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Investigation of Damping Energy in Trusses

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Abstract: Trusses are important load-bearing constructions. They are widely applied in various areas of contemporary engineering. Considering the elastic-plastic behaviour of trusses is an important step in developing the methods for analysing the mechanics of these constructions. A specific problem that appears when trusses of elastic-plastic behaviour are subjected to cyclic external loads is the damping energy. In the present paper, a general treatment of the damping problem in trusses with elastic-plastic behaviour is presented. The trusses investigated may have arbitrary geometry and external loading. First, damping in static determinate trusses is considered. A general methodology for deriving the damping energy is developed. Determination of the damping energy in statically indeterminate trusses with arbitrary number and location of the supports is also considered. The general treatment of the damping energy problem is applied to a particular static determinate truss configuration loaded by one external cyclic force. Then the damping energy in a statically indeterminate truss that has the same geometry and loading as that of the statically determinate one is analyzed. A parametric study of the damping energy is performed.

Keywords: Truss, Damping energy, Statically indeterminate structure, General treatment, Parametric investigation

Introduction

Truss constructions representing systems of rectilinear uniform bars rigidly connected at their ends are well known for their high efficiency for a long time (Huang, 1967; Aleksandrov & Potapov, 1990; Blake, 1985; Varvak, 1997). Trusses are frequently used especially for supporting heavy loads in various structural applications in civil engineering, mechanical engineering, and road and railway infrastructure (Aleksandrov & Potapov, 1990; Koltunov et al., 1983). Ensuring of safety and reliability of the truss constructions requires development and application of methods for analysis which take into account as much as possible aspects of their mechanical behaviour under various kinds of external loading (Collins, 1984; Lukash, 1997; Rabotnov, 1992; Zubchaninov, 1990). For instance, there is a variety of problems associated with analysing of truss constructions built-up by using engineering materials with elastic-plastic mechanical behaviour. When such trusses are subjected to cyclic external loadings, damping energy takes place. Thus, analyzing of damping energy in truss constructions is an up-to-date problem that needs special attention.

The present theoretical paper deals with the damping energy problem in trusses with elastic-plastic behaviour under cyclic loading (it should be mentioned that the damping energy analyses in the technical literature are concerned mainly with elastic-plastic beam structures (Collins, 1984; Dowling, 2007; Rizov, 2021). General treatment of the damping energy problem in trusses is presented here. Determination of the damping energy in statically determinate truss constructions of general configuration under arbitrary cyclic load is considered first. The problem of damping energy analysis in statically indeterminate trusses is also treated on general level. Practical applications of the general treatment are given. The damping energy in a particular statically determinate truss configuration is determined. Damping analysis is done also for a statically indeterminate truss having the same geometry and loading as the statically determinate one. Parametric investigation of the damping energy is carried-out to evaluate the influence of various parameters.

Damping Energy Studying

First, the problem of damping energy in statically determinate trusses with elastic-plastic behaviour under cyclic loading will be treated. We assume that the truss under consideration has an arbitrary number of bars. Besides, the bars have individual material properties, length and cross-section. The configuration of the truss is arbitrary. The truss is under cyclic external forces, F_{ai}, about zero mean. All external forces are applied at the joints of the truss. By considering the equilibrium of the joints, the forces, N_{ai}, in the bars of the truss can be written as

$$N_{ai} = \sum_{j=1}^{j=n} \alpha_{ij} F_{aj}, \tag{1}$$

where

$$i = 1, 2, ..., p. \tag{2}$$

In the above formulas, n is the number of external forces, p is the number of bars. The stresses, σ_{ai}, in the bars are

$$\sigma_{ai} = \frac{N_{ai}}{A_i}, \tag{3}$$

where A_i is the area of the bar cross-section. After calculation of the stresses in the bars, the damping energy, ΔU, in the truss is found as

$$\Delta U = \sum_{i=1}^{i=p} \Delta u_i l_i A_i, \tag{4}$$

where Δu_i is the unit damping energy, l_i is the length of the *i*-th bar, p is the number of bars. The problem of damping energy in statically indeterminate trusses with elastic-plastic behaviour is treated too. In this case, the forces in the bars of a truss can be expressed as

$$N_{ai} = \sum_{j=1}^{j=n} \alpha_{ij} F_{aj} + \sum_{k=1}^{k=m} \alpha_{ik} X_k, \tag{5}$$

where

$$i = 1, 2, ..., p. \tag{6}$$

In formula (5), X_k is the *k*-th hyperstatic unknown, m is the degree of the static indeterminacy of the truss under consideration. In addition to the static equilibrium equations of the truss, m equations can be written by using the theorem of Castigliano to resolve the static indeterminacy. The theorem Castigliano is written as

$$\frac{\partial U^*}{\partial X_k} = 0, \tag{7}$$

where

$$k = 1, 2, ..., m, \tag{8}$$

In formula (7), U^* is the complementary strain energy (it should be specified here that the complementary strain energy is used since the truss has elastic-plastic behaviour). Formula (9) is applied to derive the complementary strain energy, i.e.

$$U^* = \sum_{i=1}^{i=p} u_{0i}^* l_i A_i \ , \tag{9}$$

where u_{0i}^* is the unit complementary strain energy. The latter is defined by

$$u_{0i}^* = \sigma_{ai}\varepsilon_{ai} - \int_0^{\varepsilon_{ai}} \sigma_{ai} d\varepsilon_{ai} \ , \tag{10}$$

where ε_{ai} is the amplitude of the strain. It should be noted that stresses, σ_{ai}, which is involved in (10) is a continuous function of the amplitude of the strain representing the constitutive law, i.e.

$$\sigma_{ai} = \sigma_{ai}(\varepsilon_{ai}). \tag{11}$$

The equations written by applying (7) can be solved together with the static equilibrium equations by using the MatLab. After determination of forces in the bars, the stresses can be obtained by using dependence (3). Then the damping energy is calculated by (4).

Numerical Results

The general treatment of the damping energy problem in trusses with elastic-plastic behaviour presented in section 2 of this paper is applied here by using the following constitutive law (Dowling, 2007):

$$\varepsilon_{ai} = \frac{\sigma_{ai}}{E_i} + \left(\frac{\sigma_{ai}}{H_i}\right)^{\frac{1}{q_i}} \ , \tag{12}$$

where E_i is the modulus of elasticity, H_i, and q_i are material properties. The unit damping energies is given by Dowling (2007)

$$\Delta u_i = \frac{4(1-q_i)\sigma_{ai}^{1+\frac{1}{q_i}}}{(1+q_i)(H_i)^{\frac{1}{q_i}}} \ . \tag{13}$$

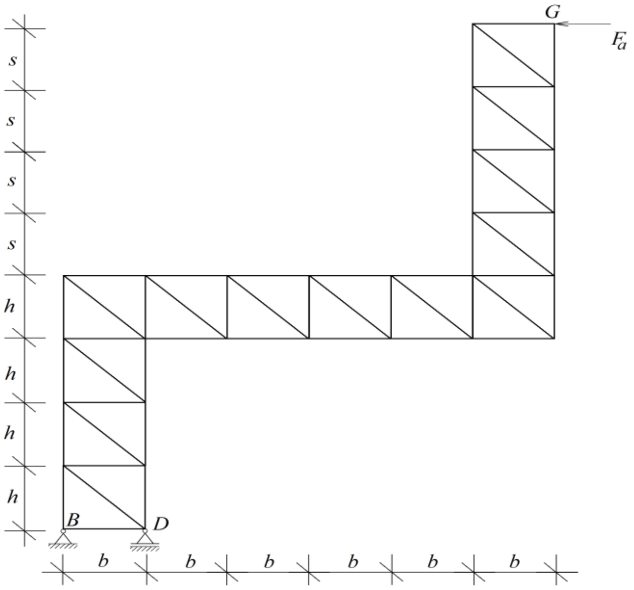

Figure 1. Statically determinate truss.

First, the damping energy in the statically determinate truss configuration shown in Fig. 1 is analyzed.

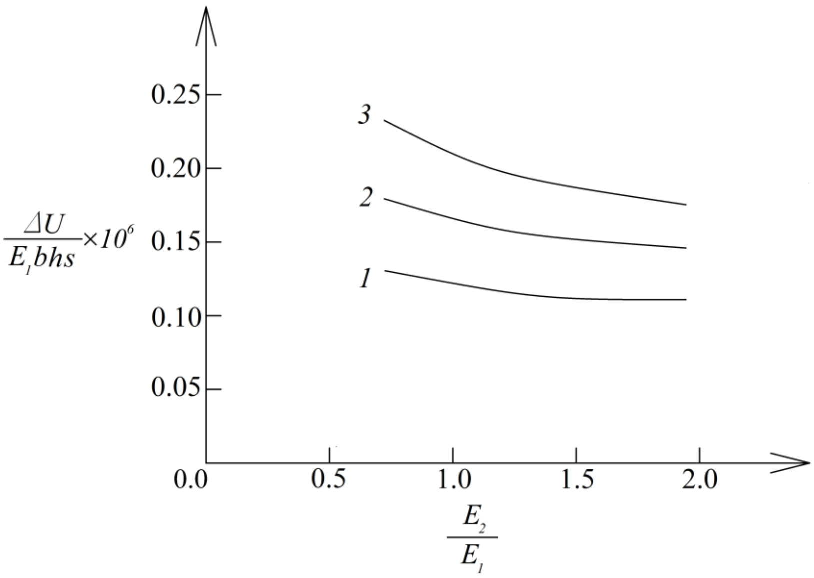

Figure 2. Change of damping energy with varying of E_2 / E_1 ratio (curve 1 – at $F_a = 2$ N, curve 2 – at $F_a = 3$ N and curve 3 – at $F_a = 4$ N).

The truss is restrained in points, B and D, as illustrated in Fig. 1. The external loading on the truss consists of a cyclic horizontal force, F_a, applied in point, G. It is assumed that $h = 3$ m, $s = 5$ m, $b = 2$ m and $F_a = 3$ N. A parametric investigation of the damping energy is carried-out.

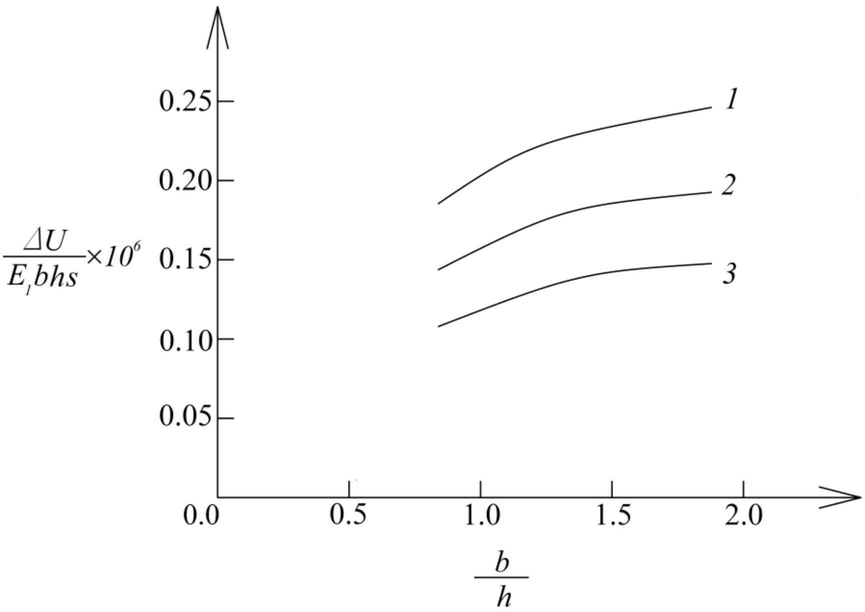

Figure 3. Change of damping energy with varying of b / h ratio (curve 1 – at $H_2 / H_1 = 0.5$, curve 2 – at $H_2 / H_1 = 1.0$ and curve 3 – at $H_2 / H_1 = 2.0$).

The purpose of the investigation is to study the dependence of the damping energy on various parameters of the truss geometry, properties and loading conditions. These dependences are presented in form of graphs in Fig. 2 and Fig. 3. The change of the damping energy with increasing of E_2 / E_1 ratio at three values of F_a is shown in Fig. 2.

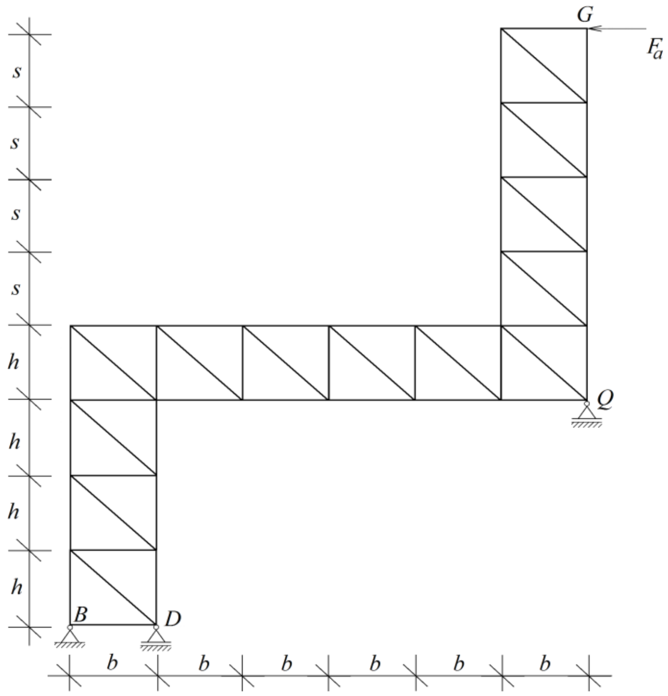

Figure 4. Statically indeterminate truss.

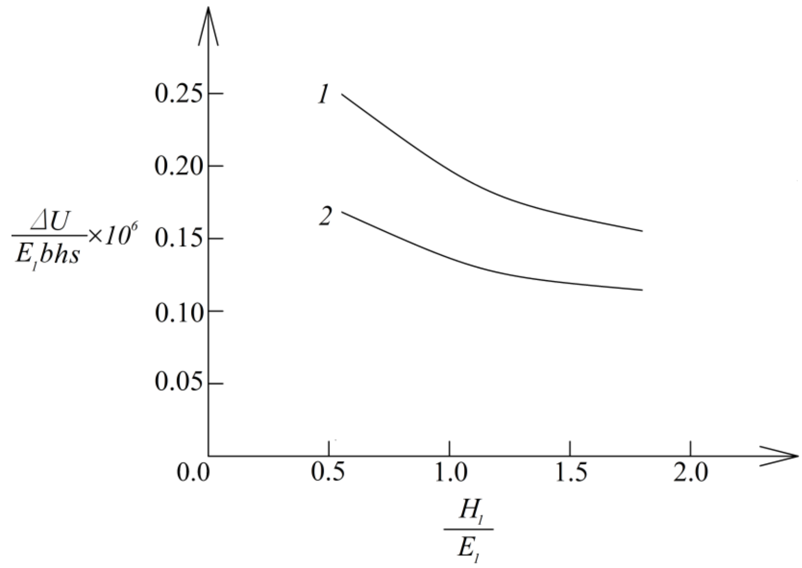

Figure 5. Change of damping energy with varying of H_1 / E_1 ratio (curve 1 – in statically determinate truss and curve 2 – in statically indeterminate truss).

It can be observed in Fig. 2 that the damping energy continuously reduces with increase of E_2 / E_1 ratio. As expected, growth of F_a cause a rise of the damping energy in the trust (Fig. 2). Figure 3 illustrates the dependence of the damping energy in the truss on b/h and H_2 / H_1 ratios.As can be seen, increase of b/h ratio generates growth of the damping energy (Fig. 3). However, increase of H_2 / H_1 ratio generates a substantial reduction of the damping energy as shown in Fig. 3. The damping energy in a truss with one degree of static indeterminacy is also studied. This truss is shown in Fig. 4. The geometry and the loading conditions of this truss are the same as these of the statically determinate truss depicted in Fig. 1. The static indeterminacy of the truss in Fig. 4 is due the roller introduced in point, Q, (actually, this the only difference between truss configurations in Fig. 1 and Fig. 4).

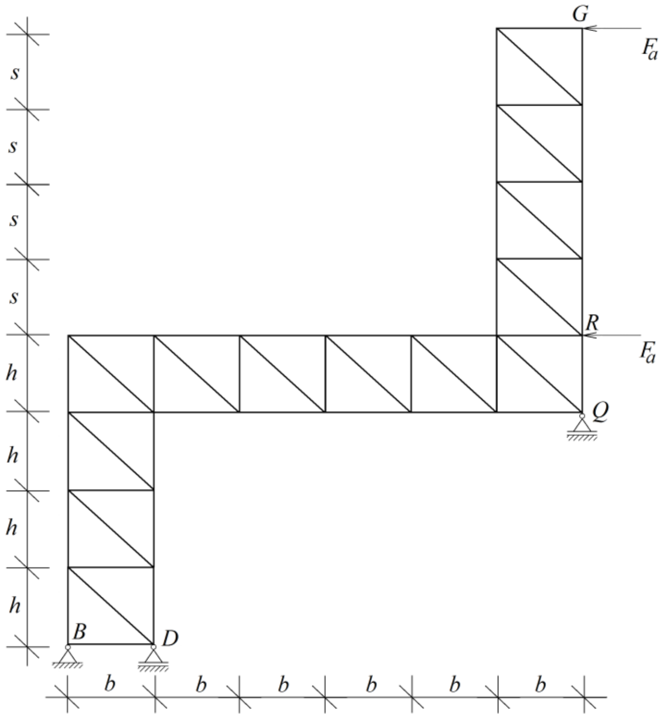

Figure 6. Statically indeterminate truss loaded by two forces.

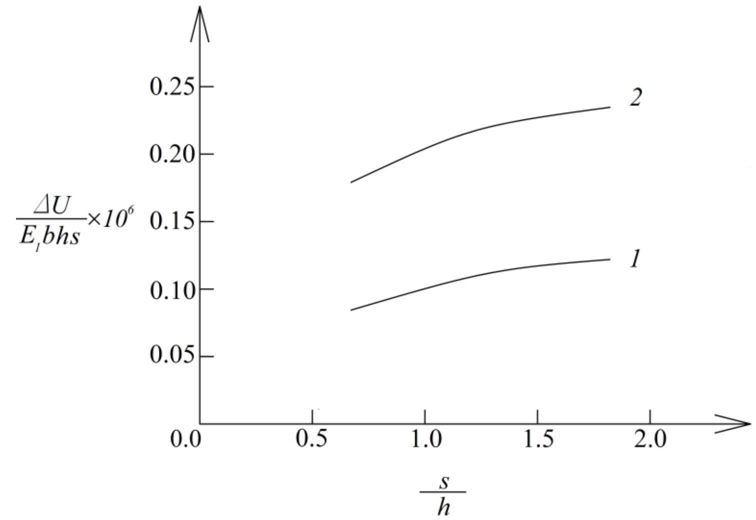

Figure 7. Change of damping energy with varying of s/h ratio (curve 1 – in truss loaded by one force and curve 2 – in truss loaded by two forces).

The change of the damping energy in the statically indeterminate truss with growth of H_1/E_1 ratio is presented in Fig. 5. It can be seen in Fig. 5 that growth of H_1/E_1 ratio induces reduction of the damping energy. The change of the damping energy with growth of H_1/E_1 ratio is shown in Fig. 5 also for the statically determinate truss. The inspection of curves in Fig. 5 indicates that the damping energy in the static indeterminate truss is lower than that in the determinate one. Finally, a damping energy analysis is performed also for the case of a statically indeterminate truss that is loaded by two horizontal forces of value, F_a, applied in points, G and R, as shown in Fig. 6. For this case, the effect of s/h ratio on the damping energy is evaluated. The results are depicted in Fig. 7. As can be seen, rise of s/h ratio generates growth of the damping energy (Fig. 7). The effect of s/h ratio on the damping energy is evaluated also for the statically indeterminate truss loaded only by one horizontal force applied in point, G, and the corresponding curve is depicted in Fig. 7. It can be observed that loading by two forces leads to increase of the damping energy (Fig. 7).

Conclusion

General treatment of the damping energy problem in trusses with elastic-plastic behaviour is presented. The analysis performed indicates that:

- the damping energy reduces with increase of E_2 / E_1 ratio;
- growth of F_a cause a rise of the damping energy;
- increase of b/h and s/h ratios generate growth of the damping energy;
- increase of H_2 / H_1 and H_1 / E_1 ratios generate a substantial reduction of the damping energy;
- the damping energy in the static indeterminate truss is lower than that in the determinate one.

Recommendations

The general treatment of the damping energy problem developed in this paper can be recommended for application in analyses of trusses under cycling loading.

Scientific Ethics Declaration

The author declares that the scientific ethical and legal responsibility of this article published in EPSTEM Journal belongs to the author.

Acknowledgements or Notes

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