-Top-Space$. Then the following statements are equivalent:

(1). $(\tilde{X},\tilde{\tau},E)$ is $S_{ft}g*\beta-R_{\mathbf{0}}$.

(2). For any non-empty soft sets (F,E) and $(G,E)\in S_{ft}g*\beta-O(\tilde{X})$ such that $(F,E)\cap(G,E)\neq\tilde{\phi}$, there exists $(K,E)\in S_{ft}g*\beta-C(\tilde{X})$ such that $(K,E)\cap(F,E)\neq\tilde{\phi}$ and $(K,E)\subseteq(G,E)$.

(3). For any $(G,E)\in S_{ft}g*\beta-O(\tilde{X})$, $(G,E)=\bigcup\left\{(K,E)\in S_{ft}g*\beta-C(\tilde{X}):(K,E)\subseteq(G,E)\right\}$

(4). For any $(K,E)\in S_{ft}g*\beta-C(\tilde{X})$, $(K,E)=\bigcap\left\{(G,E)\in S_{ft}g*\beta-O(\tilde{X}):(K,E)\subseteq(G,E)\right\}$

(5). For any $x_\alpha\in\tilde{X}$, $S_{ft}g*\beta-Cl\left(\{x_\alpha\}\right)\subseteq S_{ft}g*\beta-Ker\left(\{x_\alpha\}\right)$.

Proof. $(1)\Rightarrow(2)$: Let (F,E) be a non-empty soft subset of $\tilde{X}$ and $(G,E)\in S_{ft}g*\beta-O(\tilde{X})$ such that $(F,E)\cap(G,E)\neq\tilde{\phi}$. Let $x_\alpha\in(F,E)\cap(G,E)$. Since $x_\alpha\in(G,E)\in S_{ft}g*\beta-O(\tilde{X})$, so by (1), we have $S_{ft}g*\beta-Cl\left(\{x_\alpha\}\right)\subseteq(G,E)$. set $(K,E)=S_{ft}g*\beta-Cl\left(\{x_\alpha\}\right)$ then $(K,E)\in SS_{ft}g*\beta-C(\tilde{X})$ such that $(K,E)\subseteq(G,E)$ and $(F,E)\cap(K,E)\neq\tilde{\phi}$.

$(2)\Rightarrow(3)$: Let $(G,E)\in S_{ft}g*\beta-O(\tilde{X})$. Then $\bigcup\left\{(K,E)\in S_{ft}g*\beta-C(X):(K,E)\subseteq(G,E)\right\}\subseteq(G,E)$. Now let x_α be any soft point of (G,E). By (2) there exists $(K,E)\in S_{ft}g*\beta-C(\tilde{X})$, such that $x_\alpha\in(K,E)$ and $(K,E)\subseteq(G,E)$. So $x_\alpha\in(K,E)\subseteq\bigcup\left\{(K,E)\in S_{ft}g*\beta-C(\tilde{X}):(K,E)\subseteq(G,E)\right\}$. Thus, $(G,E)=\bigcup\left\{(K,E)\in S_{ft}g*\beta-C(\tilde{X}):(K,E)\subseteq(G,E)\right\}$.

$(3)\Rightarrow(4)$: Obvious.

$(4)\Rightarrow(5)$: Let x_α be any soft point of $\tilde{X}$ and $y_\beta\notin S_{ft}g*\beta-Ker\left(\{x_\alpha\}\right)$. So there exists $(H,E)\in S_{ft}g*\beta-O(\tilde{X})$ such that $x_\alpha\in(H,E)$ and $y_\beta\notin(H,E)$. Hence $\left[S_{ft}g*\beta-Cl\left(\{y_\beta\}\right)\right]\cap(H,E)=\tilde{\phi}$. By (4), we have $\left[\bigcap\left\{\ (G,E)\in S_{ft}g*\beta-O(\tilde{X}):\right.\right.$

$S_{ft}g*\beta - Cl(\{y_\beta\}) \subseteq (G,E)\}] \cap (H,E) = \tilde{\phi}$, Where $(G,E) \in S_{ft}g*\beta - O(\tilde{X})$ such that $x_\alpha \notin (G,E)$ and $S_{ft}g*\beta - Cl(\{y_\beta\}) \subseteq (G,E)$. Therefore $S_{ft}g*\beta - Cl(\{x_\alpha\}) \cap (G,E) = \tilde{\phi}$ and hence $y_\beta \notin S_{ft}g*\beta - Cl(\{x_\alpha\})$. Consequently, we obtain $S_{ft}g*\beta - Cl(\{x_\alpha\}) \subseteq S_{ft}g*\beta - Ker(\{x_\alpha\})$.

$(5) \Rightarrow (1)$: Let $(G,E) \in S_{ft}g*\beta - O(\tilde{X})$ and $x_\alpha \in (G,E)$, let $y_\beta \in S_{ft}g*\beta - Ker(\{x_\alpha\})$. Then $x_\alpha \in S_{ft}g*\beta - Cl(\{y_\beta\})$ and $y_\beta \in (G,E)$. This implies that $S_{ft}g*\beta - Ker(\{x_\alpha\}) \subseteq (G,E)$. Therefore, we get $S_{ft}g*\beta - Cl(\{x_\alpha\}) \subseteq S_{ft}g*\beta - Ker(\{x_\alpha\}) \subseteq (G,E)$. This shows that $(\tilde{X}, \tilde{\tau}, E)$ is $S_{ft}g*\beta - R_0$.

Theorem 3.9. Let $(\tilde{X}, \tilde{\tau}, E)$ be a $S_{ft} - Top - Space$. Then the following statements are equivalent:

(1). $(\tilde{X}, \tilde{\tau}, E)$ is a $S_{ft}g*\beta - R_0$ space.

(2). $S_{ft}g*\beta - Cl(\{x_\alpha\}) = S_{ft}g*\beta - Ker(\{x_\alpha\})$, for all $x_\alpha \in \tilde{X}$.

Proof $(1) \Rightarrow (2)$: Suppose that $(\tilde{X}, \tilde{\tau}, E)$ is a $S_{ft}g*\beta - R_0$ space. By Theorem 3.8 $S_{ft}g*\beta - Cl(\{x_\alpha\}) \subseteq S_{ft}g*\beta - Ker(\{x_\alpha\})$, for all $x_\alpha \in \tilde{X}$. Let $y_\beta \in S_{ft}g*\beta - Ker(\{x_\alpha\})$. Then $x_\alpha \in S_{ft}g*\beta - Cl(\{y_\beta\})$, and by Theorem 3.4 $S_{ft}g*\beta - Cl(\{x_\alpha\}) = S_{ft}g*\beta - Cl(\{y_\beta\})$. Therefore $y_\beta \in S_{ft}g*\beta - Cl(\{x_\alpha\})$ and hence $S_{ft}g*\beta - Ker(\{x_\alpha\}) \subseteq S_{ft}g*\beta - Cl(\{x_\alpha\})$, This shows that $S_{ft}g*\beta - Cl(\{x_\alpha\}) = S_{ft}g*\beta - Ker(\{x_\alpha\})$, for all $x_\alpha \in \tilde{X}$.

$(2) \Rightarrow (1)$: Follows from Theorem 3.8.

Theorem 3.10. Let $(\tilde{X}, \tilde{\tau}, E)$ be a $S_{ft} - Top - Space$. Then the following statements are equivalent:

(1). $(\tilde{X}, \tilde{\tau}, E)$ is a $S_{ft}g*\beta - R_0$ space.

(2). $x_\alpha \in S_{ft}g*\beta - Cl(\{y_\beta\})$ if and only if $y_\beta \in S_{ft}g*\beta - Cl(\{x_\alpha\})$ for any two distinct soft points $x_\alpha, y_\beta \in \tilde{X}$.

Proof. $(1) \Rightarrow (2)$: Assume that $(\tilde{X}, \tilde{\tau}, E)$ is a $S_{ft}g*\beta - R_0$ space. Let $x_\alpha \in S_{ft}g*\beta - Cl(\{y_\beta\})$ and (H,E) be any $S_{ft}g*\beta$-open set containing y_β. By (1) $S_{ft}g*\beta - Cl(\{y_\beta\}) \subseteq (H,E)$, hence $x_\alpha \in (H,E)$. Therefore, every $S_{ft}g*\beta$ *-open* set containing y_β contains x_α, so $y_\beta \in S_{ft}g*\beta - Cl(\{x_\alpha\})$.

$(2) \Rightarrow (1)$: Let (G,E) be any $S_{ft}g*\beta$ *-open* set containing x_α, if $y_\beta \notin (G,E)$, then $x_\alpha \notin S_{ft}g*\beta - Cl(\{y_\beta\})$ and by (2), we have $y_\beta \notin S_{ft}g*\beta - Cl(\{x_\alpha\})$. This implies that $S_{ft}g*\beta - Cl(\{x_\alpha\}) \subseteq (G,E)$, hence $(\tilde{X}, \tilde{\tau}, E)$ is a $S_{ft}g*\beta - R_0$ space.

Theorem 3.11. A $S_{ft} - Top - Space$ $(\tilde{X}, \tilde{\tau}, E)$ is a $S_{ft}g*\beta - R_0$ space if and only if $S_{ft}g*\beta - Ker(\{x_\alpha\}) \neq S_{ft}g*\beta - Ker(\{y_\beta\})$, for all $x_\alpha \neq y_\beta$.

Proof. Follows from Theorem 3.9 and Theorem 3.10.

Lemma 3.12. Let $(\tilde{X},\tilde{\tau},E)$ be a $S_{ft}-\text{Top}-\text{Space}$ and $(F,E)\subseteq\tilde{X}$.
Then $S_{ft}g*\beta-Ker(F,E)=\{x_\alpha\in\tilde{X}:S_{ft}g*\beta-Cl(\{x_\alpha\})\cap(F,E)\neq\tilde{\phi}\}$.

Proof. Let $x_\alpha\in S_{ft}g*\beta-Ker(F,E)$ and suppose $[S_{ft}g*\beta-Cl(\{x_\alpha\})]\cap(F,E)=\tilde{\phi}$. Hence $x_\alpha\notin\tilde{X}-[S_{ft}g*\beta-Cl(\{x_\alpha\})]$ which is a $S_{ft}g*\beta-$open set containing (F,E) and this is impossible, since $x_\alpha\in S_{ft}g*\beta-Ker(F,E)$, hence $S_{ft}g*\beta-Cl(\{x_\alpha\})\cap(F,E)\neq\tilde{\phi}$. Again let $x_\alpha\in\tilde{X}$ such that $[S_{ft}g*\beta-Cl(\{x_\alpha\})]\cap(F,E)\neq\tilde{\phi}$ and suppose that $x_\alpha\notin S_{ft}g*\beta-Ker(F,E)$. Then there exists a $S_{ft}g*\beta-$*open* set (G,E), such that $x_\alpha\notin(G,E)$ and $(F,E)\subseteq(G,E)$. Let $y_\beta\in[S_{ft}g*\beta-Cl(\{x_\alpha\})]\cap(F,E)$. Hence (G,E) is a $S_{ft}g*\beta-$*open* neighborhood of y_β which does not contain x_α. This contradicts that $x_\alpha\in S_{ft}g*\beta-Ker(F,E)$ so the claim.

Theorem 3.13. Let $(\tilde{X},\tilde{\tau},E)$ be a $S_{ft}-\text{Top}-\text{Space}$. Then the following statements are equivalent:
(1). $(\tilde{X},\tilde{\tau},E)$ is a $S_{ft}g*\beta-R_0$ space.
(2). If (K,E) is $S_{ft}g*\beta-$*closed*, then $(K,E)=S_{ft}g*\beta-Ker(K,E)$.
(3). If (K,E) is $S_{ft}g*\beta-$*closed*, and $x_\alpha\in(K,E)$, then $S_{ft}g*\beta-Ker(\{x_\alpha\})\subseteq(K,E)$.
(4). If $x_\alpha\in\tilde{X}$, then $S_{ft}g*\beta-Ker(\{x_\alpha\})\subseteq S_{ft}g*\beta-Cl(\{x_\alpha\})$.

Proof. $(1)\Rightarrow(2)$. Let (K,E) be a $S_{ft}g*\beta-$closed set and $x_\alpha\notin(K,E)$. Thus $\tilde{X}-(K,E)$ is $S_{ft}g*\beta-$open set containing x_α. Since $\tilde{X}$ is a $S_{ft}g*\beta-R_0$ space, so $S_{ft}g*\beta-Cl(\{x_\alpha\})\subseteq\tilde{X}-(K,E)$, thus $[S_{ft}g*\beta-Cl(\{x_\alpha\})]\cap(K,E)=\tilde{\phi}$, by Lemma 3.12 $x_\alpha\notin S_{ft}g*\beta-Ker(K,E)$. Therefore $S_{ft}g*\beta-Ker(\{x_\alpha\})\subseteq(K,E)$, hence $(K,E)=S_{ft}g*\beta-Ker((K,E))$.
$(2)\Rightarrow(3)$. In general, $(F,E)\subseteq(G,E)$ implies that $S_{ft}g*\beta-Ker((F,E))\subseteq S_{ft}g*\beta-Ker((G,E))$. Therefore, it follows from (2) that $S_{ft}g*\beta-Ker(\{x_\alpha\})\subseteq S_{ft}g*\beta-Ker((K,E))=(K,E)$.
$(3)\Rightarrow(4)$. Since $x_\alpha\in S_{ft}g*\beta-Cl(\{x_\alpha\})$ and $S_{ft}g*\beta-Cl(\{x_\alpha\})$ is $S_{ft}g*\beta-$closed, so by (3), we obtain $S_{ft}g*\beta-Ker(\{x_\alpha\})\subseteq S_{ft}g*\beta-Cl(\{x_\alpha\})$.
$(4)\Rightarrow(1)$. Let $x_\alpha\in S_{ft}g*\beta-Cl(\{y_\beta\})$, then by Theorem 3.3 $y_\beta\in S_{ft}g*\beta-Ker(\{x_\alpha\})$. $x_\alpha\in S_{ft}g*\beta-Cl(\{x_\alpha\})$ and $S_{ft}g*\beta-Cl(\{x_\alpha\})$ is $S_{ft}g*\beta-$closed. So by (4) we obtain $y_\beta\in S_{ft}g*\beta-Ker(\{x_\alpha\})\subseteq S_{ft}g*\beta-Cl(\{x_\alpha\})$. Therefore $x_\alpha\in S_{ft}g*\beta-Cl(\{y_\beta\})$ implies that $y_\beta\in S_{ft}g*\beta-Cl(\{x_\alpha\})$, on the same way, if $y_\beta\in S_{ft}g*\beta-Cl(\{x_\alpha\})$, we get $x_\alpha\in S_{ft}g*\beta-Cl(\{y_\beta\})$, so by Theorem 3.10 $(\tilde{X},\tilde{\tau},E)$ is a $S_{ft}g*\beta-R_0$ space.

Definition 3.14. A soft filter base $\Im$ is called $S_{ft}g*\beta-$*convergent* to a point x_α in $\tilde{X}$, if for any $S_{ft}g*\beta-$*open* set (H,E) of $\tilde{X}$ containing x_α there exists (G,E) in $\Im$ such that $(G,E)\subseteq(H,E)$.

Lemma 3.15. Let $(\tilde{X},\tilde{\tau},E)$ be a S_{ft}-Top-Space and let x_α and y_β be any two soft points in $\tilde{X}$ such that every net $\{x_{\alpha_i} : i \in \Lambda\}$ in $\tilde{X}$ $S_{ft}g*\beta$-*converging* to y_β $S_{ft}g*\beta$-*converges* to x_α. Then $x_\alpha \in S_{ft}g*\beta-Cl(\{y_\beta\})$.

Proof. Suppose that $x_\alpha = y_\beta$ for each $i \in \Lambda$. Then $\{x_{\alpha_i} : i \in \Lambda\}$ is a net in $S_{ft}g*\beta-Cl(\{y_\beta\})$. By the fact that $\{x_{\alpha_i} : i \in \Lambda\}$ $S_{ft}g*\beta$-*converges* to y_β, then $\{x_{\alpha_i} : i \in \Lambda\}$ $S_{ft}g*\beta$-*converges* to x_α and this means that $x_\alpha \in S_{ft}g*\beta-Cl(\{y_\beta\})$.

Theorem 3.16. Let $(\tilde{X},\tilde{\tau},E)$ be a S_{ft}-Top-Space. Then the following statements are equivalent:

(1). $(\tilde{X},\tilde{\tau},E)$ is $S_{ft}g*\beta-R_0$.

(2). If x_α, $y_\beta \in \tilde{X}$, then $y_\beta \in S_{ft}g*\beta-Cl(\{x_\alpha\})$ if and only if every net in $\tilde{X}$ $S_{ft}g*\beta$-*converging* to y_β $S_{ft}g*\beta$-*converges* to x_α.

Proof $(1)\Rightarrow(2)$. Let x_α, y_β $\in \tilde{X}$ such that $y_\beta \in S_{ft}g*\beta-Cl(\{x_\alpha\})$. Let $\{x_{\alpha_i} : i \in \Lambda\}$ be a net in $\tilde{X}$ such that $\{x_{\alpha_i} : i \in \Lambda\}$ $S_{ft}g*\beta$-*converges* to y_β. Since $y_\beta \in S_{ft}g*\beta-Cl(\{x_\alpha\})$, by Theorem 3.5 we have $S_{ft}g*\beta-Cl(\{x_\alpha\}) = S_{ft}g*\beta-Cl(\{y_\beta\})$. Therefore $x_\alpha \in S_{ft}g*\beta-Cl(\{y_\alpha\})$, This means that $\{x_{\alpha_i} : i \in \Lambda\}$ $S_{ft}g*\beta$-*converges* to x_α.

Conversely, let x_α, y_β $\in \tilde{X}$ such that every net in $\tilde{X}$ $S_{ft}g*\beta$-*converging* to y_β $S_{ft}g*\beta$-*converges* to x_α. Then $x_\alpha \in S_{ft}g*\beta-Cl(\{y_\beta\})$ by Lemma 3.12. By Theorem 3.6, we have $S_{ft}g*\beta-Cl(\{x_\alpha\}) = S_{ft}g*\beta-Cl(\{y_\beta\})$. Therefore $y_\beta \in S_{ft}g*\beta-Cl(\{x_\alpha\})$.

$(2)\Rightarrow(1)$. Assume that x_α and y_β are any two distinct soft points of $\tilde{X}$ such that $S_{ft}g*\beta-Cl(\{x_\alpha\}) \cap S_{ft}g*\beta-Cl(\{y_\beta\}) \neq \tilde{\phi}$. Let $z_\gamma \in S_{ft}g*\beta-Cl(\{x_\alpha\}) \cap S_{ft}g*\beta-Cl(\{y_\beta\})$. So there exists a net $\{x_{\alpha_i} : i \in \Lambda\}$ in $S_{ft}g*\beta-Cl(\{x_\alpha\})$ such that $\{x_{\alpha_i} : i \in \Lambda\}$ $S_{ft}g*\beta$-*converges* to z_γ. Since $z_\gamma \in S_{ft}g*\beta-Cl(\{y_\beta\})$ then $\{x_{\alpha_i} : i \in \Lambda\}$ $S_{ft}g*\beta$-*converges* to y_β. It follows that $y_\beta \in S_{ft}g*\beta-Cl(\{x_\alpha\})$. By the similarity we obtain $x_\alpha \in S_{ft}g*\beta-Cl(\{y_\beta\})$. Therefore $S_{ft}g*\beta-Cl(\{x_\alpha\}) = S_{ft}g*\beta-Cl(\{y_\beta\})$ and by Theorem 3.6 $(\tilde{X},\tilde{\tau},E)$ is a $S_{ft}g*\beta-R_0$ space.

Definition 3.17. A S_{ft}-Top-Space $(\tilde{X},\tilde{\tau},E)$ is called $S_{ft}g*\beta-R_1$ if for any two distinct soft points x_α, $y_\beta \in \tilde{X}$ with $S_{ft}g*\beta-Cl(\{x_\alpha\}) \neq S_{ft}g*\beta-Cl(\{y_\beta\})$ there exist disjoint $S_{ft}g*\beta$-open sets (F,E) and (G,E) such that $S_{ft}g*\beta-Cl(\{x_\alpha\}) \subseteq (F,E)$ and $S_{ft}g*\beta-Cl(\{y_\beta\}) \subseteq (G,E)$.

Theorem 3.18. Suppose that a S_{ft}-Top-Space $(\tilde{X},\tilde{\tau},E)$ is $S_{ft}g*\beta-R_1$, then it is a $S_{ft}g*\beta-R_0$ space.

Proof. Suppose that $(\tilde{X},\tilde{\tau},E)$ is $S_{ft}g*\beta-R_1$. Let (H,E) be any $S_{ft}g*\beta$-*open* set containing a soft point x_α. Then for each $y_\beta \in \tilde{X}-(H,E)$, $S_{ft}g*\beta-Cl(\{x_\alpha\}) \neq S_{ft}g*\beta-Cl(\{y_\beta\})$. Since $(\tilde{X},\tilde{\tau},E)$ is

$S_{ft}g*\beta-R_1$, there exist disjoint $S_{ft}g*\beta-$open sets (K,E) and (G,E) such that $S_{ft}g*\beta-Cl(\{\mathbf{x}_\alpha\})\subseteq(K,E)$ and $S_{ft}g*\beta-Cl(\{y_\beta\})\subseteq(G,E)$. Let $(F,E)=\bigcup\{(G,E): y_\beta\in\tilde{X}-(H,E)\}$, then $\tilde{X}-(H,E)\subseteq(F,E)$, $x_\alpha\notin(F,E)$ and (F,E) is a $S_{ft}S_{emi}{}^{\#}g\alpha-open$ set. Therefore, $S_{ft}g*\beta-Cl(\{\mathbf{x}_\alpha\})\subseteq\tilde{X}-(F,E)\subseteq(H,E)$. Hence (X,τ,E) is $S_{ft}g*\beta-R_0$.

Theorem 3.19. A space $\tilde{X}$ is $S_{ft}g*\beta-R_0$ if and only if for every $S_{ft}g*\beta-closed$ set (K,E) and $x_\alpha\notin(K,E)$, there exists a $S_{ft}g*\beta-open$ open set (G,E) such that $x_\alpha\notin(G,E)$ and $(K,E)\subseteq(G,E)$.

Proof. Let $\tilde{X}$ be $S_{ft}g*\beta-R_0$ space and (K,E) be $S_{ft}g*\beta-closed$ subset of $\tilde{X}$ not containing $x_\alpha\in\tilde{X}$. Then $\tilde{X}-(K,E)$ is $S_{ft}g*\beta-open$ set containing x_α. Since $\tilde{X}$ is $S_{ft}g*\beta-R_0$ space implies that $S_{ft}g*\beta-Cl(\{x_\alpha\})\subseteq\tilde{X}-(K,E)$ and then $(K,E)\subseteq\tilde{X}-[S_{ft}g*\beta-Cl(\{x_\alpha\})]$. Now let $(G,E)=\tilde{X}-[S_{ft}g*\beta-Cl(\{x_\alpha\})]$, then (G,E) is $S_{ft}g*\beta-open$ set not containing x_α and $(K,E)\subseteq(G,E)$.

Conversely: Let $x_\alpha\in(G,E)$ where (G,E) is $S_{ft}g*\beta-open$ set in $\tilde{X}$. Then $\tilde{X}-(G,E)$ is $S_{ft}g*\beta-closed$ set and $x_\alpha\notin\tilde{X}-(G,E)$, by hypothesis there exists a $S_{ft}g*\beta-open$ set (H,E) such that $x_\alpha\notin(H,E)$ and $\tilde{X}-(G,E)\subseteq(H,E)$. Now $\tilde{X}-(H,E)\subseteq(G,E)$ and $x_\alpha\in\tilde{X}-(H,E)$, but $\tilde{X}-(H,E)$ is $S_{ft}g*\beta-closed$ set, then $S_{ft}g*\beta-Cl(\{x_\alpha\})\subseteq\tilde{X}-(H,E)\subseteq(G,E)$. This implies that $\tilde{X}$ is a $S_{ft}g*\beta-R_0$ space.

4. $S_{ft}g*\beta-T_i$ Spaces for $(i=0,1,2)$

In this section, we define $S_{ft}g*\beta-T_i$ spaces for $(i=0,1,2)$ by using $S_{ft}g*\beta-$open and separating the $S_{ft}-$Top$-$Space. Several relations between these $S_{ft}-$Spaces and other types of soft separation axioms are investigated.

Definition 4.1. A $S_{ft}-$Top$-$Space $(\tilde{X},\tilde{\tau},E)$ is said to be $S_{ft}g*\beta-T_0$ if for each pair of distinct soft points $x_\alpha, y_\beta\in\tilde{X}$, there exist $S_{ft}g*\beta-$open sets (F,E) and (G,E) such that $x_\alpha\in(F,E)$ and $y_\beta\notin(F,E)$ or $y_\beta\in(G,E)$ and $x_\alpha\notin(G,E)$.

Definition 4.2. A $S_{ft}-$Top$-$Space $(\tilde{X},\tilde{\tau},E)$ is said to be $S_{ft}g*\beta-T_1$ if for each pair of distinct soft points $x_\alpha, y_\beta\in\tilde{X}$, there exist $S_{ft}g*\beta-$open sets (F,E) and (G,E) such that $x_\alpha\in(F,E)$ but $y_\beta\notin(F,E)$ and $y_\beta\in(G,E)$ but $x_\alpha\notin(G,E)$.

Definition 4.3. A S_{ft} $-$Top $-$Space $(\tilde{X},\tilde{\tau},E)$ is said to be $S_{ft}g*\beta-T_2$ if for each pair of distinct soft points $x_\alpha, y_\beta \in \tilde{X}$, there exist two disjoint $S_{ft}g*\beta-$open sets (F,E) and (G,E) containing x_α and y_β, respectively.

Proposition 4.4. A S_{ft} $-$Top $-$Space $(\tilde{X},\tilde{\tau},E)$ is $S_{ft}g*\beta-T_0$ if and only if $S_{ft}g*\beta-$closure of any two soft points is distinct.

Proof. Let $(\tilde{X},\tilde{\tau},E)$ be a $S_{ft}g*\beta-T_0$ space and $x_\alpha, y_\beta \in \tilde{X}$ with $x_\alpha \neq y_\beta$. Then, there exists a $S_{ft}g*\beta-$open set (F,E) containing one of the soft points, say x_α, but not the other. Then, $\tilde{X}-(F,E)$ is a $S_{ft}g*\beta-$closed set which does not contain x_α but contains y_β. Since, $S_{ft}g*\beta-Cl(\{y_\beta\})$ is the smallest $S_{ft}g*\beta-$*closed* set containing y_β, $S_{ft}g*\beta-Cl(\{y_\beta\}) \subseteq \tilde{X}-(F,E)$ and therefore $x_\alpha \notin S_{ft}g*\beta-Cl(\{y_\beta\})$. Consequently, $S_{ft}g*\beta-Cl(\{x_\alpha\}) \neq S_{ft}g*\beta-Cl(\{y_\beta\})$.

Conversely, suppose that $x_\alpha, y_\beta \in \tilde{X}$ is such that $x_\alpha \neq y_\beta$ and $S_{ft}g*\beta-Cl(\{x_\alpha\}) \neq S_{ft}g*\beta-Cl(\{y_\beta\})$. Let z_γ be a soft point in $\tilde{X}$ such that $z_\gamma \in S_{ft}g*\beta-Cl(\{x_\alpha\})$, but $z_\gamma \notin S_{ft}g*\beta-Cl(\{y_\beta\})$. We claim that $x_\alpha \in S_{ft}g*\beta-Cl(\{y_\beta\})$. For, if $x_\beta \notin S_{ft}g*\beta-Cl(\{y_\beta\})$, then $S_{ft}g*\beta-Cl(\{x_\alpha\}) \subseteq S_{ft}g*\beta-Cl(\{y_\beta\})$. This contradicts the fact that $z_\gamma \notin S_{ft}g*\beta-Cl(\{y_\beta\})$. Consequently, x_α belongs to the $S_{ft}g*\beta-$open set $(G,E) = \tilde{X} - [S_{ft}g*\beta-Cl(\{y_\beta\})]$. Then (G,E) is the complement of $S_{ft}g*\beta-$closed set. Thus (G,E) is a $S_{ft}g*\beta-$open set which contains x_α but not y_β. Hence, $(\tilde{X},\tilde{\tau},E)$ is $S_{ft}g*\beta-T_0$ space.

Proposition 4.5. If $(\tilde{X},\tilde{\tau},E)$ is $S_{ft}g*\beta-T_0$, space, then $[S_{ft}g*\beta-Cl(\{x_\alpha\})] \cap [S_{ft}g*\beta-Cl(\{y_\beta\})] = \tilde{\phi}$, for each pair of distinct soft points $x_\alpha, y_\beta \in \tilde{X}$.

Proof. Let $(\tilde{X},\tilde{\tau},E)$ be a $S_{ft}g*\beta-T_0$ space and $x_\alpha, y_\beta \in \tilde{X}$ such that $x_\alpha \neq y_\beta$. Then, there exists a $S_{ft}g*\beta-$open set (F,E) containing x_α or y_β, which implies that $x_\alpha \in (F,E)$ and $y_\beta \notin (F,E)$, then $y_\beta \in \tilde{X}-(F,E)$ and $\tilde{X}-(F,E)$ is $S_{ft}g*\beta-$closed. Now $S_{ft}g*\beta-Int(\{y_\beta\}) \subseteq S_{ft}g*\beta-Cl[S_{ft}g*\beta-Int(\{y_\beta\})] \subseteq \tilde{X}-(F,E)$, which implies that $(F,E) \cap S_{ft}g*\beta-Cl[S_{ft}g*\beta-Int(\{y_\beta\})] = \tilde{\phi}$, then $(F,E) \subseteq \tilde{X} - S_{ft}g*\beta-Cl[S_{ft}g*\beta-Int(\{y_\beta\})]$. Since $x_\alpha \in (F,E) \subseteq \tilde{X} - S_{ft}g*\beta-Cl[S_{ft}g*\beta-Int(\{y_\beta\})]$, then $S_{ft}g*\beta-Cl(\{x_\alpha\}) \subseteq \tilde{X} - S_{ft}g*\beta-Cl[S_{ft}g*\beta-Int(\{y_\beta\})]$, which implies that $S_{ft}g*\beta-Cl[S_{ft}g*\beta-Int(\{x_\alpha\})] \subseteq S_{ft}g*\beta-Cl(\{x_\alpha\}) \subseteq \tilde{X} - S_{ft}g*\beta-Cl[S_{ft}g*\beta-Int(\{y_\beta\})]$. Thereore, $S_{ft}g*\beta-Cl(\{x_\alpha\}) \cap S_{ft}g*\beta-Cl(\{y_\beta\}) = \tilde{\phi}$.

Theorem 4.6. A $S_{ft}-\text{Top}-\text{Space}$ $(\tilde{X},\tilde{\tau},E)$ is a $S_{ft}g*\beta-T_1$ space if and only if for each $x_\alpha \in \tilde{X}$, every soft singleton $\{x_\alpha\}$ is a $S_{ft}g*\beta-\text{closed}$ set.

Proof. Necessity. Suppose that $(\tilde{X},\tilde{\tau},E)$ is $S_{ft}g*\beta-T_1$ space and $x_\alpha \in \tilde{X}$. We have to prove that the soft singleton $\{x_\alpha\}$ is a $S_{ft}g*\beta-\text{closed}$ set. Since $(\tilde{X},\tilde{\tau},E)$ is a $S_{ft}g*\beta-T_1$ space, for each soft point $y_\beta \in \tilde{X}-\{x_\alpha\}$ we can find a $S_{ft}g*\beta-\text{open}$ set (F,E) of y_β such that $x_\alpha \notin (F,E)$. The union of all these $S_{ft}g*\beta-\text{open}$ sets is a $S_{ft}g*\beta-\text{open}$ set and it is the complement of $\{x_\alpha\}$ in $\tilde{X}$. Hence $\{x_\alpha\}$ is a $S_{ft}g*\beta-\text{closed}$ set.

Sufficiency. Suppose that $x_\alpha \in \tilde{X}$, every soft singleton $\{x_\alpha\}$ is a $S_{ft}g*\beta-\text{closed}$ set in $(\tilde{X},\tilde{\tau},E)$. We have to prove that $(\tilde{X},\tilde{\tau},E)$ is a $S_{ft}g*\beta-T_1$ space. Let $x_\alpha, y_\beta \in \tilde{X}$ and $x_\alpha \neq y_\beta$ such that $\{x_\alpha\}$ and $\{y_\beta\}$ are $S_{ft}g*\beta-\text{closed}$ and hence $\tilde{X}-\{x_\alpha\}$ and $\tilde{X}-\{y_\beta\}$ are $S_{ft}g*\beta-\text{open}$ sets. Therefore, $y_\beta \in \tilde{X}-\{x_\alpha\}$ but $x_\alpha \notin \tilde{X}-\{x_\alpha\}$ and $x_\alpha \in \tilde{X}-\{y_\beta\}$ but $y_\beta \notin \tilde{X}-\{y_\beta\}$. Thus (X,τ,E) is a $S_{ft}g*\beta-T_1$ space.

Theorem 4.7. Let $(\tilde{X},\tilde{\tau},E)$ be a $S_{ft}-\text{Top}-\text{Space}$. Then following statements are equivalent:

(i) $(\tilde{X},\tilde{\tau},E)$ is a $S_{ft}g*\beta-T_1$ space.

(ii) Each $S_{ft}-\text{subset}$ of $\tilde{X}$ is the soft intersection of all $S_{ft}g*\beta-\text{open}$ sets containing it.

(iii) The soft intersection of all $S_{ft}g*\beta-\text{open}$ sets containing the soft point $x_\alpha \in \tilde{X}$ is $\{x_\alpha\}$.

Proof. $(\text{i})\Rightarrow(\text{ii})$. Let $(\tilde{X},\tilde{\tau},E)$ be a $S_{ft}g*\beta-T_1$ space and (F,E) be a $S_{ft}-\text{subset}$ of $\tilde{X}$. Then for each $y_\beta \in \tilde{X}-(F,E)$ there exists a $S_{ft}-\text{set}$ $\tilde{X}-\{y_\beta\}$ such that $(F,E)\subseteq \tilde{X}-\{y_\beta\}$. By Theorem 4.6, the soft set $\tilde{X}-\{y_\beta\}$ is $S_{ft}g*\beta-\text{open}$ set for every y_β. This implies that $(F,E)=\bigcap\{\tilde{X}-\{y_\beta\}: y_\beta \in \tilde{X}-(F,E)\}$. So the soft intersection of all $S_{ft}g*\beta-\text{open}$ sets containing (F,E) is (F,E) itself.

$(\text{ii})\Rightarrow(\text{iii})$. Let $x_\alpha \in \tilde{X}$. Then $\{x_\alpha\}$ is a soft subset of $\tilde{X}$. By (ii), the soft intersection of all $S_{ft}g*\beta-\text{open}$ sets containing $\{x_\alpha\}$ is $\{x_\alpha\}$ itself.

$(\text{iii})\Rightarrow(\text{i})$. Let $x_\alpha, y_\beta \in \tilde{X}$ such that $x_\alpha \neq y_\beta$. By (iii), the soft intersection of all $S_{ft}g*\beta-\text{open}$ sets containing x_α and y_β are $\{x_\alpha\}$ and $\{y_\beta\}$ respectively. Then there exist $S_{ft}g*\beta-\text{open}$ sets (F,E) and (G,E) such that $x_\alpha \in (F,E)$ and $y_\beta \notin (F,E)$ and $x_\alpha \notin (G,E)$ and $y_\beta \in (G,E)$. Therefore, (X,τ,E) is a a $S_{ft}g*\beta-T_1$ space.

Theorem 4.8. Let $(\tilde{X},\tilde{\tau},E)$ be a $S_{ft}g*\beta-T_2$ space. Then the following statements are equivalent:

(i) $(\tilde{X},\tilde{\tau},E)$ is a $S_{ft}g*\beta-T_2$ space.

(ii) If $x_\alpha \in \tilde{X}$, then for each $y_\beta \neq x_\alpha$, there exists a $S_{ft}g*\beta-\text{open}$ set (F,A) containing x_α such that

$y_\beta \notin S_{ft}g*\beta-Cl(F,A)$.

(iii) $\{x_\alpha\}=\bigcap\{S_{ft}g*\beta-Cl(F,A): x_\alpha \in (F,A)$ and (F,A) is $S_{ft}g*\beta-$open set in $\tilde{X}\}$ for every $x_\alpha \in \tilde{X}$.

Proof. $(i)\Rightarrow(ii)$. Let $(\tilde{X},\tilde{\tau},E)$ be a $S_{ft}g*\beta-T_2$ space and x_α, y_β are any two distinct soft points of $\tilde{X}$. Then there exist disjoint $S_{ft}g*\beta-$open sets (F,A) and (G,B) such that $x_\alpha \in (F,A)$, $y_\beta \in (G,B)$, Then $(G,B)^C$ is a $S_{ft}g*\beta-$closed set such that $(F,A)\subseteq(G,B)^C$. By the definition of $S_{ft}g*\beta-$closure, $y_\beta \notin S_{ft}g*\beta-Cl(F,A)$, since $S_{ft}g*\beta-Cl(F,A)$ is the smallest $S_{ft}g*\beta-$closed set containing (F,A).

$(ii)\Rightarrow(iii)$. Suppose that $x_\alpha \in \tilde{X}$. By hypothesis, for any soft point $y_\beta(\neq x_\alpha)\in\tilde{X}$, there exists a $S_{ft}g*\beta-$open set (F,A) containing x_α such that $y_\beta \notin S_{ft}g*\beta-Cl(F,A)$. Therefore $y_\beta \notin \bigcap\{S_{ft}g*\beta-Cl(F,A): x_\alpha \in (F,A)\in S_{ft}g*\beta-O(\tilde{X})\}$. Thus consequently, we obtain $\{x_\alpha\}=\bigcap\{S_{ft}g*\beta-Cl(F,A): x_\alpha \in (F,A)\in S_{ft}g*\beta-O(\tilde{X})\}$.

$(iii)\Rightarrow(i)$. Suppose that x_α and y_β are any two distinct soft points in $\tilde{X}$. By hypothesis, there exists a $S_{ft}g*\beta-$open set (F,A) containing x_α such that $y_\beta \notin S_{ft}g*\beta-Cl(F,A)$. Let $(G,B)=\tilde{X}-S_{ft}g*\beta-Cl(F,A)$, then $y_\beta \in (G,B)$, $x_\alpha \in (F,A)$ and $(F,A)\bigcap(G,B)=\tilde{\phi}$. Therefore, (X,τ,E) is a $S_{ft}g*\beta-T_2$ space.

Proposition 4.9. Every $S_{ft}S_{emi}{}^{\#}g\alpha-T_1$ *space* $(\tilde{X},\tilde{\tau},E)$ is $S_{ft}g*\beta-R_0$.

Proof. Obvious.

Theorem 4.10. Let $(\tilde{X},\tilde{\tau},E)$ be a $S_{ft}-$Top$-$Space. Then the following implications are true:
(X,τ,E) is $S_{ft}g*\beta-T_2$ space $\Rightarrow (X,\tau,E)$ is $S_{ft}g*\beta-T_1$ space $\Rightarrow (X,\tau,E)$ is $S_{ft}g*\beta-T_0$ space.

Proof. $\underline{S_{ft}g*\beta-T_2 \text{ space} \Rightarrow S_{ft}g*\beta-T_1 \text{ space}}$: Let (X,τ,E) be a $S_{ft}g*\beta-T_2$ space. Then, for every $x_\alpha, y_\beta \in \tilde{X}$ and $y_\beta \neq x_\alpha$, there exist $S_{ft}g*\beta-$open sets (F,A) and (G,B) of x_α and y_β such that $(F,A)\bigcap(G,B)=\tilde{\phi}$. $x_\alpha \in (F,A) \Rightarrow x_\alpha \notin (G,B)$ as $(F,A)\bigcap(G,B)=\tilde{\phi}$. Similarly, $y_\beta \in (G,B)$. This implies $y_\beta \notin (F,A)$. Hence, $x_\alpha \in (F,A)$ but $y_\beta \notin (F,A)$ and $y_\beta \in (G,B)$ but $x_\alpha \notin (G,B)$. Therefore, $(\tilde{X},\tilde{\tau},E)$ is a $S_{ft}g*\beta-T_1$ space.

$\underline{S_{ft}g*\beta-T_1 \text{ space} \Rightarrow S_{ft}g*\beta-T_0 \text{ space}}$: Let (X,τ,E) be a $S_{ft}g*\beta-T_1$ space. Then for every $x_\alpha, y_\beta \in \tilde{X}$ with $x_\alpha \neq y_\beta$, there exist $S_{ft}g*\beta-$open sets (F,A) and (G,B) such that $x_\alpha \in (F,A)$ but $y_\beta \notin (F,A)$ and $y_\beta \in (G,B)$ but $x_\alpha \notin (G,B)$. Therefore, $(\tilde{X},\tilde{\tau},E)$ is a $S_{ft}g*\beta-T_0$ space.

Proposition 4.11. A $S_{ft}-$Top$-$Space $(\tilde{X},\tilde{\tau},E)$ is $S_{ft}g*\beta-T_1$ if and only if $\tilde{X}$ is both $S_{ft}g*\beta-T_0$ and $S_{ft}g*\beta-R_0$.

Proof. Necessity. Let $\tilde{X}$ be $S_{ft}g*\beta-T_1$, then by Proposition 3.19, $\tilde{X}$ is $S_{ft}g*\beta-R_0$ and since every $S_{ft}g*\beta-T_1$ is $S_{ft}g*\beta-T_0$ that completes the proof.

Sufficiency. Assume that $\tilde{X}$ is both $S_{ft}g*\beta-T_0$ and $S_{ft}g*\beta-R_0$. Let $x_\alpha, y_\beta \in \tilde{X}$ be any pair of distinct soft points, since $\tilde{X}$ is $S_{ft}g*\beta-T_0$, there exists a $S_{ft}g*\beta-open$ set (H,E) such that $x_\alpha \in (H,E)$ and $y_\beta \notin (H,E)$ or there exists a $S_{ft}g*\beta-open$ set (G,E) such that $y_\beta \in (G,E)$ and $x_\alpha \notin (G,E)$. Suppose that $x_\alpha \in (H,E)$ and $y_\beta \notin (H,E)$. Since $\tilde{X}$ is $S_{ft}g*\beta-R_0$, then $S_{ft}g*\beta-Cl\{x_\alpha\} \subseteq (H,E)$. As $y_\beta \notin (H,E)$ implies $y_\beta \notin S_{ft}g*\beta-Cl\{x_\alpha\}$. Hence $y_\beta \in (G,E) = \tilde{X} - [S_{ft}g*\beta-Cl\{x_\alpha\}]$ and it is clear that $x_\alpha \notin (G,E)$, this implies that there exist $S_{ft}g*\beta-open$ sets (G,E) and (H,E) containing x_α and y_β respectively such that $x_\alpha \notin (G,E)$ and $y_\beta \notin (H,E)$. Therefore $(\tilde{X},\tilde{\tau},E)$ is a $S_{ft}g*\beta-T_1$ space.

Theorem 4.12. A space $\tilde{X}$ is $S_{ft}g*\beta-T_2$ if and only if it is $S_{ft}g*\beta-R_1$ and $S_{ft}g*\beta-T_0$.

Proof. Let $\tilde{X}$ be $S_{ft}g*\beta-T_2$. Then from Theorem 4.10, $\tilde{X}$ is $S_{ft}g*\beta-T_0$ and to show $\tilde{X}$ is $S_{ft}g*\beta-R_1$ space, let $x_\alpha, y_\beta \in \tilde{X}$ such that $S_{ft}g*\beta-Cl\{x_\alpha\} \neq S_{ft}g*\beta-Cl\{y_\beta\}$ and since $\tilde{X}$ is $S_{ft}g*\beta-T_1$ space so by Theorem 4.6, every singleton set in $\tilde{X}$ is $S_{ft}g*\beta-closed$, this means $S_{ft}g*\beta-Cl(\{x_\alpha\}) = \{x_\alpha\}$ and $S_{ft}g*\beta-Cl(\{y_\beta\}) = \{y_\beta\}$ implies that $\{x_\alpha\} \neq \{y_\beta\}$ and since $\tilde{X}$ is $S_{ft}g*\beta-T_2$ space so there exist two disjoint $S_{ft}g*\beta-open$ sets (G,E) and (H,E) such that $x_\alpha \in (G,E)$ and $y_\beta \in (H,E)$ implies that $S_{ft}g*\beta-Cl(\{x_\alpha\}) \subseteq (G,E)$ and $S_{ft}g*\beta-Cl(\{y_\beta\}) \subseteq (H,E)$. Thus $\tilde{X}$ is a $S_{ft}g*\beta-R_1$ space.

Conversely, let $\tilde{X}$ be $S_{ft}g*\beta-R_1$ and $S_{ft}g*\beta-T_0$ space and $x_\alpha, y_\beta \in \tilde{X}$ such that $x_\alpha \neq y_\beta$. Now since $\tilde{X}$ is $S_{ft}g*\beta-T_0$ so by Definition 4.1 there exists a $S_{ft}g*\beta-open$ set (G,E) such that $x_\alpha \in (G,E)$ and $y_\beta \notin (G,E)$ or $y_\beta \in (G,E)$ and $x_\alpha \notin (G,E)$, take $x_\alpha \in (G,E)$ and $y_\beta \notin (G,E)$ implies that $(G,E) \cap \{y_\beta\} = \tilde{\phi}$, and then $S_{ft}g*\beta-Cl(\{y_\beta\})$. This implies that $S_{ft}g*\beta-Cl(\{x_\alpha\}) \neq S_{ft}g*\beta-Cl(\{y_\beta\})$ and since $\tilde{X}$ is $S_{ft}g*\beta-R_1$ so there exist two disjoint $S_{ft}g*\beta-open$ sets (G,E) and (H,E) such that $S_{ft}g*\beta-Cl(\{x_\alpha\}) \subseteq (G,E)$ and $S_{ft}g*\beta-Cl(\{y_\beta\}) \subseteq (H,E)$ implies that $x_\alpha \in (G,E)$ and $y_\beta \in (H,E)$. Thus $\tilde{X}$ is $S_{ft}g*\beta-T_2$.

Theorem 4.13. Let $(\tilde{X},\tilde{\tau},E)$ be a $S_{ft}g*\beta-T_2$ space. If for any $x_\alpha, y_\beta \in \tilde{X}$ such that $x_\alpha \neq y_\beta$, then there exist $S_{ft}g*\beta-$closed sets (F_1,E) and (F_2,E) such that $x_\alpha \in (F_1,E)$, $y_\beta \notin (F_1,E)$ and $x_\alpha \notin (F_2,E)$, $y_\beta \in (F_2,E)$, and $(F_1,E) \cup (F_2,E) = \tilde{X}$.

Proof. Since $(\tilde{X},\tilde{\tau},E)$ is a $S_{ft}g*\beta-T_2$ space and $x_\alpha, y_\beta \in \tilde{X}$ such that $x_\alpha \neq y_\beta$, there exist $S_{ft}g*\beta-$open sets (G_1,E) and (G_2,E) such that $x_\alpha \in (G_1,E)$ and $y_\beta \in (G_2,E)$ and $(G_1,E) \cap (G_2,E) = \tilde{\phi}$. Clearly $(G_1,E) \subseteq (G_2,E)^C$ and $(G_2,E) \subseteq (G_1,E)^C$. Hence $x_\alpha \in (G_2,E)^C$. Put

$(G_2,E)^C = (F_1,E)$. This gives $x_\alpha \in (F_1,E)$ and $y_\beta \notin (F_1,E)$. Also $y_\beta \in (G_1,E)^C$. Put $(G_1,E)^C = (F_2,E)$. Therefore $x_\alpha \in (F_1,E)$ and $y_\beta \in (F_2,E)$. Moreover $(F_1,E) \cup (F_2,E) = (G_2,E)^C \cup (G_1,E)^C = \tilde{X}$.

5. $S_{ft}g*\beta$ –Regular, $S_{ft}g*\beta$ –Normal Spaces and $S_{ft}g*\beta$ –T_i Spaces for $(i = 3,4)$

In this section, we redefine $S_{ft}g*\beta$ –regular spaces, $S_{ft}g*\beta$ –normal spaces and $S_{ft}g*\beta$ –T_i spaces for $(i = 3,4)$. We investigate properties as well as characterizations of these spaces in soft topological spaces.

Definition 5.1. Let $(\tilde{X},\tilde{\tau},E)$ be a S_{ft} –Top –Space. Let (F,A) be a $S_{ft}g*\beta$ –closed set and $x_\alpha \in \tilde{X}$ such that $x_\alpha \notin (F,A)$. If there exist $S_{ft}g*\beta$ –open sets (G,B) and (H,C) such that $x_\alpha \in (G,B)$, $(F,A) \subseteq (H,C)$ and $(G,B) \cap (H,C) = \tilde{\phi}$, then $(\tilde{X},\tilde{\tau},E)$ is called a $S_{ft}g*\beta$ –regular space.

Theorem 5.2. Let $(\tilde{X},\tilde{\tau},E)$ be a S_{ft} –Top –Space. Then the following statements are equivalent:

(i) $(\tilde{X},\tilde{\tau},E)$ is a $S_{ft}g*\beta$ –regular space.

(ii) For any $S_{ft}g*\beta$ –open set (F,A) in $(\tilde{X},\tilde{\tau},E)$ and $x_\alpha \in (F,A)$, there is a $S_{ft}g*\beta$ –open set (G,B) containing x_α such that $x_\alpha \in S_{ft}g*\beta\text{–}Cl(G,B) \subseteq (F,A)$.

(iii) Each soft point in $(\tilde{X},\tilde{\tau},E)$ has a $S_{ft}g*\beta$ –neighbourhood base consisting of $S_{ft}g*\beta$ –closed sets.

Proof. $(i) \Rightarrow (ii)$. Let (F,A) be a $S_{ft}g*\beta$ –open set in $(\tilde{X},\tilde{\tau},E)$ and $x_\alpha \in (F,A)$. Then $(F,A)^C$ is a $S_{ft}g*\beta$ –closed set such that $x_\alpha \notin (F,A)^C$. By the $S_{ft}g*\beta$ –regularity of $(\tilde{X},\tilde{\tau},E)$, there are $S_{ft}g*\beta$ –open sets (G,B), (H,C) such that $x_\alpha \in (H,B)$, $(F,A)^C \subseteq (H,C)$ and $(G,B) \cap (H,C) = \tilde{\phi}$. Clearly $(H,C)^C$ is a $S_{ft}g*\beta$ –closed set contained in (F,A). Thus $(G,B) \subseteq (H,C)^C \subseteq (F,A)$. This gives $S_{ft}g*\beta\text{–}Cl(G,B) \subseteq (H,C)^C \subseteq (F,A)$. Consequently, $x_\alpha \in (G,B) \subseteq S_{ft}g*\beta\text{–}Cl(G,B) \subseteq (F,A)$.

$(ii) \Rightarrow (iii)$. Let $x_\alpha \in \tilde{X}$ and $S_{ft}g*\beta$ –open set (F,A) in $(\tilde{X},\tilde{\tau},E)$ such that $x_\alpha \in (F,A)$. Then there is a is a $S_{ft}g*\beta$ –open set (G,B) containing x_α such that $x_\alpha \in S_{ft}g*\beta\text{–}Cl(G,B) \subseteq (F,A)$. Thus for each $x_\alpha \in \tilde{X}$, the sets $S_{ft}g*\beta\text{–}Cl(G,B)$ form a $S_{ft}g*\beta$ –neighbourhood base consisting of $S_{ft}g*\beta$ –closed sets of $(\tilde{X},\tilde{\tau},E)$.

$(iii) \Rightarrow (i)$. Let (F,A) be a $S_{ft}g*\beta$ –closed set such that $x_\alpha \notin (F,A)$. Then $(F,A)^C$ is a $S_{ft}g*\beta$ –open neighborhood of x_α. By (iii), there is a $S_{ft}g*\beta$ –closed set (G,B) which contains x_α and is a $S_{ft}g*\beta$ –neighbourhood of x_α with $(G,B) \subseteq (F,A)^C$. Then $x_\alpha \notin (G,B)^C$, $(F,A) \subseteq (G,B)^C = (H,C)$ and $(G,B) \cap (H,C) = \tilde{\phi}$. Therefore $(\tilde{X},\tilde{\tau},E)$ is $S_{ft}g*\beta$ –regular space.

Theorem 5.3. Prove that $S_{ft}-\text{Top}-\text{Space}\ (\tilde{X},\tilde{\tau},E)$ is $S_{ft}g*\beta-$regular if and only if for each $x_\alpha \in \tilde{X}$ and a $S_{ft}g*\beta-$closed set (F,A) in $(\tilde{X},\tilde{\tau},E)$ such that $x_\alpha \notin (F,A)$, there exist $S_{ft}g*\beta-$open sets (G,B), (H,C) in $(\tilde{X},\tilde{\tau},E)$ such that $x_\alpha \in (G,B)$ and $(F,A)\subseteq(H,C)$ and $S_{ft}g*\beta-\text{Cl}(G,B)\cap S_{ft}g*\beta-\text{Cl}(H,C)=\tilde{\phi}$.

Proof. For each $x_\alpha \in \tilde{X}$ and a $S_{ft}g*\beta-$closed set (F,A) such that $x_\alpha \notin (F,A)$, by Theorem 5.2 (ii), there is a $S_{ft}g*\beta-$open set (G,B) such that $x_\alpha \in (G,B)$, $S_{ft}g*\beta-\text{Cl}(G,B)\subseteq(F,A)^C$. Again by Theorem 5.2 (ii), there is a $S_{ft}g*\beta-$open set (H,C) containing x_α such that $S_{ft}g*\beta-\text{Cl}(H,C)\subseteq(G,B)$. Let $(M,K)=\left[S_{ft}g*\beta-open(G,B)\right]^C$. Then $S_{ft}g*\beta-\text{Cl}(H,C)\subseteq\ (G,B)\subseteq S_{ft}g*\beta-\text{Cl}(G,B)\subseteq(F,A)^C$ implies $(F,A)\subseteq\left[S_{ft}S_{emi}{}^{\#}g\alpha-\text{Cl}(G,B)\right]^C=\ (M,K)$ or $(F,A)\subseteq(M,K)$. Also

$S_{ft}g*\beta-\text{Cl}(H,C)\cap S_{ft}g*\beta-\text{Cl}(M,K)=$

$S_{ft}g*\beta-\text{Cl}(H,C)\cap S_{ft}g*\beta-\text{Cl}\left[\left(\left(S_{ft}g*\beta-\text{Cl}(G,B)\right)\right)^C\right]\subseteq(G,B)\cap S_{ft}g*\beta-\text{Cl}\left[\left(S_{ft}g*\beta-\text{Cl}(G,B)\right)^C\right]\subseteq$

$S_{ft}g*\beta-\text{Cl}\left[(G,B)\cap\left(S_{ft}g*\beta-\text{Cl}(G,B)\right)^C\right]=S_{ft}g*\beta-\text{Cl}(\tilde{\phi})=\tilde{\phi}.$

Thus (H,C) and (M,K) are the required $S_{ft}g*\beta-$open sets in $(\tilde{X},\tilde{\tau},E)$.
This proves the necessity. The sufficiency is immediate.

Definition 5.4. Let $(\tilde{X},\tilde{\tau},E)$ be a $S_{ft}-\text{Top}-\text{Space}$. Then $(\tilde{X},\tilde{\tau},E)$ is said to be a $S_{ft}g*\beta-T_3$ space, if it is a $S_{ft}g*\beta-$regular and a $S_{ft}g*\beta-T_1$ space.

Definition 5.5. Let $(\tilde{X},\tilde{\tau},E)$ be a $S_{ft}-\text{Top}-\text{Space}$, (F,A) and (G,B) be $S_{ft}g*\beta-$closed sets such that $(F,A)\cap(G,B)=\tilde{\phi}$. If there exist $S_{ft}g*\beta-$open sets (H,C) and (M,K) such that $(F,A)\subseteq(H,C)$, $(G,B)\subseteq(M,K)$ and $(H,C)\cap(M,K)=\tilde{\phi}$, then (X,τ,E) is called a $S_{ft}g*\beta-$normal space.

Definition 5.6. Let $(\tilde{X},\tilde{\tau},E)$ be a $S_{ft}-\text{Top}-\text{Space}$. Then $(\tilde{X},\tilde{\tau},E)$ is said to be a $S_{ft}g*\beta-T_4$ space, if it is a $S_{ft}S_{emi}{}^{\#}g\alpha-$normal space and a $S_{ft}g*\beta-T_1$ space.

Theorem 5.7. A $S_{ft}-\text{Top}-\text{Space}\ (\tilde{X},\tilde{\tau},E)$ is $S_{ft}g*\beta-$normal if and only if for any $S_{ft}g*\beta-$closed set (F,A) and a $S_{ft}g*\beta-$open set (G,B) such that $(F,A)\subseteq(G,B)$, there exists at least one $S_{ft}g*\beta-$open set (H,C) containing (F,A) such that $(F,A)\subseteq(H,C)\subseteq S_{ft}g*\beta-\text{Cl}(H,C)\subseteq(G,B)$.

Proof. Suppose that $(\tilde{X},\tilde{\tau},E)$ is a $S_{ft}g*\beta-$normal space and (F,A) is any $S_{ft}g*\beta-$closed subset of $(\tilde{X},\tilde{\tau},E)$ and (G,B) be a $S_{ft}g*\beta-$open set such that $(F,A)\subseteq(G,B)$. Then $(G,B)^C$ is $S_{ft}g*\beta-$closed and $(F,A)\cap(G,B)^C=\tilde{\phi}$. So by hypothesis, there are $S_{ft}g*\beta-$open sets (H,C) and (M,K) such that $(F,A)\subseteq(H,C)$, $(G,B)^C\subseteq(M,K)$ and $(H,C)\cap(M,K)=\tilde{\phi}$. Since $(H,C)\cap(M,K)=\tilde{\phi}$,

$(H,C)\subseteq(M,K)^C$. But $(M,K)^C$ is $S_{ft}g*\beta$-closed, so that $(F,A)\subseteq(H,C)\subseteq S_{ft}g*\beta\text{-}Cl(H,C)\subseteq(M,K)^C\subseteq(G,B)$. Hence $(F,A)\subseteq(H,C)\subseteq S_{ft}g*\beta\text{-}Cl(H,C)\subseteq(G,B)$.

Conversely, suppose that for every $S_{ft}g*\beta$-closed (F,A) and a $S_{ft}g*\beta$-open set (G,B) such that $(F,A)\subseteq(G,B)$, there is a $S_{ft}g*\beta$-open set (H,C) such that $(F,A)\subseteq(H,C)\subseteq S_{ft}g*\beta\text{-}Cl(H,C)\subseteq(G,B)$. Let (J,L) and (M,K) be any two disjoint $S_{ft}g*\beta$-closed sets. Then $(J,L)\subseteq(M,K)^C$, where $(M,K)^C$ is $S_{ft}g*\beta$-open set. Hence there is a $S_{ft}g*\beta$-open set (H,C) such that $(J,L)\subseteq(H,C)\subseteq S_{ft}g*\beta\text{-}Cl(H,C)\subseteq(M,K)^C$. But then $(M,K)\subseteq\left[S_{ft}g*\beta\text{-}Cl(H,C)\right]^C$ and $(H,C)\cap\left[S_{ft}g*\beta\text{-}Cl(H,C)\right]^C=\tilde{\phi}$. Hence $(J,L)\subseteq(H,C)$, $(M,K)\subseteq\left[S_{ft}g*\beta\text{-}Cl(H,C)\right]^C$ with $(H,C)\cap\left[S_{ft}g*\beta\text{-}Cl(H,C)\right]^C=\tilde{\phi}$. Hence $(\tilde{X},\tilde{\tau},E)$ is a $S_{ft}g*\beta$-normal space.

Theorem 5.8. Prove that a $S_{ft}g*\beta$-closed subspace $(\tilde{Y},\tilde{\tau}_Y,E)$ of a $S_{ft}g*\beta$-normal space $(\tilde{X},\tilde{\tau},E)$ is $S_{ft}g*\beta$-normal.

Proof. Let $(\tilde{Y},\tilde{\tau}_Y,E)$ be a $S_{ft}g*\beta$-closed subspace of a $S_{ft}g*\beta$-normal space $(\tilde{X},\tilde{\tau},E)$. Let (F,A) and (G,B) be a disjoint pair of $S_{ft}g*\beta$-closed subsets of $(\tilde{Y},\tilde{\tau}_Y,E)$. Since $(\tilde{Y},\tilde{\tau}_Y,E)$ is a $S_{ft}g*\beta$-closed subspace of $(\tilde{Y},\tilde{\tau}_Y,E)$. The intersection of two $S_{ft}g*\beta$-closed subsets of $(\tilde{X},\tilde{\tau},E)$ is $S_{ft}g*\beta$-closed. Therefore (F,A) and (G,B) are disjoint $S_{ft}g*\beta$-closed subsets of $(\tilde{Y},\tilde{\tau}_Y,E)$. By hypothesis, $(\tilde{X},\tilde{\tau},E)$ is a $S_{ft}g*\beta$-normal space. There exist (J,L) and (M,K) $S_{ft}g*\beta$-open subsets of $(\tilde{X},\tilde{\tau},E)$ such that $(F,A)\subseteq(J,L)$, $(G,B)\subseteq(M,K)$ and $(J,L)\cap(M,K)=\tilde{\phi}$. Let $(P,Q)=(J,L)\cap\tilde{Y}$ and $(S,T)=(M,K)\cap\tilde{Y}$. Then (P,Q) and (S,T) are $S_{ft}g*\beta$-open subsets of $(\tilde{Y},\tilde{\tau}_Y,E)$ such that $(F,A)\subseteq(P,Q)$, $(G,B)\subseteq(S,T)$ and $(P,Q)\cap(S,T)=\tilde{\phi}$. Thus $(\tilde{Y},\tilde{\tau}_Y,E)$ is a $S_{ft}g*\beta$-normal space.

Theorem 5.9. Let $f:(\tilde{X},\tilde{\tau},E)\to(\tilde{Y},\tilde{\sigma},E)$ be a surjective function which is both $S_{ft}g*\beta$-irresolute and $S_{ft}g*\beta$-open where $(\tilde{X},\tilde{\tau},E)$ and $(\tilde{Y},\tilde{\sigma},E)$ are S_{ft}-Top-Spaces. If $(\tilde{X},\tilde{\tau},E)$ is $S_{ft}g*\beta$-normal, then $(\tilde{Y},\tilde{\sigma},E)$ is also $S_{ft}g*\beta$-normal.

Proof. Let (F,A) and (G,B) be a disjoint pair of $S_{ft}g*\beta$-closed subsets of $(\tilde{Y},\tilde{\sigma},E)$. As f is $S_{ft}g*\beta$-irresolute $f^{-1}(F,A)$ and $f^{-1}(G,B)$ are disjoint $S_{ft}S_{emi}{}^{\#}g\alpha$-closed subsets of $(\tilde{X},\tilde{\tau},E)$. This implies there exist disjoint $S_{ft}g*\beta$-open sets (J,L) and (M,K) such that $f^{-1}(F,A)\subseteq(J,L)$ and $f^{-1}(G,B)\subseteq(M,K)$, since $S_{ft}g*\beta$-normal. Then $(F,A)\subseteq f(J,L)$ and $(G,B)\subseteq f(M,K)$. This implies $f(J,L)$ and $f(M,K)$ are disjoint $S_{ft}g*\beta$-open sets of $(\tilde{Y},\tilde{\sigma},E)$ containing (F,A) and (G,B) respectively. Condequently we conclude that $(\tilde{Y},\tilde{\sigma},E)$ is also $S_{ft}g*\beta$-normal.

Theorem 5.10. $S_{ft}g*\beta\text{-}T_4\Rightarrow S_{ft}g*\beta\text{-}T_3\Rightarrow S_{ft}g*\beta\text{-}T_2$.

Proof. $\underline{S_{ft}g*\beta-T_4 \Rightarrow S_{ft}g*\beta-T_3}$**:** Let $(\tilde{X},\tilde{\tau},E)$ be a $S_{ft}g*\beta-T_4$ space. Let x_α and (F,A) be a pair of soft points and $S_{ft}g*\beta-$closed subsets of $(\tilde{X},\tilde{\tau},E)$ such that $x_\alpha \notin (F,A)$. Then $S_{ft}g*\beta-Cl(\{x_\alpha\})$ is the $S_{ft}g*\beta-$closed set containing x_α. As $S_{ft}g*\beta-T_4$ space is $S_{ft}g*\beta-T_1$ space and hence every point in $(\tilde{X},\tilde{\tau},E)$ is $S_{ft}g*\beta-$closed. Hence $S_{ft}g*\beta-Cl(\{x_\alpha\})$ and (F,A) are disjoint $S_{ft}g*\beta-$closed sets of $(\tilde{X},\tilde{\tau},E)$. Then by the definition of $S_{ft}g*\beta-$normal there exist two disjoint $S_{ft}g*\beta-$open sets (J,L) and (M,K) such that $x_\alpha \in (J,L)$ and $(F,A) \subseteq (M,K)$. Therefore $(\tilde{X},\tilde{\tau},E)$ is a $S_{ft}g*\beta-T_3$ space.

$\underline{S_{ft}g*\beta-T_3 \Rightarrow S_{ft}g*\beta-T_2}$: Let $(\tilde{X},\tilde{\tau},E)$ be a $S_{ft}g*\beta-T_3$ space. Let x_α and y_β be two distinct soft points of $(\tilde{X},\tilde{\tau},E)$. Then by the definition of $S_{ft}g*\beta-T_1$ space $\{x_\alpha\}$ is a $S_{ft}g*\beta-$closed set. Hence, we have a soft point and a $S_{ft}g*\beta-$closed set such that $y_\beta \notin S_{ft}g*\beta-Cl(\{x_\alpha\})$. Hence there exist disjoint $S_{ft}g*\beta-$open sets (J,L) and (M,K) such that $x_\alpha \in (J,L)$ and $y_\beta \in (M,K)$. Consequently $(\tilde{X},\tilde{\tau},E)$ is a $S_{ft}g*\beta-T_2$ space.

Conclusion

Soft set theory is very important during the study towards possible applications in classical and nonclassical logic. In recent years many researchers worked on the findings of structures of soft sets theory initiated by Molodtsov and applied to many problems having uncertainties. It is worth mentioning that soft topological spaces based on soft set theory which is a collection of information granules is the mathematical formulation of approximate reasoning about information systems. In the last two decades the soft set theory, new definitions, examples, new classes of soft sets, and properties for mappings between different classes of soft sets are introduced and studied. After then, the theory of soft topological spaces is investigated. These soft separation axioms would be useful for the development of the theory of soft topology to solve the complicated problems containing uncertainties in economics, engineering, medical, environment and in general man-machine systems of various types. These findings are the addition for strengthening the toolbox of soft topology. Soft separation axioms are among the most widespread and important concepts in soft topology because they are utilized to classify the objects of study and to construct different families of soft topological spaces. We have introduced some soft separation axioms called $S_{ft}g*\beta-R_0$ space, $S_{ft}g*\beta-R_1$ space, $S_{ft}g*\beta-T_0$ space, $S_{ft}g*\beta-T_1$ space, $S_{ft}g*\beta-T_2$ space, $S_{ft}g*\beta-$regular space and $S_{ft}g*\beta-$normal space in soft topological spaces. We have investigated their properties and characterizations in soft topological spaces. In the future I plan to study $S_{ft}g*\beta-$compact space, $S_{ft}g*\beta-$Lindelof space, countably $S_{ft}g*\beta-$compact space, $S_{ft}g*\beta-$connected spaces as well as $S_{ft}g*\beta-$continuous function, $S_{ft}g*\beta-$open function, $S_{ft}g*\beta-$closed function and $S_{ft}g*\beta-$irresolute function in soft topological spaces. We hope that the concepts initiated herein will be beneficial for researchers and scholars to promote and progress the study of soft topology and decision-making problems with applications in many fields soon.

Scientific Ethics Declaration

The author declares that the scientific ethical and legal responsibility of this article published in EPSTEM journal belongs to the author.

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