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Multilayered Inhomogeneous Beams under Torsion and Bending: An Analytical Study of Energy Dissipation

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Abstract: The present theoretical paper is devoted to the problem of energy dissipation in multilayered inhomogeneous beam structures of visoelastic behaviour. In particular, our attention is concentrated on studying the energy dissipation for the case of a beam that is loaded simultaneously in torsion and bending. Both torsion and bending moments which are applied on the beam change with time so that the angles of twist and rotation of the beam end vary continuously with time. The viscoelastic mechanical behaviour of the beam under torsion is treated by a viscoelastic model that has two linear springs and a linear dashpot. The model is under time-dependent shear stress. The viscoelastic behaviour of the beam under bending is modelled in a similar way. An analytical approach of deriving the energy dissipation is applied. The approach treats a multilayered beam that has an arbitrary number of layers of different thicknesses and material properties. The layers are longitudinally inhomogeneous. An investigation of the energy dissipation is carried-out to clarify the effect of the parameters of the time-dependent bending and torsion moments, the material inhomogeneity and the beam size on the energy dissipation.

Keywords: Energy dissipation, Beam structure, Multilayered material, Viscoelastic behaviour, Torsion

Introduction

The multilayered structural materials are produced usually by adhesively bonded layers of different materials (Kim et al., 1999; Finot & Suresh, 1996; Rzhanitsyn, 1986). The layers may have also different thickness (Kaul, 2014; Lloyd & Molina-Aldareguia, 2003; Rizov, 2018). The multilayered materials are known with various useful properties which make them especially suitable for use in engineering applications with complex requirements. For instance, they have relatively high strength-to-weight and stiffness-to-weight ratios. Thus, the multilayered materials are applied with great success for building-up light-weight structures. Multilayered materials are often an object of study of engineering science (Dolgov, 2005, 2016; Rizov, 2021; Sy et al., 2015; Sy et al., 2020). Although the concept of multilayered materials is not a new one, there are some aspects of the mechanical behaviour of the multilayered materials especially when they are used for manufacturing of members of various mechanisms and load-bearing engineering structures which need further clarification.

Such aspect is the energy dissipation in multilayered structural members of viscoelastic behaviour when loaded simultaneously in torsion and bending. It should be noted here that publications on energy dissipation usually are concerned with multilayered viscoelastic beam structures loaded in pure bending (Narisawa, 1987; Rizov, 2021). However, beams under simultaneous action of torsion and bending moments are frequently used as members of various engineering structures. Besides, in some cases the loading varies continuously with time.

The aim of the present paper is to study analytically the energy dissipation in a multilayered viscoelastic beam structure under torsion and bending. The layers of the beam are continuously inhomogeneous in longitudinal direction. Thus, the material properties (in particular, the moduli of elasticity and the coefficient of viscosity of the viscoelastic model) are continuously distributed in each layer along the beam length. The torsion and

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bending moments which act on the beam change with time so that the angles of twist and rotation of the end of the beam are continuous functions of time. The energy dissipation is derived. An investigation of the effects of the parameters of the time-dependent bending and torsion moments, the material inhomogeneity and the beam size on the energy dissipation is carried-out.

Theoretical Analysis

Consider the multilayered beam structure whose configuration is shown in Fig. 1.

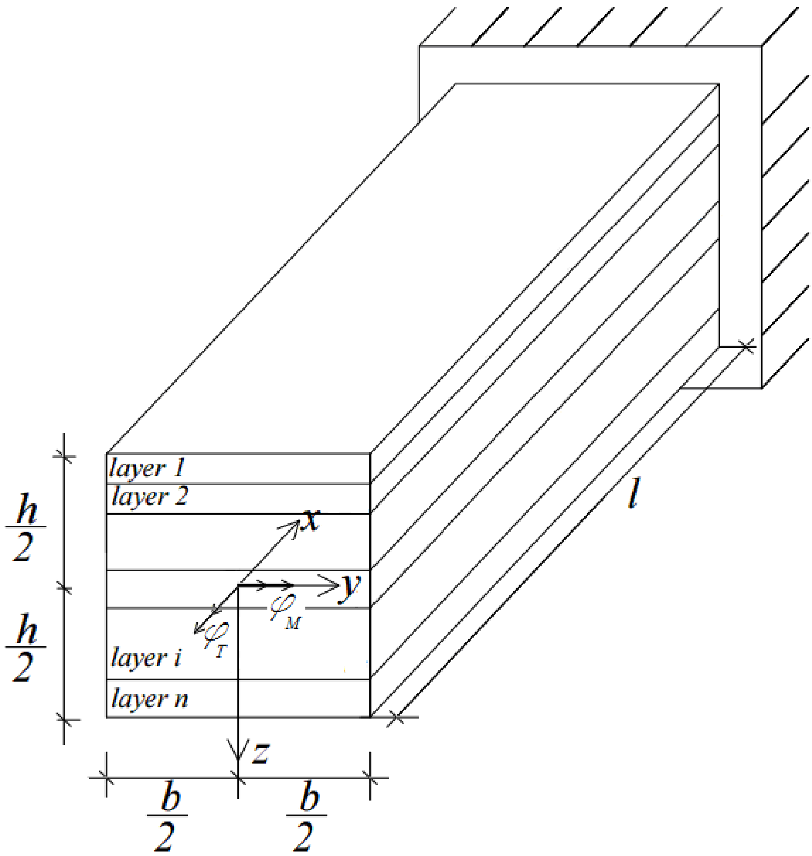

Figure 1. Multilayered beam structure.

The beam has n layers. Each layer has individual thickness and material properties. The width and thickness of the beam are b and h, respectively. The beam is subjected to torsion so that the variation of the angle of twist, φ_T, (Fig. 1) with time, t, is given by

$$\varphi_T = \varphi_{T0} \cos(\omega t), \tag{1}$$

where φ_{T0} and ω are parameters. Besides, the beam is also under bending moment so that the variation of the angle of rotation, φ_M, with time can be described by

$$\varphi_M = \varphi_{M0} \cos(\omega t), \tag{2}$$

where φ_{M0} is a parameter.

The beam under consideration has linear viscoelastic mechanical behaviour. The beam response to torsion is treated by the viscoelastic model shown in Fig. 2.

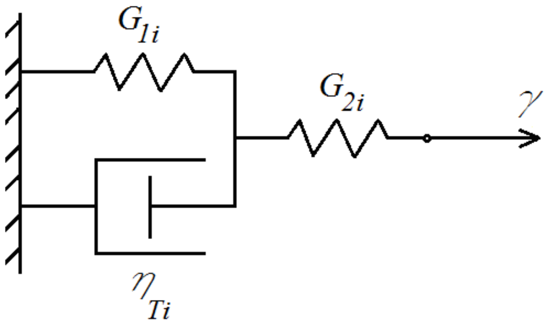

Figure 2. Viscoelastic model.

The two springs of the viscoelastic model have shear moduli, G_{1i} and G_{2i}. The coefficient of viscosity of the dashpot is η_{Ti} (Fig. 2). It should be specified here that the subscript, i, refers to the *i*-th layer of the beam structure. The layers of the beam are made by materials exhibiting continuous inhomogeneity in longitudinal direction. Thus, G_{1i}, G_{2i} and η_{Ti} change continuously along the *x*-axis. The changes of G_{1i}, G_{2i} and η_{Ti} are defined by

$$G_{1i} = G_{1Bi} e^{\delta_{1i} \frac{x}{l}}, \tag{3}$$

$$G_{2i} = G_{2Bi} e^{\delta_{2i} \frac{x}{l}}, \tag{4}$$

$$\eta_{Ti} = \eta_{TBi} e^{\delta_{3i} \frac{x}{l}}, \tag{5}$$

where

$$0 \leq x \leq l, \tag{6}$$

$$i = 1, 2, ..., n. \tag{7}$$

In formulas (3) – (6), G_{1Bi}, G_{2Bi}, η_{TBi}, δ_{1i}, δ_{2i} and δ_{3i} are parameters, l is the longitudinal size of the beam structure. The model in Fig. 2 is under shear strain, γ, that is given by

$$\gamma = \gamma_0 \cos(\omega t), \tag{8}$$

where γ_0 is a parameter. The time-dependent shear modulus, $\tilde{G}_i(t)$, of the viscoelastic model is defined by

$$\tilde{G}_i(t) = \frac{\tau_i}{\gamma}, \tag{9}$$

where τ_i is the shear stress in the model.

The bending response of the beam structure is treated by a viscoelastic model that has the same components like the model in Fig. 2. The only difference is that G_{1i}, G_{2i}, η_{Ti}, γ and γ_0 are replaced with E_{1i}, E_{2i}, η_{Mi}, ε and ε_0, respectively (here, E_{1i} and E_{2i} are the moduli of elasticity of the two springs, η_{Mi} is the coefficient of viscosity, ε is the linear strain, ε_0 is a parameter). The change of E_{1i}, E_{2i} and η_{Mi} along the beam length is given by

$$E_{1i} = E_{1Bi} e^{\delta_{4i} \frac{x}{l}} , \tag{10}$$

$$E_{2i} = E_{2Bi} e^{\delta_{5i} \frac{x}{l}} , \tag{11}$$

$$\eta_{Mi} = \eta_{MBi} e^{\delta_{6i} \frac{x}{l}} , \tag{12}$$

where E_{1Bi}, E_{2Bi}, η_{MBi}, δ_{4i}, δ_{5i} and δ_{6i} are parameters. The time-dependent modulus of elasticity, $\tilde{E}_i(t)$, of the viscoelastic model in this case is presented by

$$\tilde{E}_i(t) = \frac{\sigma_i}{\varepsilon} , \tag{13}$$

where σ_i is the normal stress in the model.

The unit dissipated energy, u_{0Ti}, in the *i*-th layer of the beam due to torsion is defined by

$$u_{0Ti} = \int_0^t \tau_{\eta Ti} \dot{\gamma}_{\eta T} dt , \tag{14}$$

where $\tau_{\eta Ti}$ and $\gamma_{\eta Ti}$ are the shear stress and the shear strain in the dashpot of the viscoelastic model, respectively.

The dissipated energy, U_T, due to the torsion of the beam is obtained by

$$U_T = l \sum_{i=1}^{i=n} \iint_{(A_i)} u_{0Ti} dA, \tag{15}$$

where A_i is the area of the cross-section of the *i*-th layer.

The unit dissipated energy, u_{0Mi}, in the *i*-th layer in the context of the beam response to bending is given by

$$u_{0Mi} = \int_0^t \sigma_{\eta Mi} \dot{\varepsilon}_{\eta M} dt , \tag{16}$$

where $\sigma_{\eta Mi}$ and $\varepsilon_{\eta M}$ are the normal stress and the linear strain in the dashpot of the viscoelastic model, respectively.

The unit dissipated energy (16) is integrated to derive the dissipated energy, U_M, in the beam due to bending, i.e.

$$U_M = l \sum_{i=1}^{i=n} \iint_{(A_i)} u_{0Mi} dA . \tag{17}$$

The dissipated energy, U, due to torsion and bending is equal to the sum of U_T and U_M, i.e.

$$U = U_T + U_M \,, \tag{18}$$

where the integrals (15) and (17) are solved by the MatLab.

Numerical Results

Numerical results are derived assuming that $n = 4$, $b = 0.008$ m, $h = 0.012$ m, $l = 0.450$ m, $\varphi_{T0} = 0.004$ rad, $\varphi_{M0} = 0.006$ rad and $\omega = 0.0003$.

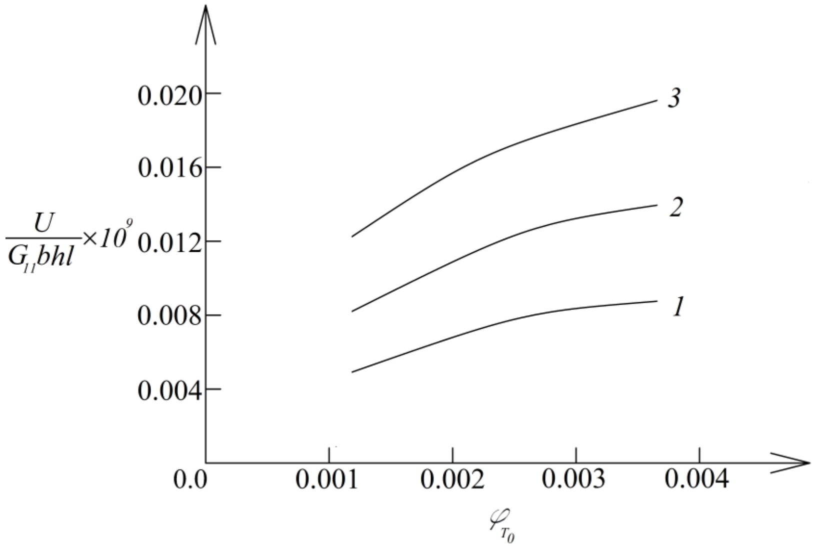

Figure 3. Dissipated energy versus φ_{T0} (curve 1 – at $\delta_{11} = 0.3$, curve 2 – at $\delta_{11} = 0.5$ and curve 3 – at $\delta_{11} = 0.7$).

The results are shown in form of various graphs in Fig. 3, Fig. 4 and Fig. 5 illustrating how the dissipated energy in the beam structure changes with varying of different parameters of the loading, the beam size, the distribution of moduli of elasticity and the coefficients of viscosity in the length direction of the layers.

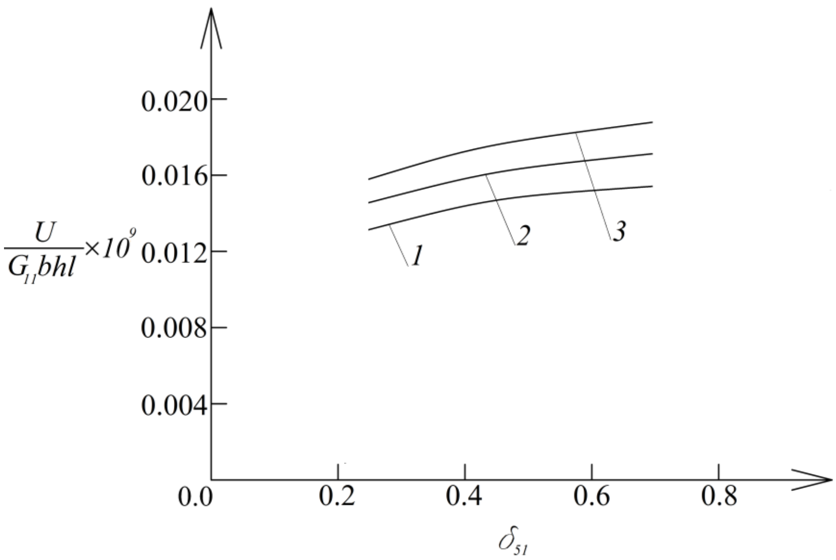

Figure 4. Dissipated energy versus δ_{51} (curve 1 – at $\varphi_{M0} = 0.002$ rad, curve 2 – at $\varphi_{M0} = 0.004$ rad and curve 3 – at rad).

Results illustrating variation of the dissipated energy induced by changing the parameter, φ_{T0}, at different values of the parameter, δ_{11}, are depicted in Fig. 3. From the results in Fig. 3 it becomes clear that there is a significant influence of both parameters, φ_{T0} and δ_{11}, on the dissipated energy. It can be seen that the dissipated energy grows with growing of φ_{T0} (Fig. 3). The growth of δ_{11} also causes growth of the dissipated energy.

Figure 4 gives plots of the dissipated energy versus the para $\varphi_{M0} = 0.006$meter, δ_{51}, for three values of the parameter, φ_{M0}. The results show that increase of δ_{51} produces an increase of the dissipated energy (Fig. 4).

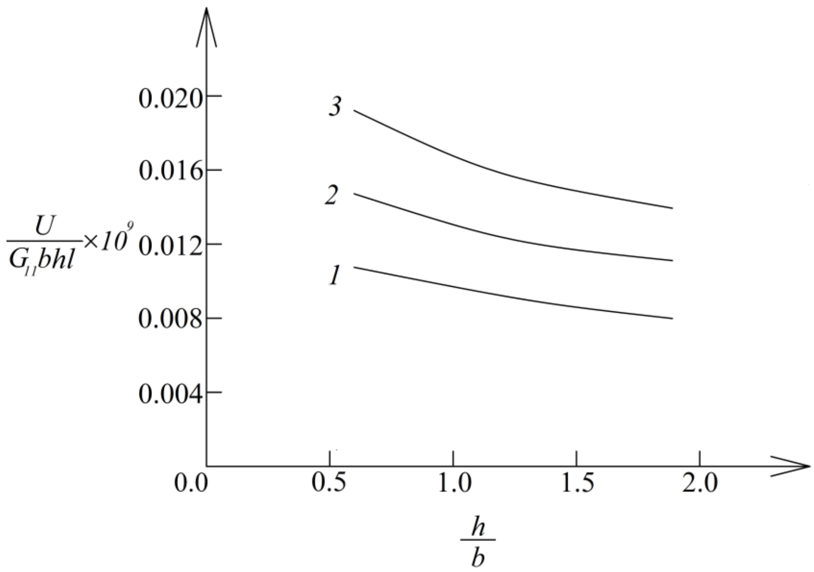

Figure 5. Dissipated energy versus h/b ratio (curve 1 – at $l/b = 20$, curve 2 – at $l/b = 40$ and curve 3 – at $l/b = 60$).

The same character has the dependence of the dissipated energy on the parameter, φ_{M0}, i.e. increase of φ_{M0} leads to growth of the dissipated energy (Fig. 4).

Figure 5 visualizes the dependence of the dissipated energy upon the beam sizes (the latter are represented by h/b and l/b ratios). It can be observed in Fig. 5 that as a result of increase of h/b ratio the dissipated energy reduces. However, increase of l/b ratio induces a growth of the dissipated energy (Fig. 5).

Conclusion

A theoretical study of the energy dissipation in a multilayered inhomogeneous beam structure of viscoelastic behaviour under time-dependent bending and torsion is performed. The study reveals that:

1) the dissipated energy grows with growing of φ_{T0};

2) increase of φ_{M0} leads also to growth of the dissipated energy;

3) the growth of the parameter, δ_{11}, causes growth of the dissipated energy;

4) increase of δ_{51} produces an increase of the dissipated energy;

5) the dissipated energy reduces when h/b ratio increases;

6) increase of l/b ratio induces a growth of the dissipated energy.

Recommendations

It is recommendable in a future work to develop further the present dissipation analysis by considering non-linear viscoelastic behavior.

Scientific Ethics Declaration

The author declares that the scientific ethical and legal responsibility of this article published in EPSTEM journal belongs to the author.

Acknowledgements or Notes

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