The Eurasia Proceedings of Science, Technology,
Engineering & Mathematics (EPSTEM)
ISSN: 2602-3199

The Eurasia Proceedings of Science, Technology, Engineering & Mathematics (EPSTEM), 2023

Volume 26, Pages 554-577

IConTES 2023: International Conference on Technology, Engineering and Science

Some 3D-Determinant Properties for Calculating of Cubic-Matrix of Order 2 and Order 3

Armend Salihu
Universum International College

Orgest Zaka
Agricultural University of Tirana

Abstract: In this paper, as a continuation of our work on the determinant of cubic matrices, we have studied some properties of calculation of the determinant of cubic matrices for the cubic matrix of the order 2×2, as well as the order 3×3. Like properties of square determinants, these properties are analogous to some properties for determinants of the square matrix we have proved and noted that these properties also are applicable (or not in some details) to this concept for the determinant of the cubic matrix of orders 2×2 and order 3×3. All properties that have been proved in this paper, are proved for each case for orders 2×2 and order 3×3, by including all elements of the matrix, as well as all horizontal plans, horizontal layers, and vertical layers. All results in this paper, are presented in detail during the theorem proofs.

Keywords: Cubic-matrix, Determinant of cubic-matrix, Determinant properties.

Introduction and Preliminaries

In this paper we prove some properties for determinant of cubic-matrix of order 2 and order 3. In the paper Salihu et.al. (2023a), we have defined the concept of determinant for cubic-matrix of order 2 and order 3, and we have prove some basic properties for calculating this determinants. This idea for developing this concept, it came simply from the determinant of 2D square matrices (Salihu et.al., 2021; Salihu et al., 2019b; Salihu, 2018; Artin, 1991; Bretscher 2005, Schneide et.al. 1973), as well as determinant of rectangular matrices (Salihu et al., 2022a, Salihu et al., 2023b, Salihu et al., 2022b, Salihu et al., 2022c; Salihu et al., 2019a; Amiri et al., 2010; Radic, 1966; Radic, 2005; Makarewicz et al., 2014).

In paper Zaka et al. (2023) we have prove that the Laplace expansion method is valid for calculating the determinant of cubic-matrix for orders 2 and 3. Encouraged by geometric intuition, in this paper we are trying to give an idea and visualize the meaning of the determinants for the cubic-matrix. Our early research mainly lies between geometry, algebra, matrix theory, etc., (see Peters et al., 2023; Zaka 2019a; Zaka et al., 2016, Filipi et al., 2019; Zaka, 2018a, 2018b, Zaka 2016b, Zaka et al., 2019b, 2019c, Zaka et al., 2020a, 2020b).

This paper is continuation of the ideas that arise based on previous researches of 3D matrix ring with element from any whatever field F see Zaka (2019d), but here we study the case when the field F is the field of real numbers $\mathbb{R}$ also is continuation of our research Salihu et al. (2023a) and Zaka et.al (2023) related to the study of the properties of determinants for cubic-matrix of order 2 and 3. In this paper we follow a different method from method which is studied in Zaka (2017).

Results for More Properties of Determinants of Cubic-Matrix of Order 2 and Order 3

- Selection and peer-review under responsibility of the Organizing Committee of the Conference

In this section, all proofs of our theorems are based on the definition of determinant for cubic-matrix of orders 2 and 3, presented in the papers Salihu et al. (2023a) and Zaka et al. (2023), and results obtained in these papers. The proofs of the following Theorems are too loaded with indices to calculate, and we are trying to make them a little simpler by separating them case by case, to avoid the difficulty of calculations!

Theorem 1:

Let's be A and B 3D-cubic matrix with same order (second and third order matrices), then we have that:

$$\det(A+B)=\det(A)+\det(B).$$

Proof:

Case 1: The cubic-matrix A of order 2, (and B has order 2), we will proof the case 1 for each "horizontal layer", "vertical page" and "vertical layer", as following:

1. For plan $i=1$: Let A and B be cubic-matrix of order 2, where all elements on the plan $i=1$ are identical in both matrices, then based on definition of determinant of cubic-matrix presented in Salihu et.al (2023a) and Zaka et.al (2023) we have:

$$det\left(A_{[2\times2\times2]}\right)+\det\left(B_{[2\times2\times2]}\right)=det\begin{pmatrix}a_{111} & a_{121} & a_{112} & a_{122}\\ a_{211} & a_{221} & a_{212} & a_{222}\end{pmatrix}+det\begin{pmatrix}a_{111} & a_{121} & a_{112} & a_{122}\\ b_{211} & b_{221} & b_{212} & b_{222}\end{pmatrix}$$
$$=a_{111}\cdot a_{222}-a_{112}\cdot a_{221}-a_{121}\cdot a_{212}+a_{122}\cdot a_{211}+a_{111}\cdot b_{222}-a_{112}\cdot b_{221}-a_{121}\cdot b_{212}+a_{122}\cdot b_{211},$$

while,

$$det\left(A_{[2\times2\times2]}+B_{[2\times2\times2]}\right)=det\begin{pmatrix}a_{111} & a_{121} & a_{112} & a_{122}\\ a_{211}+b_{211} & a_{221}+b_{221} & a_{212}+b_{212} & a_{222}+b_{222}\end{pmatrix}$$
$$=a_{111}\cdot(a_{222}+b_{222})-a_{112}\cdot(a_{221}+b_{221})-a_{121}\cdot(a_{212}+b_{212})+a_{122}\cdot(a_{211}+b_{211})$$
$$=a_{111}\cdot a_{222}+a_{111}\cdot b_{222}-a_{112}\cdot a_{221}-a_{112}\cdot b_{221}-a_{121}\cdot a_{212}-a_{121}\cdot b_{212}+a_{122}\cdot a_{211}+a_{122}\cdot b_{211}.$$

If we compare results of above equations, we can see that we have the same result in both cases. Similarly, we will proof for all other cases.

2. For plan $i=2$: Let A and B be cubic-matries of order 2, where all elements on the plan $i=2$ are identical in both matrices, then we have:

$$det\left(A_{[2\times2\times2]}\right)+det\left(B_{[2\times2\times2]}\right)=det\begin{pmatrix}a_{111} & a_{121} & a_{112} & a_{122}\\ a_{211} & a_{221} & a_{212} & a_{222}\end{pmatrix}+det\begin{pmatrix}b_{111} & b_{121} & b_{112} & b_{122}\\ a_{211} & a_{221} & a_{212} & a_{222}\end{pmatrix}$$
$$=a_{111}\cdot a_{222}-a_{112}\cdot a_{221}-a_{121}\cdot a_{212}+a_{122}\cdot a_{211}+b_{111}\cdot a_{222}-b_{112}\cdot a_{221}-b_{121}\cdot a_{212}+b_{122}\cdot a_{211},$$

while,

$$det\left(A_{[2\times2\times2]}+B_{[2\times2\times2]}\right)=det\begin{pmatrix}a_{111}+b_{111} & a_{121}+b_{121} & a_{112}+b_{112} & a_{122}+b_{122}\\ a_{211} & a_{221} & a_{212} & a_{222}\end{pmatrix}$$
$$=(a_{111}+b_{111})\cdot a_{222}-(a_{112}+b_{112})\cdot a_{221}-(a_{121}+b_{121})\cdot a_{212}+(a_{122}+b_{122})\cdot a_{211}$$
$$=a_{111}\cdot a_{222}+b_{111}\cdot a_{222}-a_{112}\cdot a_{221}-b_{112}\cdot a_{221}-a_{121}\cdot a_{212}-b_{121}\cdot a_{212}+a_{122}\cdot a_{211}+b_{122}\cdot a_{211}.$$

If we compare results of above equations, we can see that we have the same result in both cases.

3. For plan $j=1$: Let A and B be cubic-matries of order 2, where all elements on the plan $j=1$ are identical in both matrices, then we have:

$$det\left(A_{[2\times2\times2]}\right)+det\left(B_{[2\times2\times2]}\right)=det\begin{pmatrix}a_{111} & a_{121} & a_{112} & a_{122}\\ a_{211} & a_{221} & a_{212} & a_{222}\end{pmatrix}+det\begin{pmatrix}a_{111} & b_{121} & a_{112} & b_{122}\\ a_{211} & b_{221} & a_{212} & b_{222}\end{pmatrix}$$
$$=a_{111}\cdot a_{222}-a_{112}\cdot a_{221}-a_{121}\cdot a_{212}+a_{122}\cdot a_{211}+a_{111}\cdot b_{222}-a_{112}\cdot b_{221}-b_{121}\cdot a_{212}+b_{122}\cdot a_{211},$$

while,

$$det\left(A_{[2\times2\times2]}+B_{[2\times2\times2]}\right)=det\begin{pmatrix}a_{111} & a_{121}+b_{121} & a_{112} & a_{122}+b_{122}\\ a_{211} & a_{221}+b_{221} & a_{212} & a_{222}+b_{222}\end{pmatrix}$$

$$=a_{111}\cdot(a_{222}+b_{222})-a_{112}\cdot(a_{221}+b_{221})-(a_{121}+b_{121})\cdot a_{212}+(a_{122}+b_{122})\cdot a_{211}$$

$$=a_{111}\cdot a_{222}+a_{111}\cdot b_{222}-a_{112}\cdot a_{221}-a_{112}\cdot b_{221}-a_{121}\cdot a_{212}-b_{121}\cdot a_{212}+a_{122}\cdot a_{211}+b_{122}\cdot a_{211}.$$

If we compare results of above equations, we can see that we have the same result in both cases.

4. For plan $j=2$: Let A and B be cubic-matries of order 2, where all elements on the plan $j=2$ are identical in both matrices, then we have:

$$det\left(A_{[2\times2\times2]}\right)+det\left(B_{[2\times2\times2]}\right)=det\begin{pmatrix}a_{111} & a_{121} & a_{112} & a_{122}\\ a_{211} & a_{221} & a_{212} & a_{222}\end{pmatrix}+det\begin{pmatrix}b_{111} & a_{121} & b_{112} & a_{122}\\ b_{211} & a_{221} & b_{212} & a_{222}\end{pmatrix}$$

$$=a_{111}\cdot a_{222}-a_{112}\cdot a_{221}-a_{121}\cdot a_{212}+a_{122}\cdot a_{211}+b_{111}\cdot a_{222}-b_{112}\cdot a_{221}-a_{121}\cdot b_{212}+a_{122}\cdot b_{211},$$

while,

$$det\left(A_{[2\times2\times2]}+B_{[2\times2\times2]}\right)=det\begin{pmatrix}a_{111}+b_{111} & a_{121} & a_{112}+b_{112} & a_{122}\\ a_{211}+b_{211} & a_{221} & a_{212}+b_{212} & a_{222}\end{pmatrix}$$

$$=(a_{111}+b_{111})\cdot a_{222}-(a_{112}+b_{112})\cdot a_{221}-a_{121}\cdot(a_{212}+b_{212})+a_{122}\cdot(a_{211}+b_{211})$$

$$=a_{111}\cdot a_{222}+b_{111}\cdot a_{222}-a_{112}\cdot a_{221}-b_{112}\cdot a_{221}-a_{121}\cdot a_{212}-a_{121}\cdot b_{212}+a_{122}\cdot a_{211}+a_{122}\cdot b_{211}.$$

If we compare results of above equations, we can see that we have the same result in both cases.

5. For plan $k=1$: Let A and B be cubic-matries of order 2, where all elements on the plan $k=1$ are identical in both matrices, then we have:

$$det\left(A_{[2\times2\times2]}\right)+det\left(B_{[2\times2\times2]}\right)=det\begin{pmatrix}a_{111} & a_{121} & a_{112} & a_{122}\\ a_{211} & a_{221} & a_{212} & a_{222}\end{pmatrix}+det\begin{pmatrix}a_{111} & a_{121} & b_{112} & b_{122}\\ a_{211} & a_{221} & b_{212} & b_{222}\end{pmatrix}$$

$$=a_{111}\cdot a_{222}-a_{112}\cdot a_{221}-a_{121}\cdot a_{212}+a_{122}\cdot a_{211}+a_{111}\cdot b_{222}-b_{112}\cdot a_{221}-a_{121}\cdot b_{212}+b_{122}\cdot a_{211},$$

while,

$$det\left(A_{[2\times2\times2]}+B_{[2\times2\times2]}\right)=det\begin{pmatrix}a_{111} & a_{121} & a_{112}+b_{112} & a_{122}+b_{122}\\ a_{211} & a_{221} & a_{212}+b_{212} & a_{222}+b_{222}\end{pmatrix}$$

$$=a_{111}\cdot(a_{222}+b_{222})-(a_{112}+b_{112})\cdot a_{221}-a_{121}\cdot(a_{212}+b_{212})+(a_{122}+b_{122})\cdot a_{211}$$

$$=a_{111}\cdot a_{222}+a_{111}\cdot b_{222}-a_{112}\cdot a_{221}-b_{112}\cdot a_{221}-a_{121}\cdot a_{212}-a_{121}\cdot b_{212}+a_{122}\cdot a_{211}+b_{122}\cdot a_{211}.$$

If we compare results of above equations, we can see that we have the same result in both cases.

6. For plan $k=2$: Let A and B be cubic-matries of order 2, where all elements on the plan $k=2$ are identical in both matrices, then we have:

$$det\left(A_{[2\times2\times2]}\right)+det\left(B_{[2\times2\times2]}\right)=det\begin{pmatrix}a_{111} & a_{121} & a_{112} & a_{122}\\ a_{211} & a_{221} & a_{212} & a_{222}\end{pmatrix}+det\begin{pmatrix}b_{111} & b_{121} & a_{112} & a_{122}\\ b_{211} & b_{221} & a_{212} & a_{222}\end{pmatrix}$$

$$=a_{111}\cdot a_{222}-a_{112}\cdot a_{221}-a_{121}\cdot a_{212}+a_{122}\cdot a_{211}+b_{111}\cdot a_{222}-a_{112}\cdot b_{221}-b_{121}\cdot a_{212}+a_{122}\cdot b_{211},$$

while,

$$det\left(A_{[2\times2\times2]}+B_{[2\times2\times2]}\right)=det\begin{pmatrix}a_{111}+b_{111} & a_{121}+b_{121} & a_{112} & a_{122}\\ a_{211}+b_{211} & a_{221}+b_{221} & a_{212} & a_{222}\end{pmatrix}$$

$$=(a_{111}+b_{111})\cdot a_{222}-(a_{112}+b_{112})\cdot a_{221}-(a_{121}+b_{121})\cdot a_{212}+(a_{122}+b_{122})\cdot a_{211}$$

$$=a_{111}\cdot a_{222}+b_{111}\cdot a_{222}-a_{112}\cdot a_{221}-b_{112}\cdot a_{221}-a_{121}\cdot a_{212}-b_{121}\cdot a_{212}+a_{122}\cdot a_{211}+b_{122}\cdot a_{211}.$$

If we compare results of above equations, we can see that we have the same result in both cases.

Case 2: The cubic-matrix A of order 3, (and B has order 3), we will proof the case 1 for each "horizontal layer", "vertical page" and "vertical layer", as following:

1. For plan $i = 1$: Let A and B be cubic-matries of order 3, where all elements on the plan $i = 1$ and $i = 2$ are identical in both matrices, then we have:

$$det(A_{[3\times3\times3]}) + det(B_{[3\times3\times3]}) = det\left(\begin{array}{ccc|ccc|ccc} a_{111} & a_{121} & a_{131} & a_{112} & a_{122} & a_{132} & a_{113} & a_{123} & a_{133} \\ a_{211} & a_{221} & a_{231} & a_{212} & a_{222} & a_{232} & a_{213} & a_{223} & a_{233} \\ a_{311} & a_{321} & a_{331} & a_{312} & a_{322} & a_{332} & a_{313} & a_{323} & a_{333} \end{array}\right)$$
$$+det\left(\begin{array}{ccc|ccc|ccc} a_{111} & a_{121} & a_{131} & a_{112} & a_{122} & a_{132} & a_{113} & a_{123} & a_{133} \\ a_{211} & a_{221} & a_{231} & a_{212} & a_{222} & a_{232} & a_{213} & a_{223} & a_{233} \\ b_{311} & b_{321} & b_{331} & b_{312} & b_{322} & b_{332} & b_{313} & b_{323} & b_{333} \end{array}\right)$$
$$= \{a_{111}\cdot a_{222}\cdot a_{333} - a_{111}\cdot a_{232}\cdot a_{323} - a_{111}\cdot a_{223}\cdot a_{332} + a_{111}\cdot a_{233}\cdot a_{322} - a_{112}\cdot a_{221}\cdot a_{333}$$
$$+a_{112}\cdot a_{223}\cdot a_{331} + a_{112}\cdot a_{231}\cdot a_{323} - a_{112}\cdot a_{233}\cdot a_{321} + a_{113}\cdot a_{221}\cdot a_{332} - a_{113}\cdot a_{222}\cdot a_{331}$$
$$-a_{113}\cdot a_{231}\cdot a_{322} + a_{113}\cdot a_{232}\cdot a_{321} - a_{121}\cdot a_{212}\cdot a_{333} + a_{121}\cdot a_{213}\cdot a_{332} + a_{121}\cdot a_{232}\cdot a_{313}$$
$$-a_{121}\cdot a_{233}\cdot a_{312} + a_{122}\cdot a_{211}\cdot a_{333} - a_{122}\cdot a_{213}\cdot a_{331} - a_{122}\cdot a_{231}\cdot a_{313} + a_{122}\cdot a_{233}\cdot a_{311}$$
$$-a_{123}\cdot a_{211}\cdot a_{332} + a_{123}\cdot a_{212}\cdot a_{331} + a_{123}\cdot a_{231}\cdot a_{312} - a_{123}\cdot a_{232}\cdot a_{311} + a_{131}\cdot a_{212}\cdot a_{323}$$
$$-a_{131}\cdot a_{213}\cdot a_{322} - a_{131}\cdot a_{222}\cdot a_{313} + a_{131}\cdot a_{223}\cdot a_{312} - a_{132}\cdot a_{211}\cdot a_{323} + a_{132}\cdot a_{213}\cdot a_{321}$$
$$+a_{132}\cdot a_{221}\cdot a_{313} - a_{132}\cdot a_{223}\cdot a_{311} + a_{133}\cdot a_{211}\cdot a_{322} - a_{133}\cdot a_{212}\cdot a_{321} - a_{133}\cdot a_{221}\cdot a_{312}$$
$$+a_{133}\cdot a_{222}\cdot a_{311}\} + \{a_{111}\cdot a_{222}\cdot b_{333} - a_{111}\cdot a_{232}\cdot b_{323} - a_{111}\cdot a_{223}\cdot b_{332} + a_{111}\cdot a_{233}\cdot b_{322}$$
$$-a_{112}\cdot a_{221}\cdot b_{333} + a_{112}\cdot a_{223}\cdot b_{331} + a_{112}\cdot a_{231}\cdot b_{323} - a_{112}\cdot a_{233}\cdot b_{321} + a_{113}\cdot a_{221}\cdot b_{332}$$
$$-a_{113}\cdot a_{222}\cdot b_{331} - a_{113}\cdot a_{231}\cdot b_{322} + a_{113}\cdot a_{232}\cdot b_{321} - a_{121}\cdot a_{212}\cdot b_{333} + a_{121}\cdot a_{213}\cdot b_{332}$$
$$+a_{121}\cdot a_{232}\cdot b_{313} - a_{121}\cdot a_{233}\cdot b_{312} + a_{122}\cdot a_{211}\cdot b_{333} - a_{122}\cdot a_{213}\cdot b_{331} - a_{122}\cdot a_{231}\cdot b_{313}$$
$$+a_{122}\cdot a_{233}\cdot b_{311} - a_{123}\cdot a_{211}\cdot b_{332} + a_{123}\cdot a_{212}\cdot b_{331} + a_{123}\cdot a_{231}\cdot b_{312} - a_{123}\cdot a_{232}\cdot b_{311}$$
$$+a_{131}\cdot a_{212}\cdot b_{323} - a_{131}\cdot a_{213}\cdot b_{322} - a_{131}\cdot a_{222}\cdot b_{313} + a_{131}\cdot a_{223}\cdot b_{312} - a_{132}\cdot a_{211}\cdot b_{323}$$
$$+a_{132}\cdot a_{213}\cdot b_{321} + a_{132}\cdot a_{221}\cdot b_{313} - a_{132}\cdot a_{223}\cdot b_{311} + a_{133}\cdot a_{211}\cdot b_{322} - a_{133}\cdot a_{212}\cdot b_{321}$$
$$-a_{133}\cdot a_{221}\cdot b_{312} + a_{133}\cdot a_{222}\cdot b_{311}\},$$

while,

$$det(A_{[3\times3\times3]}) + det(B_{[3\times3\times3]}) = det\left(\begin{array}{ccc|ccc|ccc} a_{111} & a_{121} & a_{131} & a_{112} & a_{122} & a_{132} & a_{113} & a_{123} & a_{133} \\ a_{211} & a_{221} & a_{231} & a_{212} & a_{222} & a_{232} & a_{213} & a_{223} & a_{233} \\ a_{311}+b_{311} & a_{321}+b_{321} & a_{331}+b_{331} & a_{312}+b_{312} & a_{322}+b_{322} & a_{332}+b_{332} & a_{313}+b_{313} & a_{323}+b_{323} & a_{333}+b_{333} \end{array}\right)$$
$$= a_{111}\cdot a_{222}\cdot(a_{333}+b_{333}) - a_{111}\cdot a_{232}\cdot(a_{323}+b_{323}) - a_{111}\cdot a_{223}\cdot(a_{332}+b_{332})$$
$$+a_{111}\cdot a_{233}\cdot(a_{322}+b_{322}) - a_{112}\cdot a_{221}\cdot(a_{333}+b_{333}) + a_{112}\cdot a_{223}\cdot(a_{331}+b_{331})$$
$$+a_{112}\cdot a_{231}\cdot(a_{323}+b_{323}) - a_{112}\cdot a_{233}\cdot(a_{321}+b_{321}) + a_{113}\cdot a_{221}\cdot(a_{332}+b_{332})$$
$$-a_{113}\cdot a_{222}\cdot(a_{331}+b_{331}) - a_{113}\cdot a_{231}\cdot(a_{322}+b_{322}) + a_{113}\cdot a_{232}\cdot(a_{321}+b_{321})$$
$$-a_{121}\cdot a_{212}\cdot(a_{333}+b_{333}) + a_{121}\cdot a_{213}\cdot(a_{332}+b_{332}) + a_{121}\cdot a_{232}\cdot(a_{313}+b_{313})$$
$$-a_{121}\cdot a_{233}\cdot(a_{312}+b_{312}) + a_{122}\cdot a_{211}\cdot(a_{333}+b_{333}) - a_{122}\cdot a_{213}\cdot(a_{331}+b_{331})$$
$$-a_{122}\cdot a_{231}\cdot(a_{313}+b_{313}) + a_{122}\cdot a_{233}\cdot(a_{311}+b_{311}) - a_{123}\cdot a_{211}\cdot(a_{332}+b_{332})$$
$$+a_{123}\cdot a_{212}\cdot(a_{331}+b_{331}) + a_{123}\cdot a_{231}\cdot(a_{312}+b_{312}) - a_{123}\cdot a_{232}\cdot(a_{311}+b_{311})$$
$$+a_{131}\cdot a_{212}\cdot(a_{323}+b_{323}) - a_{131}\cdot a_{213}\cdot(a_{322}+b_{322}) - a_{131}\cdot a_{222}\cdot(a_{313}+b_{313})$$
$$+a_{131}\cdot a_{223}\cdot(a_{312}+b_{312}) - a_{132}\cdot a_{211}\cdot(a_{323}+b_{323}) + a_{132}\cdot a_{213}\cdot(a_{321}+b_{321})$$
$$+a_{132}\cdot a_{221}\cdot(a_{313}+b_{313}) - a_{132}\cdot a_{223}\cdot(a_{311}+b_{311}) + a_{133}\cdot a_{211}\cdot(a_{322}+b_{322})$$
$$-a_{133}\cdot a_{212}\cdot(a_{321}+b_{321}) - a_{133}\cdot a_{221}\cdot(a_{312}+b_{312}) + a_{133}\cdot a_{222}\cdot(a_{311}+b_{311}).$$

Hence,

$$
\begin{aligned}
det\left(A_{[3\times3\times3]}\right)+det\left(B_{[3\times3\times3]}\right) &= a_{111}\cdot a_{222}\cdot a_{333}+a_{111}\cdot a_{222}\cdot b_{333}\\
&-a_{111}\cdot a_{232}\cdot a_{323}-a_{111}\cdot a_{232}\cdot b_{323}-a_{111}\cdot a_{223}\cdot a_{332}-a_{111}\cdot a_{223}\cdot b_{332}+a_{111}\cdot a_{233}\cdot a_{322}\\
&+a_{111}\cdot a_{233}\cdot b_{322}-a_{112}\cdot a_{221}\cdot a_{333}-a_{112}\cdot a_{221}\cdot b_{333}+a_{112}\cdot a_{223}\cdot a_{331}+a_{112}\cdot a_{223}\cdot b_{331}\\
&+a_{112}\cdot a_{231}\cdot a_{323}+a_{112}\cdot a_{231}\cdot b_{323}-a_{112}\cdot a_{233}\cdot a_{321}-a_{112}\cdot a_{233}\cdot b_{321}+a_{113}\cdot a_{221}\cdot a_{332}\\
&+a_{113}\cdot a_{221}\cdot b_{332}-a_{113}\cdot a_{222}\cdot a_{331}-a_{113}\cdot a_{222}\cdot b_{331}-a_{113}\cdot a_{231}\cdot a_{322}-a_{113}\cdot a_{231}\cdot b_{322}\\
&+a_{113}\cdot a_{232}\cdot a_{321}+a_{113}\cdot a_{232}\cdot b_{321}-a_{121}\cdot a_{212}\cdot a_{333}-a_{121}\cdot a_{212}\cdot b_{333}+a_{121}\cdot a_{213}\cdot a_{332}\\
&+a_{121}\cdot a_{213}\cdot b_{332}+a_{121}\cdot a_{232}\cdot a_{313}+a_{121}\cdot a_{232}\cdot b_{313}-a_{121}\cdot a_{233}\cdot a_{312}-a_{121}\cdot a_{233}\cdot b_{312}\\
&+a_{122}\cdot a_{211}\cdot a_{333}+a_{122}\cdot a_{211}\cdot b_{333}-a_{122}\cdot a_{213}\cdot a_{331}-a_{122}\cdot a_{213}\cdot b_{331}-a_{122}\cdot a_{231}\cdot a_{313}\\
&-a_{122}\cdot a_{231}\cdot b_{313}+a_{122}\cdot a_{233}\cdot a_{311}+a_{122}\cdot a_{233}\cdot b_{311}-a_{123}\cdot a_{211}\cdot a_{332}-a_{123}\cdot a_{211}\cdot b_{332}\\
&+a_{123}\cdot a_{212}\cdot a_{331}+a_{123}\cdot a_{212}\cdot b_{331}+a_{123}\cdot a_{231}\cdot a_{312}+a_{123}\cdot a_{231}\cdot b_{312}-a_{123}\cdot a_{232}\cdot a_{311}\\
&-a_{123}\cdot a_{232}\cdot b_{311}+a_{131}\cdot a_{212}\cdot a_{323}+a_{131}\cdot a_{212}\cdot b_{323}-a_{131}\cdot a_{213}\cdot a_{322}-a_{131}\cdot a_{213}\cdot b_{322}\\
&-a_{131}\cdot a_{222}\cdot a_{313}-a_{131}\cdot a_{222}\cdot b_{313}+a_{131}\cdot a_{223}\cdot a_{312}+a_{131}\cdot a_{223}\cdot b_{312}-a_{132}\cdot a_{211}\cdot a_{323}\\
&-a_{132}\cdot a_{211}\cdot b_{323}+a_{132}\cdot a_{213}\cdot a_{321}+a_{132}\cdot a_{213}\cdot b_{321}+a_{132}\cdot a_{221}\cdot a_{313}+a_{132}\cdot a_{221}\cdot b_{313}\\
&-a_{132}\cdot a_{223}\cdot a_{311}-a_{132}\cdot a_{223}\cdot b_{311}+a_{133}\cdot a_{211}\cdot a_{322}+a_{133}\cdot a_{211}\cdot b_{322}-a_{133}\cdot a_{212}\cdot a_{321}\\
&-a_{133}\cdot a_{212}\cdot b_{321}-a_{133}\cdot a_{221}\cdot a_{312}-a_{133}\cdot a_{221}\cdot b_{312}+a_{133}\cdot a_{222}\cdot a_{311}+a_{133}\cdot a_{222}\cdot b_{311}.
\end{aligned}
$$

If we compare results of above equations, we can see that we have the same result in both cases.

2. For plan $i = 2$: Let A and B be cubic-matries of order 3, where all elements on the plan $i = 1$ and $i = 3$ are identical in both matrices, then we have:

$$
\begin{aligned}
det\left(A_{[3\times3\times3]}\right)+det\left(B_{[3\times3\times3]}\right) &= det\left(\begin{array}{ccc|ccc|ccc} a_{111} & a_{121} & a_{131} & a_{112} & a_{122} & a_{132} & a_{113} & a_{123} & a_{133}\\ a_{211} & a_{221} & a_{231} & a_{212} & a_{222} & a_{232} & a_{213} & a_{223} & a_{233}\\ a_{311} & a_{321} & a_{331} & a_{312} & a_{322} & a_{332} & a_{313} & a_{323} & a_{333} \end{array}\right)\\
&+det\left(\begin{array}{ccc|ccc|ccc} a_{111} & a_{121} & a_{131} & a_{112} & a_{122} & a_{132} & a_{113} & a_{123} & a_{133}\\ b_{211} & b_{221} & b_{231} & b_{212} & b_{222} & b_{232} & b_{213} & b_{223} & b_{233}\\ a_{311} & a_{321} & a_{331} & a_{312} & a_{322} & a_{332} & a_{313} & a_{323} & a_{333} \end{array}\right)\\
&= \{a_{111}\cdot a_{222}\cdot a_{333}-a_{111}\cdot a_{232}\cdot a_{323}-a_{111}\cdot a_{223}\cdot a_{332}+a_{111}\cdot a_{233}\cdot a_{322}-a_{112}\cdot a_{221}\cdot a_{333}\\
&+a_{112}\cdot a_{223}\cdot a_{331}+a_{112}\cdot a_{231}\cdot a_{323}-a_{112}\cdot a_{233}\cdot a_{321}+a_{113}\cdot a_{221}\cdot a_{332}-a_{113}\cdot a_{222}\cdot a_{331}\\
&-a_{113}\cdot a_{231}\cdot a_{322}+a_{113}\cdot a_{232}\cdot a_{321}-a_{121}\cdot a_{212}\cdot a_{333}+a_{121}\cdot a_{213}\cdot a_{332}+a_{121}\cdot a_{232}\cdot a_{313}\\
&-a_{121}\cdot a_{233}\cdot a_{312}+a_{122}\cdot a_{211}\cdot a_{333}-a_{122}\cdot a_{213}\cdot a_{331}-a_{122}\cdot a_{231}\cdot a_{313}+a_{122}\cdot a_{233}\cdot a_{311}\\
&-a_{123}\cdot a_{211}\cdot a_{332}+a_{123}\cdot a_{212}\cdot a_{331}+a_{123}\cdot a_{231}\cdot a_{312}-a_{123}\cdot a_{232}\cdot a_{311}+a_{131}\cdot a_{212}\cdot a_{323}\\
&-a_{131}\cdot a_{213}\cdot a_{322}-a_{131}\cdot a_{222}\cdot a_{313}+a_{131}\cdot a_{223}\cdot a_{312}-a_{132}\cdot a_{211}\cdot a_{323}+a_{132}\cdot a_{213}\cdot a_{321}\\
&+a_{132}\cdot a_{221}\cdot a_{313}-a_{132}\cdot a_{223}\cdot a_{311}+a_{133}\cdot a_{211}\cdot a_{322}-a_{133}\cdot a_{212}\cdot a_{321}-a_{133}\cdot a_{221}\cdot a_{312}\\
&+a_{133}\cdot a_{222}\cdot a_{311}\}+\{a_{111}\cdot b_{222}\cdot a_{333}-a_{111}\cdot b_{232}\cdot a_{323}-a_{111}\cdot b_{223}\cdot a_{332}+a_{111}\cdot b_{233}\cdot a_{322}\\
&-a_{112}\cdot b_{221}\cdot a_{333}+a_{112}\cdot b_{223}\cdot a_{331}+a_{112}\cdot b_{231}\cdot a_{323}-a_{112}\cdot b_{233}\cdot a_{321}+a_{113}\cdot b_{221}\cdot a_{332}\\
&-a_{113}\cdot b_{222}\cdot a_{331}-a_{113}\cdot b_{231}\cdot a_{322}+a_{113}\cdot b_{232}\cdot a_{321}-a_{121}\cdot b_{212}\cdot a_{333}+a_{121}\cdot b_{213}\cdot a_{332}\\
&+a_{121}\cdot b_{232}\cdot a_{313}-a_{121}\cdot b_{233}\cdot a_{312}+a_{122}\cdot b_{211}\cdot a_{333}-a_{122}\cdot b_{213}\cdot a_{331}-a_{122}\cdot b_{231}\cdot a_{313}\\
&+a_{122}\cdot b_{233}\cdot a_{311}-a_{123}\cdot b_{211}\cdot a_{332}+a_{123}\cdot b_{212}\cdot a_{331}+a_{123}\cdot b_{231}\cdot a_{312}-a_{123}\cdot b_{232}\cdot a_{311}\\
&+a_{131}\cdot b_{212}\cdot a_{323}-a_{131}\cdot b_{213}\cdot a_{322}-a_{131}\cdot b_{222}\cdot a_{313}+a_{131}\cdot b_{223}\cdot a_{312}-a_{132}\cdot b_{211}\cdot a_{323}\\
&+a_{132}\cdot b_{213}\cdot a_{321}+a_{132}\cdot b_{221}\cdot a_{313}-a_{132}\cdot b_{223}\cdot a_{311}+a_{133}\cdot b_{211}\cdot a_{322}-a_{133}\cdot b_{212}\cdot a_{321}\\
&-a_{133}\cdot b_{221}\cdot a_{312}+a_{133}\cdot b_{222}\cdot a_{311}\},
\end{aligned}
$$

while,

$$
det\left(A_{[3\times3\times3]}\right)+det\left(B_{[3\times3\times3]}\right) = det\left(\begin{array}{ccc|} a_{111} & a_{121} & a_{131}\\ a_{211}+b_{211} & a_{221}+b_{221} & a_{231}+b_{231}\\ a_{311} & a_{321} & a_{331} \end{array}\right.
$$

$$\left.\begin{array}{ccc|ccc} a_{112} & a_{122} & a_{132} & a_{113} & a_{123} & a_{133} \\ a_{212}+b_{212} & a_{222}+b_{222} & a_{232}+b_{232} & a_{213}+b_{213} & a_{223}+b_{223} & a_{233}+b_{233} \\ a_{312} & a_{322} & a_{332} & a_{313} & a_{323} & a_{333} \end{array}\right)$$

$$= a_{111}\cdot(a_{222}+b_{222})\cdot a_{333} - a_{111}\cdot(a_{232}+b_{232})\cdot a_{323} - a_{111}\cdot(a_{223}+b_{223})\cdot a_{332}$$
$$+a_{111}\cdot(a_{233}+b_{233})\cdot a_{322} - a_{112}\cdot(a_{221}+b_{221})\cdot a_{333} + a_{112}\cdot(a_{223}+b_{223})\cdot a_{331}$$
$$+a_{112}\cdot(a_{231}+b_{231})\cdot a_{323} - a_{112}\cdot(a_{233}+b_{233})\cdot a_{321} + a_{113}\cdot(a_{221}+b_{221})\cdot a_{332}$$
$$-a_{113}\cdot(a_{222}+b_{222})\cdot a_{331} - a_{113}\cdot(a_{231}+b_{231})\cdot a_{322} + a_{113}\cdot(a_{232}+b_{232})\cdot a_{321}$$
$$-a_{121}\cdot(a_{212}+b_{212})\cdot a_{333} + a_{121}\cdot(a_{213}+b_{213})\cdot a_{332} + a_{121}\cdot(a_{232}+b_{232})\cdot a_{313}$$
$$-a_{121}\cdot(a_{233}+b_{233})\cdot a_{312} + a_{122}\cdot(a_{211}+b_{211})\cdot a_{333} - a_{122}\cdot(a_{213}+b_{213})\cdot a_{331}$$
$$-a_{122}\cdot(a_{231}+b_{231})\cdot a_{313} + a_{122}\cdot(a_{233}+b_{233})\cdot a_{311} - a_{123}\cdot(a_{211}+b_{211})\cdot a_{332}$$
$$+a_{123}\cdot(a_{212}+b_{212})\cdot a_{331} + a_{123}\cdot(a_{231}+b_{231})\cdot a_{312} - a_{123}\cdot(a_{232}+b_{232})\cdot a_{311}$$
$$+a_{131}\cdot(a_{212}+b_{212})\cdot a_{323} - a_{131}\cdot(a_{213}+b_{213})\cdot a_{322} - a_{131}\cdot(a_{222}+b_{222})\cdot a_{313}$$
$$+a_{131}\cdot(a_{223}+b_{223})\cdot a_{312} - a_{132}\cdot(a_{211}+b_{211})\cdot a_{323} + a_{132}\cdot(a_{213}+b_{213})\cdot a_{321}$$
$$+a_{132}\cdot(a_{221}+b_{221})\cdot a_{313} - a_{132}\cdot(a_{223}+b_{223})\cdot a_{311} + a_{133}\cdot(a_{211}+b_{211})\cdot a_{322}$$
$$-a_{133}\cdot(a_{212}+b_{212})\cdot a_{321} - a_{133}\cdot(a_{221}+b_{221})\cdot a_{312} + a_{133}\cdot(a_{222}+b_{222})\cdot a_{311}.$$

Hence,

$$det\left(A_{[3\times3\times3]}\right) + det\left(B_{[3\times3\times3]}\right) = a_{111}\cdot a_{222}\cdot a_{333} + a_{111}\cdot b_{222}\cdot a_{333}$$
$$-a_{111}\cdot a_{232}\cdot a_{323} - a_{111}\cdot b_{232}\cdot a_{323} - a_{111}\cdot a_{223}\cdot a_{332} - a_{111}\cdot b_{223}\cdot a_{332} + a_{111}\cdot a_{233}\cdot a_{322}$$
$$+a_{111}\cdot b_{233}\cdot a_{322} - a_{112}\cdot a_{221}\cdot a_{333} - a_{112}\cdot b_{221}\cdot a_{333} + a_{112}\cdot a_{223}\cdot a_{331} + a_{112}\cdot b_{223}\cdot a_{331}$$
$$+a_{112}\cdot a_{231}\cdot a_{323} + a_{112}\cdot b_{231}\cdot a_{323} - a_{112}\cdot a_{233}\cdot a_{321} - a_{112}\cdot b_{233}\cdot a_{321} + a_{113}\cdot a_{221}\cdot a_{332}$$
$$+a_{113}\cdot b_{221}\cdot a_{332} - a_{113}\cdot a_{222}\cdot a_{331} - a_{113}\cdot b_{222}\cdot a_{331} - a_{113}\cdot a_{231}\cdot a_{322} - a_{113}\cdot b_{231}\cdot a_{322}$$
$$+a_{113}\cdot a_{232}\cdot a_{321} + a_{113}\cdot b_{232}\cdot a_{321} - a_{121}\cdot a_{212}\cdot a_{333} - a_{121}\cdot b_{212}\cdot a_{333} + a_{121}\cdot a_{213}\cdot a_{332}$$
$$+a_{121}\cdot b_{213}\cdot a_{332} + a_{121}\cdot a_{232}\cdot a_{313} + a_{121}\cdot b_{232}\cdot a_{313} - a_{121}\cdot a_{233}\cdot a_{312} - a_{121}\cdot b_{233}\cdot a_{312}$$
$$+a_{122}\cdot a_{211}\cdot a_{333} + a_{122}\cdot b_{211}\cdot a_{333} - a_{122}\cdot a_{213}\cdot a_{331} - a_{122}\cdot b_{213}\cdot a_{331} - a_{122}\cdot a_{231}\cdot a_{313}$$
$$-a_{122}\cdot b_{231}\cdot a_{313} + a_{122}\cdot a_{233}\cdot a_{311} + a_{122}\cdot b_{233}\cdot a_{311} - a_{123}\cdot a_{211}\cdot a_{332} - a_{123}\cdot b_{211}\cdot a_{332}$$
$$+a_{123}\cdot a_{212}\cdot a_{331} + a_{123}\cdot b_{212}\cdot a_{331} + a_{123}\cdot a_{231}\cdot a_{312} + a_{123}\cdot b_{231}\cdot a_{312} - a_{123}\cdot a_{232}\cdot a_{311}$$
$$-a_{123}\cdot b_{232}\cdot a_{311} + a_{131}\cdot a_{212}\cdot a_{323} + a_{131}\cdot b_{212}\cdot a_{323} - a_{131}\cdot a_{213}\cdot a_{322} - a_{131}\cdot b_{213}\cdot a_{322}$$
$$-a_{131}\cdot a_{222}\cdot a_{313} - a_{131}\cdot b_{222}\cdot a_{313} + a_{131}\cdot a_{223}\cdot a_{312} + a_{131}\cdot b_{223}\cdot a_{312} - a_{132}\cdot a_{211}\cdot a_{323}$$
$$-a_{132}\cdot b_{211}\cdot a_{323} + a_{132}\cdot a_{213}\cdot a_{321} + a_{132}\cdot b_{213}\cdot a_{321} + a_{132}\cdot a_{221}\cdot a_{313} + a_{132}\cdot b_{221}\cdot a_{313}$$
$$-a_{132}\cdot a_{223}\cdot a_{311} - a_{132}\cdot b_{223}\cdot a_{311} + a_{133}\cdot a_{211}\cdot a_{322} + a_{133}\cdot b_{211}\cdot a_{322} - a_{133}\cdot a_{212}\cdot a_{321}$$
$$-a_{133}\cdot b_{212}\cdot a_{321} - a_{133}\cdot a_{221}\cdot a_{312} - a_{133}\cdot b_{221}\cdot a_{312} + a_{133}\cdot a_{222}\cdot a_{311} + a_{133}\cdot b_{222}\cdot a_{311}.$$

If we compare results of above equations, we can see that we have the same result in both cases.

3. For plan $i = 3$: Let A and B be cubic-matries of order 3, where all elements on the plan $i = 2$ and $i = 3$ are identical in both matrices, then we have:

$$det\left(A_{[3\times3\times3]}\right) + det\left(B_{[3\times3\times3]}\right) = det\begin{pmatrix} a_{111} & a_{121} & a_{131} & a_{112} & a_{122} & a_{132} & a_{113} & a_{123} & a_{133} \\ a_{211} & a_{221} & a_{231} & a_{212} & a_{222} & a_{232} & a_{213} & a_{223} & a_{233} \\ a_{311} & a_{321} & a_{331} & a_{312} & a_{322} & a_{332} & a_{313} & a_{323} & a_{333} \end{pmatrix}$$
$$+det\begin{pmatrix} b_{111} & b_{121} & b_{131} & b_{112} & b_{122} & b_{132} & b_{113} & b_{123} & b_{133} \\ a_{211} & a_{221} & a_{231} & a_{212} & a_{222} & a_{232} & a_{213} & a_{223} & a_{233} \\ a_{311} & a_{321} & a_{331} & a_{312} & a_{322} & a_{332} & a_{313} & a_{323} & a_{333} \end{pmatrix}$$
$$= \{a_{111}\cdot a_{222}\cdot a_{333} - a_{111}\cdot a_{232}\cdot a_{323} - a_{111}\cdot a_{223}\cdot a_{332} + a_{111}\cdot a_{233}\cdot a_{322} - a_{112}\cdot a_{221}\cdot a_{333}$$
$$+a_{112}\cdot a_{223}\cdot a_{331} + a_{112}\cdot a_{231}\cdot a_{323} - a_{112}\cdot a_{233}\cdot a_{321} + a_{113}\cdot a_{221}\cdot a_{332} - a_{113}\cdot a_{222}\cdot a_{331}$$
$$-a_{113}\cdot a_{231}\cdot a_{322} + a_{113}\cdot a_{232}\cdot a_{321} - a_{121}\cdot a_{212}\cdot a_{333} + a_{121}\cdot a_{213}\cdot a_{332} + a_{121}\cdot a_{232}\cdot a_{313}$$

$$-a_{121}\cdot a_{233}\cdot a_{312}+a_{122}\cdot a_{211}\cdot a_{333}-a_{122}\cdot a_{213}\cdot a_{331}-a_{122}\cdot a_{231}\cdot a_{313}+a_{122}\cdot a_{233}\cdot a_{311}$$
$$-a_{123}\cdot a_{211}\cdot a_{332}+a_{123}\cdot a_{212}\cdot a_{331}+a_{123}\cdot a_{231}\cdot a_{312}-a_{123}\cdot a_{232}\cdot a_{311}+a_{131}\cdot a_{212}\cdot a_{323}$$
$$-a_{131}\cdot a_{213}\cdot a_{322}-a_{131}\cdot a_{222}\cdot a_{313}+a_{131}\cdot a_{223}\cdot a_{312}-a_{132}\cdot a_{211}\cdot a_{323}+a_{132}\cdot a_{213}\cdot a_{321}$$
$$+a_{132}\cdot a_{221}\cdot a_{313}-a_{132}\cdot a_{223}\cdot a_{311}+a_{133}\cdot a_{211}\cdot a_{322}-a_{133}\cdot a_{212}\cdot a_{321}-a_{133}\cdot a_{221}\cdot a_{312}$$
$$+a_{133}\cdot a_{222}\cdot a_{311}\}+\{b_{111}\cdot a_{222}\cdot a_{333}-b_{111}\cdot a_{232}\cdot a_{323}-b_{111}\cdot a_{223}\cdot a_{332}+b_{111}\cdot a_{233}\cdot a_{322}$$
$$-b_{112}\cdot a_{221}\cdot a_{333}+b_{112}\cdot a_{223}\cdot a_{331}+b_{112}\cdot a_{231}\cdot a_{323}-b_{112}\cdot a_{233}\cdot a_{321}+b_{113}\cdot a_{221}\cdot a_{332}$$
$$-b_{113}\cdot a_{222}\cdot a_{331}-b_{113}\cdot a_{231}\cdot a_{322}+b_{113}\cdot a_{232}\cdot a_{321}-b_{121}\cdot a_{212}\cdot a_{333}+b_{121}\cdot a_{213}\cdot a_{332}$$
$$+b_{121}\cdot a_{232}\cdot a_{313}-b_{121}\cdot a_{233}\cdot a_{312}+b_{122}\cdot a_{211}\cdot a_{333}-b_{122}\cdot a_{213}\cdot a_{331}-b_{122}\cdot a_{231}\cdot a_{313}$$
$$+b_{122}\cdot a_{233}\cdot a_{311}-b_{123}\cdot a_{211}\cdot a_{332}+b_{123}\cdot a_{212}\cdot a_{331}+b_{123}\cdot a_{231}\cdot a_{312}-b_{123}\cdot a_{232}\cdot a_{311}$$
$$+b_{131}\cdot a_{212}\cdot a_{323}-b_{131}\cdot a_{213}\cdot a_{322}-b_{131}\cdot a_{222}\cdot a_{313}+b_{131}\cdot a_{223}\cdot a_{312}-b_{132}\cdot a_{211}\cdot a_{323}$$
$$+b_{132}\cdot a_{213}\cdot a_{321}+b_{132}\cdot a_{221}\cdot a_{313}-b_{132}\cdot a_{223}\cdot a_{311}+b_{133}\cdot a_{211}\cdot a_{322}-b_{133}\cdot a_{212}\cdot a_{321}$$
$$-b_{133}\cdot a_{221}\cdot a_{312}+b_{133}\cdot a_{222}\cdot a_{311},$$

while,

$$det\left(A_{[3\times3\times3]}\right)+det\left(B_{[3\times3\times3]}\right)=det\left(\begin{matrix}a_{111}+b_{111} & a_{121}+b_{121} & a_{131}+b_{131}\\ a_{211} & a_{221} & a_{231}\\ a_{311} & a_{321} & a_{331}\end{matrix}\middle|\begin{matrix}a_{112}+b_{112} & a_{122}+b_{122} & a_{132}+b_{132}\\ a_{212} & a_{222} & a_{232}\\ a_{312} & a_{322} & a_{332}\end{matrix}\middle|\begin{matrix}a_{113}+b_{113} & a_{123}+b_{123} & a_{133}+b_{133}\\ a_{213} & a_{223} & a_{233}\\ a_{313} & a_{323} & a_{333}\end{matrix}\right)$$
$$=(a_{111}+b_{111})\cdot a_{222}\cdot a_{333}-(a_{111}+b_{111})\cdot a_{232}\cdot a_{323}-(a_{111}+b_{111})\cdot a_{223}\cdot a_{332}$$
$$+(a_{111}+b_{111})\cdot a_{233}\cdot a_{322}-(a_{112}+b_{112})\cdot a_{221}\cdot a_{333}+(a_{112}+b_{112})\cdot a_{223}\cdot a_{331}$$
$$+(a_{112}+b_{112})\cdot a_{231}\cdot a_{323}-(a_{112}+b_{112})\cdot a_{233}\cdot a_{321}+(a_{113}+b_{113})\cdot a_{221}\cdot a_{332}$$
$$-(a_{113}+b_{113})\cdot a_{222}\cdot a_{331}-(a_{113}+b_{113})\cdot a_{231}\cdot a_{322}+(a_{113}+b_{113})\cdot a_{232}\cdot a_{321}$$
$$-(a_{121}+b_{121})\cdot a_{212}\cdot a_{333}+(a_{121}+b_{121})\cdot a_{213}\cdot a_{332}+(a_{121}+b_{121})\cdot a_{232}\cdot a_{313}$$
$$-(a_{121}+b_{121})\cdot a_{233}\cdot a_{312}+(a_{122}+b_{122})\cdot a_{211}\cdot a_{333}-(a_{122}+b_{122})\cdot a_{213}\cdot a_{331}$$
$$-(a_{122}+b_{122})\cdot a_{231}\cdot a_{313}+(a_{122}+b_{122})\cdot a_{233}\cdot a_{311}-(a_{123}+b_{123})\cdot a_{211}\cdot a_{332}$$
$$+(a_{123}+b_{123})\cdot a_{212}\cdot a_{331}+(a_{123}+b_{123})\cdot a_{231}\cdot a_{312}-(a_{123}+b_{123})\cdot a_{232}\cdot a_{311}$$
$$+(a_{131}+b_{131})\cdot a_{212}\cdot a_{323}-(a_{131}+b_{131})\cdot a_{213}\cdot a_{322}-(a_{131}+b_{131})\cdot a_{222}\cdot a_{313}$$
$$+(a_{131}+b_{131})\cdot a_{223}\cdot a_{312}-(a_{132}+b_{132})\cdot a_{211}\cdot a_{323}+(a_{132}+b_{132})\cdot a_{213}\cdot a_{321}$$
$$+(a_{132}+b_{132})\cdot a_{221}\cdot a_{313}-(a_{132}+b_{132})\cdot a_{223}\cdot a_{311}+(a_{133}+b_{133})\cdot a_{211}\cdot a_{322}$$
$$-(a_{133}+b_{133})\cdot a_{212}\cdot a_{321}-(a_{133}+b_{133})\cdot a_{221}\cdot a_{312}+(a_{133}+b_{133})\cdot a_{222}\cdot a_{311}.$$

Hence,

$$det\left(A_{[3\times3\times3]}\right)+det\left(B_{[3\times3\times3]}\right)=a_{111}\cdot a_{222}\cdot a_{333}+b_{111}\cdot a_{222}\cdot a_{333}$$
$$-a_{111}\cdot a_{232}\cdot a_{323}-b_{111}\cdot a_{232}\cdot a_{323}-a_{111}\cdot a_{223}\cdot a_{332}-b_{111}\cdot a_{223}\cdot a_{332}+a_{111}\cdot a_{233}\cdot a_{322}$$
$$+b_{111}\cdot a_{233}\cdot a_{322}-a_{112}\cdot a_{221}\cdot a_{333}-b_{112}\cdot a_{221}\cdot a_{333}+a_{112}\cdot a_{223}\cdot a_{331}+b_{112}\cdot a_{223}\cdot a_{331}$$
$$+a_{112}\cdot a_{231}\cdot a_{323}+b_{112}\cdot a_{231}\cdot a_{323}-a_{112}\cdot a_{233}\cdot a_{321}-b_{112}\cdot a_{233}\cdot a_{321}+a_{113}\cdot a_{221}\cdot a_{332}$$
$$+b_{113}\cdot a_{221}\cdot a_{332}-a_{113}\cdot a_{222}\cdot a_{331}-b_{113}\cdot a_{222}\cdot a_{331}-a_{113}\cdot a_{231}\cdot a_{322}-b_{113}\cdot a_{231}\cdot a_{322}$$
$$+a_{113}\cdot a_{232}\cdot a_{321}+b_{113}\cdot a_{232}\cdot a_{321}-a_{121}\cdot a_{212}\cdot a_{333}-b_{121}\cdot a_{212}\cdot a_{333}+a_{121}\cdot a_{213}\cdot a_{332}$$
$$+b_{121}\cdot a_{213}\cdot a_{332}+a_{121}\cdot a_{232}\cdot a_{313}+b_{121}\cdot a_{232}\cdot a_{313}-a_{121}\cdot a_{233}\cdot a_{312}-b_{121}\cdot a_{233}\cdot a_{312}$$
$$+a_{122}\cdot a_{211}\cdot a_{333}+b_{122}\cdot a_{211}\cdot a_{333}-a_{122}\cdot a_{213}\cdot a_{331}-b_{122}\cdot a_{213}\cdot a_{331}-a_{122}\cdot a_{231}\cdot a_{313}$$
$$-b_{122}\cdot a_{231}\cdot a_{313}+a_{122}\cdot a_{233}\cdot a_{311}+b_{122}\cdot a_{233}\cdot a_{311}-a_{123}\cdot a_{211}\cdot a_{332}-b_{123}\cdot a_{211}\cdot a_{332}$$
$$+a_{123}\cdot a_{212}\cdot a_{331}+b_{123}\cdot a_{212}\cdot a_{331}+a_{123}\cdot a_{231}\cdot a_{312}+b_{123}\cdot a_{231}\cdot a_{312}-a_{123}\cdot a_{232}\cdot a_{311}$$

$$-b_{123}\cdot a_{232}\cdot a_{311}+a_{131}\cdot a_{212}\cdot a_{323}+b_{131}\cdot a_{212}\cdot a_{323}-a_{131}\cdot a_{213}\cdot a_{322}-b_{131}\cdot a_{213}\cdot a_{322}$$
$$-a_{131}\cdot a_{222}\cdot a_{313}-b_{131}\cdot a_{222}\cdot a_{313}+a_{131}\cdot a_{223}\cdot a_{312}+b_{131}\cdot a_{223}\cdot a_{312}-a_{132}\cdot a_{211}\cdot a_{323}$$
$$-b_{132}\cdot a_{211}\cdot a_{323}+a_{132}\cdot a_{213}\cdot a_{321}+b_{132}\cdot a_{213}\cdot a_{321}+a_{132}\cdot a_{221}\cdot a_{313}+b_{132}\cdot a_{221}\cdot a_{313}$$
$$-a_{132}\cdot a_{223}\cdot a_{311}-b_{132}\cdot a_{223}\cdot a_{311}+a_{133}\cdot a_{211}\cdot a_{322}+b_{133}\cdot a_{211}\cdot a_{322}-a_{133}\cdot a_{212}\cdot a_{321}$$
$$-b_{133}\cdot a_{212}\cdot a_{321}-a_{133}\cdot a_{221}\cdot a_{312}-b_{133}\cdot a_{221}\cdot a_{312}+a_{133}\cdot a_{222}\cdot a_{311}+b_{133}\cdot a_{222}\cdot a_{311}.$$

If we compare results of above equations, we can see that we have the same result in both cases.

4. For plan $j=1$: Let A and B be cubic-matries of order 3, where all elements on the plan $j=1$ and $j=2$ are identical in both matrices, then we have:

$$det\left(A_{[3\times3\times3]}\right)+det\left(B_{[3\times3\times3]}\right)=det\left(\begin{array}{ccc|ccc|ccc} a_{111} & a_{121} & a_{131} & a_{112} & a_{122} & a_{132} & a_{113} & a_{123} & a_{133} \\ a_{211} & a_{221} & a_{231} & a_{212} & a_{222} & a_{232} & a_{213} & a_{223} & a_{233} \\ a_{311} & a_{321} & a_{331} & a_{312} & a_{322} & a_{332} & a_{313} & a_{323} & a_{333} \end{array}\right)$$
$$+det\left(\begin{array}{ccc|ccc|ccc} a_{111} & a_{121} & b_{131} & a_{112} & a_{122} & b_{132} & a_{113} & a_{123} & b_{133} \\ a_{211} & a_{221} & b_{231} & a_{212} & a_{222} & b_{232} & a_{213} & a_{223} & b_{233} \\ a_{311} & a_{321} & b_{331} & a_{312} & a_{322} & b_{332} & a_{313} & a_{323} & b_{333} \end{array}\right)$$
$$=\{a_{111}\cdot a_{222}\cdot a_{333}-a_{111}\cdot a_{232}\cdot a_{323}-a_{111}\cdot a_{223}\cdot a_{332}+a_{111}\cdot a_{233}\cdot a_{322}-a_{112}\cdot a_{221}\cdot a_{333}$$
$$+a_{112}\cdot a_{223}\cdot a_{331}+a_{112}\cdot a_{231}\cdot a_{323}-a_{112}\cdot a_{233}\cdot a_{321}+a_{113}\cdot a_{221}\cdot a_{332}-a_{113}\cdot a_{222}\cdot a_{331}$$
$$-a_{113}\cdot a_{231}\cdot a_{322}+a_{113}\cdot a_{232}\cdot a_{321}-a_{121}\cdot a_{212}\cdot a_{333}+a_{121}\cdot a_{213}\cdot a_{332}+a_{121}\cdot a_{232}\cdot a_{313}$$
$$-a_{121}\cdot a_{233}\cdot a_{312}+a_{122}\cdot a_{211}\cdot a_{333}-a_{122}\cdot a_{213}\cdot a_{331}-a_{122}\cdot a_{231}\cdot a_{313}+a_{122}\cdot a_{233}\cdot a_{311}$$
$$-a_{123}\cdot a_{211}\cdot a_{332}+a_{123}\cdot a_{212}\cdot a_{331}+a_{123}\cdot a_{231}\cdot a_{312}-a_{123}\cdot a_{232}\cdot a_{311}+a_{131}\cdot a_{212}\cdot a_{323}$$
$$-a_{131}\cdot a_{213}\cdot a_{322}-a_{131}\cdot a_{222}\cdot a_{313}+a_{131}\cdot a_{223}\cdot a_{312}-a_{132}\cdot a_{211}\cdot a_{323}+a_{132}\cdot a_{213}\cdot a_{321}$$
$$+a_{132}\cdot a_{221}\cdot a_{313}-a_{132}\cdot a_{223}\cdot a_{311}+a_{133}\cdot a_{211}\cdot a_{322}-a_{133}\cdot a_{212}\cdot a_{321}-a_{133}\cdot a_{221}\cdot a_{312}$$
$$+a_{133}\cdot a_{222}\cdot a_{311}\}+\{a_{111}\cdot a_{222}\cdot b_{333}-a_{111}\cdot b_{232}\cdot a_{323}-a_{111}\cdot a_{223}\cdot b_{332}+a_{111}\cdot b_{233}\cdot a_{322}$$
$$-a_{112}\cdot a_{221}\cdot b_{333}+a_{112}\cdot a_{223}\cdot b_{331}+a_{112}\cdot b_{231}\cdot a_{323}-a_{112}\cdot b_{233}\cdot a_{321}+a_{113}\cdot a_{221}\cdot b_{332}$$
$$-a_{113}\cdot a_{222}\cdot b_{331}-a_{113}\cdot b_{231}\cdot a_{322}+a_{113}\cdot b_{232}\cdot a_{321}-a_{121}\cdot a_{212}\cdot b_{333}+a_{121}\cdot a_{213}\cdot b_{332}$$
$$+a_{121}\cdot b_{232}\cdot a_{313}-a_{121}\cdot b_{233}\cdot a_{312}+a_{122}\cdot a_{211}\cdot b_{333}-a_{122}\cdot a_{213}\cdot b_{331}-a_{122}\cdot b_{231}\cdot a_{313}$$
$$+a_{122}\cdot b_{233}\cdot a_{311}-a_{123}\cdot a_{211}\cdot b_{332}+a_{123}\cdot a_{212}\cdot b_{331}+a_{123}\cdot b_{231}\cdot a_{312}-a_{123}\cdot b_{232}\cdot a_{311}$$
$$+b_{131}\cdot a_{212}\cdot a_{323}-b_{131}\cdot a_{213}\cdot a_{322}-b_{131}\cdot a_{222}\cdot a_{313}+b_{131}\cdot a_{223}\cdot a_{312}-b_{132}\cdot a_{211}\cdot a_{323}$$
$$+b_{132}\cdot a_{213}\cdot a_{321}+b_{132}\cdot a_{221}\cdot a_{313}-b_{132}\cdot a_{223}\cdot a_{311}+b_{133}\cdot a_{211}\cdot a_{322}-b_{133}\cdot a_{212}\cdot a_{321}$$
$$-b_{133}\cdot a_{221}\cdot a_{312}+b_{133}\cdot a_{222}\cdot a_{311}\},$$

while,

$$\det\left(A_{[3\times3\times3]}\right)+det\left(B_{[3\times3\times3]}\right)$$
$$=\det\left(\begin{array}{ccc|ccc|ccc} a_{111} & a_{121} & a_{131}+b_{131} & a_{112} & a_{122} & a_{132}+b_{132} & a_{113} & a_{123} & a_{133}+b_{133} \\ a_{211} & a_{221} & a_{231}+b_{231} & a_{212} & a_{222} & a_{232}+b_{232} & a_{213} & a_{223} & a_{233}+b_{233} \\ a_{311} & a_{321} & a_{331}+b_{331} & a_{312} & a_{322} & a_{332}+b_{332} & a_{313} & a_{323} & a_{333}+b_{333} \end{array}\right)$$
$$=a_{111}\cdot a_{222}\cdot(a_{333}+b_{333})-a_{111}\cdot(a_{232}+b_{232})\cdot a_{323}-a_{111}\cdot a_{223}\cdot(a_{332}+b_{332})$$
$$+a_{111}\cdot(a_{233}+b_{233})\cdot a_{322}-a_{112}\cdot a_{221}\cdot(a_{333}+b_{333})+a_{112}\cdot a_{223}\cdot(a_{331}+b_{331})$$
$$+a_{112}\cdot(a_{231}+b_{231})\cdot a_{323}-a_{112}\cdot(a_{233}+b_{233})\cdot a_{321}+a_{113}\cdot a_{221}\cdot(a_{332}+b_{332})$$
$$-a_{113}\cdot a_{222}\cdot(a_{331}+b_{331})-a_{113}\cdot(a_{231}+b_{231})\cdot a_{322}+a_{113}\cdot(a_{232}+b_{232})\cdot a_{321}$$
$$-a_{121}\cdot a_{212}\cdot(a_{333}+b_{333})+a_{121}\cdot a_{213}\cdot(a_{332}+b_{332})+a_{121}\cdot(a_{232}+b_{232})\cdot a_{313}$$
$$-a_{121}\cdot(a_{233}+b_{233})\cdot a_{312}+a_{122}\cdot a_{211}\cdot(a_{333}+b_{333})-a_{122}\cdot a_{213}\cdot(a_{331}+b_{331})$$
$$-a_{122}\cdot(a_{231}+b_{231})\cdot a_{313}+a_{122}\cdot(a_{233}+b_{233})\cdot a_{311}-a_{123}\cdot a_{211}\cdot(a_{332}+b_{332})$$
$$+a_{123}\cdot a_{212}\cdot(a_{331}+b_{331})+a_{123}\cdot(a_{231}+b_{231})\cdot a_{312}-a_{123}\cdot(a_{232}+b_{232})\cdot a_{311}$$

$$
\begin{aligned}
&+(a_{131}+b_{131})\cdot a_{212}\cdot a_{323}-(a_{131}+b_{131})\cdot a_{213}\cdot a_{322}-(a_{131}+b_{131})\cdot a_{222}\cdot a_{313}\\
&+(a_{131}+b_{131})\cdot a_{223}\cdot a_{312}-(a_{132}+b_{132})\cdot a_{211}\cdot a_{323}+(a_{132}+b_{132})\cdot a_{213}\cdot a_{321}\\
&+(a_{132}+b_{132})\cdot a_{221}\cdot a_{313}-(a_{132}+b_{132})\cdot a_{223}\cdot a_{311}+(a_{133}+b_{133})\cdot a_{211}\cdot a_{322}\\
&-(a_{133}+b_{133})\cdot a_{212}\cdot a_{321}-(a_{133}+b_{133})\cdot a_{221}\cdot a_{312}+(a_{133}+b_{133})\cdot a_{222}\cdot a_{311}\\
&=a_{111}\cdot a_{222}\cdot a_{333}+a_{111}\cdot a_{222}\cdot b_{333}-a_{111}\cdot a_{232}\cdot a_{323}-a_{111}\cdot b_{232}\cdot a_{323}-a_{111}\cdot a_{223}\cdot a_{332}\\
&-a_{111}\cdot a_{223}\cdot b_{332}+a_{111}\cdot a_{233}\cdot a_{322}+a_{111}\cdot b_{233}\cdot a_{322}-a_{112}\cdot a_{221}\cdot a_{333}-a_{112}\cdot a_{221}\cdot b_{333}\\
&+a_{112}\cdot a_{223}\cdot a_{331}+a_{112}\cdot b_{223}\cdot a_{331}+a_{112}\cdot a_{231}\cdot a_{323}+a_{112}\cdot b_{231}\cdot a_{323}-a_{112}\cdot a_{233}\cdot a_{321}\\
&-a_{112}\cdot b_{233}\cdot a_{321}+a_{113}\cdot a_{221}\cdot a_{332}+a_{113}\cdot a_{221}\cdot b_{332}-a_{113}\cdot a_{222}\cdot a_{331}-a_{113}\cdot a_{222}\cdot b_{331}\\
&-a_{113}\cdot a_{231}\cdot a_{322}-a_{113}\cdot b_{231}\cdot a_{322}+a_{113}\cdot a_{232}\cdot a_{321}+a_{113}\cdot b_{232}\cdot a_{321}-a_{121}\cdot a_{212}\cdot a_{333}\\
&-a_{121}\cdot a_{212}\cdot b_{333}+a_{121}\cdot a_{213}\cdot a_{332}+a_{121}\cdot a_{213}\cdot b_{332}+a_{121}\cdot a_{232}\cdot a_{313}+a_{121}\cdot b_{232}\cdot a_{313}\\
&-a_{121}\cdot a_{233}\cdot a_{312}-a_{121}\cdot b_{233}\cdot a_{312}+a_{122}\cdot a_{211}\cdot a_{333}+a_{122}\cdot a_{211}\cdot b_{333}-a_{122}\cdot a_{213}\cdot a_{331}\\
&-a_{122}\cdot a_{213}\cdot b_{331}-a_{122}\cdot a_{231}\cdot a_{313}-a_{122}\cdot b_{231}\cdot a_{313}+a_{122}\cdot a_{233}\cdot a_{311}+a_{122}\cdot b_{233}\cdot a_{311}\\
&-a_{123}\cdot a_{211}\cdot a_{332}-a_{123}\cdot a_{211}\cdot b_{332}+a_{123}\cdot a_{212}\cdot a_{331}+a_{123}\cdot a_{212}\cdot b_{331}+a_{123}\cdot a_{231}\cdot a_{312}\\
&+a_{123}\cdot b_{231}\cdot a_{312}-a_{123}\cdot a_{232}\cdot a_{311}-a_{123}\cdot b_{232}\cdot a_{311}+a_{131}\cdot a_{212}\cdot a_{323}+b_{131}\cdot a_{212}\cdot a_{323}\\
&-a_{131}\cdot a_{213}\cdot a_{322}-b_{131}\cdot a_{213}\cdot a_{322}-a_{131}\cdot a_{222}\cdot a_{313}-b_{131}\cdot a_{222}\cdot a_{313}+a_{131}\cdot a_{223}\cdot a_{312}\\
&+b_{131}\cdot a_{223}\cdot a_{312}-a_{132}\cdot a_{211}\cdot a_{323}-b_{132}\cdot a_{211}\cdot a_{323}+a_{132}\cdot a_{213}\cdot a_{321}+b_{132}\cdot a_{213}\cdot a_{321}\\
&+a_{132}\cdot a_{221}\cdot a_{313}+b_{132}\cdot a_{221}\cdot a_{313}-a_{132}\cdot a_{223}\cdot a_{311}-b_{132}\cdot a_{223}\cdot a_{311}+a_{133}\cdot a_{211}\cdot a_{322}\\
&+b_{133}\cdot a_{211}\cdot a_{322}-a_{133}\cdot a_{212}\cdot a_{321}-b_{133}\cdot a_{212}\cdot a_{321}-a_{133}\cdot a_{221}\cdot a_{312}-b_{133}\cdot a_{221}\cdot a_{312}\\
&+a_{133}\cdot a_{222}\cdot a_{311}+b_{133}\cdot a_{222}\cdot a_{311}.
\end{aligned}
$$

If we compare results of above equations, we can see that we have the same result in both cases.

5. For plan $j=2$: Let A and B be cubic-matries of order 3, where all elements on the plan $j=1$ and $j=3$ are identical in both matrices, then we have:

$$
det\left(A_{[3\times3\times3]}\right)+det\left(B_{[3\times3\times3]}\right)=det\left(\begin{array}{ccc|ccc|ccc}a_{111}&a_{121}&a_{131}&a_{112}&a_{122}&a_{132}&a_{113}&a_{123}&a_{133}\\a_{211}&a_{221}&a_{231}&a_{212}&a_{222}&a_{232}&a_{213}&a_{223}&a_{233}\\a_{311}&a_{321}&a_{331}&a_{312}&a_{322}&a_{332}&a_{313}&a_{323}&a_{333}\end{array}\right)
$$

$$
+det\left(\begin{array}{ccc|ccc|ccc}a_{111}&b_{121}&a_{131}&a_{112}&b_{122}&a_{132}&a_{113}&b_{123}&a_{133}\\a_{211}&b_{221}&a_{231}&a_{212}&b_{222}&a_{232}&a_{213}&b_{223}&a_{233}\\a_{311}&b_{321}&a_{331}&a_{312}&b_{322}&a_{332}&a_{313}&b_{323}&a_{333}\end{array}\right)
$$

$$
\begin{aligned}
&=\{a_{111}\cdot a_{222}\cdot a_{333}-a_{111}\cdot a_{232}\cdot a_{323}-a_{111}\cdot a_{223}\cdot a_{332}+a_{111}\cdot a_{233}\cdot a_{322}-a_{112}\cdot a_{221}\cdot a_{333}\\
&+a_{112}\cdot a_{223}\cdot a_{331}+a_{112}\cdot a_{231}\cdot a_{323}-a_{112}\cdot a_{233}\cdot a_{321}+a_{113}\cdot a_{221}\cdot a_{332}-a_{113}\cdot a_{222}\cdot a_{331}\\
&-a_{113}\cdot a_{231}\cdot a_{322}+a_{113}\cdot a_{232}\cdot a_{321}-a_{121}\cdot a_{212}\cdot a_{333}+a_{121}\cdot a_{213}\cdot a_{332}+a_{121}\cdot a_{232}\cdot a_{313}\\
&-a_{121}\cdot a_{233}\cdot a_{312}+a_{122}\cdot a_{211}\cdot a_{333}-a_{122}\cdot a_{213}\cdot a_{331}-a_{122}\cdot a_{231}\cdot a_{313}+a_{122}\cdot a_{233}\cdot a_{311}\\
&-a_{123}\cdot a_{211}\cdot a_{332}+a_{123}\cdot a_{212}\cdot a_{331}+a_{123}\cdot a_{231}\cdot a_{312}-a_{123}\cdot a_{232}\cdot a_{311}+a_{131}\cdot a_{212}\cdot a_{323}\\
&-a_{131}\cdot a_{213}\cdot a_{322}-a_{131}\cdot a_{222}\cdot a_{313}+a_{131}\cdot a_{223}\cdot a_{312}-a_{132}\cdot a_{211}\cdot a_{323}+a_{132}\cdot a_{213}\cdot a_{321}\\
&+a_{132}\cdot a_{221}\cdot a_{313}-a_{132}\cdot a_{223}\cdot a_{311}+a_{133}\cdot a_{211}\cdot a_{322}-a_{133}\cdot a_{212}\cdot a_{321}-a_{133}\cdot a_{221}\cdot a_{312}\\
&+a_{133}\cdot a_{222}\cdot a_{311}\}+\{a_{111}\cdot b_{222}\cdot a_{333}-a_{111}\cdot a_{232}\cdot b_{323}-a_{111}\cdot b_{223}\cdot a_{332}+a_{111}\cdot a_{233}\cdot b_{322}\\
&-a_{112}\cdot b_{221}\cdot a_{333}+a_{112}\cdot b_{223}\cdot a_{331}+a_{112}\cdot a_{231}\cdot b_{323}-a_{112}\cdot a_{233}\cdot b_{321}+a_{113}\cdot b_{221}\cdot a_{332}\\
&-a_{113}\cdot b_{222}\cdot a_{331}-a_{113}\cdot a_{231}\cdot b_{322}+a_{113}\cdot a_{232}\cdot b_{321}-b_{121}\cdot a_{212}\cdot a_{333}+b_{121}\cdot a_{213}\cdot a_{332}\\
&+b_{121}\cdot a_{232}\cdot a_{313}-b_{121}\cdot a_{233}\cdot a_{312}+b_{122}\cdot a_{211}\cdot a_{333}-b_{122}\cdot a_{213}\cdot a_{331}-b_{122}\cdot a_{231}\cdot a_{313}\\
&+b_{122}\cdot a_{233}\cdot a_{311}-b_{123}\cdot a_{211}\cdot a_{332}+b_{123}\cdot a_{212}\cdot a_{331}+b_{123}\cdot a_{231}\cdot a_{312}-b_{123}\cdot a_{232}\cdot a_{311}\\
&+a_{131}\cdot a_{212}\cdot b_{323}-a_{131}\cdot a_{213}\cdot b_{322}-a_{131}\cdot b_{222}\cdot a_{313}+a_{131}\cdot b_{223}\cdot a_{312}-a_{132}\cdot a_{211}\cdot b_{323}\\
&+a_{132}\cdot a_{213}\cdot b_{321}+a_{132}\cdot b_{221}\cdot a_{313}-a_{132}\cdot b_{223}\cdot a_{311}+a_{133}\cdot a_{211}\cdot b_{322}-a_{133}\cdot a_{212}\cdot a_{321}\\
&-a_{133}\cdot b_{221}\cdot a_{312}+a_{133}\cdot b_{222}\cdot a_{311}\},
\end{aligned}
$$

while,

$$\det\left(A_{[3\times3\times3]}\right) + det\left(B_{[3\times3\times3]}\right)$$
$$= \det\begin{pmatrix} a_{111} & a_{121}+b_{121} & a_{131} \\ a_{211} & a_{221}+b_{221} & a_{231} \\ a_{311} & a_{321}+b_{321} & a_{331} \end{pmatrix}\begin{vmatrix} a_{112} & a_{122}+b_{122} & a_{132} \\ a_{212} & a_{222}+b_{222} & a_{232} \\ a_{312} & a_{322}+b_{322} & a_{332} \end{vmatrix}\begin{pmatrix} a_{113} & a_{123}+b_{123} & a_{133} \\ a_{213} & a_{223}+b_{223} & a_{233} \\ a_{313} & a_{323}+b_{323} & a_{333} \end{pmatrix}$$
$$= a_{111}\cdot(a_{222}+b_{222})\cdot a_{333} - a_{111}\cdot a_{232}\cdot(a_{323}+b_{323}) - a_{111}\cdot(a_{223}+b_{223})\cdot a_{332}$$
$$+a_{111}\cdot a_{233}\cdot(a_{322}+b_{322}) - a_{112}\cdot(a_{221}+b_{221})\cdot a_{333} + a_{112}\cdot(a_{223}+b_{223})\cdot a_{331}$$
$$+a_{112}\cdot a_{231}\cdot(a_{323}+b_{323}) - a_{112}\cdot a_{233}\cdot(a_{321}+b_{321}) + a_{113}\cdot(a_{221}+b_{221})\cdot a_{332}$$
$$-a_{113}\cdot(a_{222}+b_{222})\cdot a_{331} - a_{113}\cdot a_{231}\cdot(a_{322}+b_{322}) + a_{113}\cdot a_{232}\cdot(a_{321}+b_{321})$$
$$-(a_{121}+b_{121})\cdot a_{212}\cdot a_{333} + (a_{121}+b_{121})\cdot a_{213}\cdot a_{332} + (a_{121}+b_{121})\cdot a_{232}\cdot a_{313}$$
$$-(a_{121}+b_{121})\cdot a_{233}\cdot a_{312} + (a_{122}+b_{122})\cdot a_{211}\cdot a_{333} - (a_{122}+b_{122})\cdot a_{213}\cdot a_{331}$$
$$-(a_{122}+b_{122})\cdot a_{231}\cdot a_{313} + (a_{122}+b_{122})\cdot a_{233}\cdot a_{311} - (a_{123}+b_{123})\cdot a_{211}\cdot a_{332}$$
$$+(a_{123}+b_{123})\cdot a_{212}\cdot a_{331} + (a_{123}+b_{123})\cdot a_{231}\cdot a_{312} - (a_{123}+b_{123})\cdot a_{232}\cdot a_{311}$$
$$+a_{131}\cdot a_{212}\cdot(a_{323}+b_{323}) - a_{131}\cdot a_{213}\cdot(a_{322}+b_{322}) - a_{131}\cdot(a_{222}+b_{222})\cdot a_{313}$$
$$+a_{131}\cdot(a_{223}+b_{223})\cdot a_{312} - a_{132}\cdot a_{211}\cdot(a_{323}+b_{323}) + a_{132}\cdot a_{213}\cdot(a_{321}+b_{321})$$
$$+a_{132}\cdot(a_{221}+b_{221})\cdot a_{313} - a_{132}\cdot(a_{223}+b_{223})\cdot a_{311} + a_{133}\cdot a_{211}\cdot(a_{322}+b_{322})$$
$$-a_{133}\cdot a_{212}\cdot(a_{321}+b_{321}) - a_{133}\cdot(a_{221}+b_{221})\cdot a_{312} + a_{133}\cdot(a_{222}+b_{222})\cdot a_{311}$$
$$= a_{111}\cdot a_{222}\cdot a_{333} + a_{111}\cdot b_{222}\cdot a_{333} - a_{111}\cdot a_{232}\cdot a_{323} - a_{111}\cdot a_{232}\cdot b_{323} - a_{111}\cdot a_{223}\cdot a_{332}$$
$$-a_{111}\cdot b_{223}\cdot a_{332} + a_{111}\cdot a_{233}\cdot a_{322} + a_{111}\cdot a_{233}\cdot b_{322} - a_{112}\cdot a_{221}\cdot a_{333} - a_{112}\cdot b_{221}\cdot a_{333}$$
$$+a_{112}\cdot a_{223}\cdot a_{331} + a_{112}\cdot b_{223}\cdot a_{331} + a_{112}\cdot a_{231}\cdot a_{323} + a_{112}\cdot a_{231}\cdot b_{323} - a_{112}\cdot a_{233}\cdot a_{321}$$
$$-a_{112}\cdot a_{233}\cdot b_{321} + a_{113}\cdot a_{221}\cdot a_{332} + a_{113}\cdot b_{221}\cdot a_{332} - a_{113}\cdot a_{222}\cdot a_{331} - a_{113}\cdot b_{222}\cdot a_{331}$$
$$-a_{113}\cdot a_{231}\cdot a_{322} - a_{113}\cdot a_{231}\cdot b_{322} + a_{113}\cdot a_{232}\cdot a_{321} + a_{113}\cdot a_{232}\cdot b_{321} - a_{121}\cdot a_{212}\cdot a_{333}$$
$$-b_{121}\cdot a_{212}\cdot a_{333} + a_{121}\cdot a_{213}\cdot a_{332} + b_{121}\cdot a_{213}\cdot a_{332} + a_{121}\cdot a_{232}\cdot a_{313} + b_{121}\cdot a_{232}\cdot a_{313}$$
$$-a_{121}\cdot a_{233}\cdot a_{312} - b_{121}\cdot a_{233}\cdot a_{312} + a_{122}\cdot a_{211}\cdot a_{333} + b_{122}\cdot a_{211}\cdot a_{333} - a_{122}\cdot a_{213}\cdot a_{331}$$
$$-b_{122}\cdot a_{213}\cdot a_{331} - a_{122}\cdot a_{231}\cdot a_{313} - b_{122}\cdot a_{231}\cdot a_{313} + a_{122}\cdot a_{233}\cdot a_{311} + b_{122}\cdot a_{233}\cdot a_{311}$$
$$-a_{123}\cdot a_{211}\cdot a_{332} - b_{123}\cdot a_{211}\cdot a_{332} + a_{123}\cdot a_{212}\cdot a_{331} + b_{123}\cdot a_{212}\cdot a_{331} + a_{123}\cdot a_{231}\cdot a_{312}$$
$$+b_{123}\cdot a_{231}\cdot a_{312} - a_{123}\cdot a_{232}\cdot a_{311} - b_{123}\cdot a_{232}\cdot a_{311} + a_{131}\cdot a_{212}\cdot a_{323} + a_{131}\cdot a_{212}\cdot b_{323}$$
$$-a_{131}\cdot a_{213}\cdot a_{322} - a_{131}\cdot a_{213}\cdot b_{322} - a_{131}\cdot a_{222}\cdot a_{313} - a_{131}\cdot b_{222}\cdot a_{313} + a_{131}\cdot a_{223}\cdot a_{312}$$
$$+a_{131}\cdot b_{223}\cdot a_{312} - a_{132}\cdot a_{211}\cdot a_{323} - a_{132}\cdot a_{211}\cdot b_{323} + a_{132}\cdot a_{213}\cdot a_{321} + a_{132}\cdot a_{213}\cdot b_{321}$$
$$+a_{132}\cdot a_{221}\cdot a_{313} + a_{132}\cdot b_{221}\cdot a_{313} - a_{132}\cdot a_{223}\cdot a_{311} - a_{132}\cdot b_{223}\cdot a_{311} + a_{133}\cdot a_{211}\cdot a_{322}$$
$$+a_{133}\cdot a_{211}\cdot b_{322} - a_{133}\cdot a_{212}\cdot a_{321} - a_{133}\cdot a_{212}\cdot b_{321} - a_{133}\cdot a_{221}\cdot a_{312} - a_{133}\cdot b_{221}\cdot a_{312}$$
$$+a_{133}\cdot a_{222}\cdot a_{311} + a_{133}\cdot b_{222}\cdot a_{311}.$$

If we compare results of above equations, we can see that we have the same result in both cases.

6. For plan $j = 3$: Let A and B be cubic-matries of order 3, where all elements on the plan $j = 2$ and $j = 3$ are identical in both matrices, then we have:

$$det\left(A_{[3\times3\times3]}\right) + det\left(B_{[3\times3\times3]}\right) = det\begin{pmatrix} a_{111} & a_{121} & a_{131} \\ a_{211} & a_{221} & a_{231} \\ a_{311} & a_{321} & a_{331} \end{pmatrix}\begin{vmatrix} a_{112} & a_{122} & a_{132} \\ a_{212} & a_{222} & a_{232} \\ a_{312} & a_{322} & a_{332} \end{vmatrix}\begin{pmatrix} a_{113} & a_{123} & a_{133} \\ a_{213} & a_{223} & a_{233} \\ a_{313} & a_{323} & a_{333} \end{pmatrix}$$
$$+det\begin{pmatrix} b_{111} & a_{121} & a_{131} \\ b_{211} & a_{221} & a_{231} \\ b_{311} & a_{321} & a_{331} \end{pmatrix}\begin{vmatrix} b_{112} & a_{122} & a_{132} \\ b_{212} & a_{222} & a_{232} \\ b_{312} & a_{322} & a_{332} \end{vmatrix}\begin{pmatrix} b_{113} & a_{123} & a_{133} \\ b_{213} & a_{223} & a_{233} \\ b_{313} & a_{323} & a_{333} \end{pmatrix}$$
$$= \{a_{111}\cdot a_{222}\cdot a_{333} - a_{111}\cdot a_{232}\cdot a_{323} - a_{111}\cdot a_{223}\cdot a_{332} + a_{111}\cdot a_{233}\cdot a_{322} - a_{112}\cdot a_{221}\cdot a_{333}$$
$$+a_{112}\cdot a_{223}\cdot a_{331} + a_{112}\cdot a_{231}\cdot a_{323} - a_{112}\cdot a_{233}\cdot a_{321} + a_{113}\cdot a_{221}\cdot a_{332} - a_{113}\cdot a_{222}\cdot a_{331}$$
$$-a_{113}\cdot a_{231}\cdot a_{322} + a_{113}\cdot a_{232}\cdot a_{321} - a_{121}\cdot a_{212}\cdot a_{333} + a_{121}\cdot a_{213}\cdot a_{332} + a_{121}\cdot a_{232}\cdot a_{313}$$

$$
\begin{gathered}
-a_{121} \cdot a_{233} \cdot a_{312} + a_{122} \cdot a_{211} \cdot a_{333} - a_{122} \cdot a_{213} \cdot a_{331} - a_{122} \cdot a_{231} \cdot a_{313} + a_{122} \cdot a_{233} \cdot a_{311} \\
-a_{123} \cdot a_{211} \cdot a_{332} + a_{123} \cdot a_{212} \cdot a_{331} + a_{123} \cdot a_{231} \cdot a_{312} - a_{123} \cdot a_{232} \cdot a_{311} + a_{131} \cdot a_{212} \cdot a_{323} \\
-a_{131} \cdot a_{213} \cdot a_{322} - a_{131} \cdot a_{222} \cdot a_{313} + a_{131} \cdot a_{223} \cdot a_{312} - a_{132} \cdot a_{211} \cdot a_{323} + a_{132} \cdot a_{213} \cdot a_{321} \\
+a_{132} \cdot a_{221} \cdot a_{313} - a_{132} \cdot a_{223} \cdot a_{311} + a_{133} \cdot a_{211} \cdot a_{322} - a_{133} \cdot a_{212} \cdot a_{321} - a_{133} \cdot a_{221} \cdot a_{312} \\
+a_{133} \cdot a_{222} \cdot a_{311}\} + \{b_{111} \cdot a_{222} \cdot a_{333} - b_{111} \cdot a_{232} \cdot a_{323} - b_{111} \cdot a_{223} \cdot a_{332} + b_{111} \cdot a_{233} \cdot a_{322} \\
-b_{112} \cdot a_{221} \cdot a_{333} + b_{112} \cdot a_{223} \cdot a_{331} + b_{112} \cdot a_{231} \cdot a_{323} - b_{112} \cdot a_{233} \cdot a_{321} + b_{113} \cdot a_{221} \cdot a_{332} \\
-b_{113} \cdot a_{222} \cdot a_{331} - b_{113} \cdot a_{231} \cdot a_{322} + b_{113} \cdot a_{232} \cdot a_{321} - a_{121} \cdot b_{212} \cdot a_{333} + a_{121} \cdot b_{213} \cdot a_{332} \\
+a_{121} \cdot a_{232} \cdot b_{313} - a_{121} \cdot a_{233} \cdot b_{312} + a_{122} \cdot b_{211} \cdot a_{333} - a_{122} \cdot b_{213} \cdot a_{331} - a_{122} \cdot a_{231} \cdot b_{313} \\
+a_{122} \cdot a_{233} \cdot b_{311} - a_{123} \cdot b_{211} \cdot a_{332} + a_{123} \cdot b_{212} \cdot a_{331} + a_{123} \cdot a_{231} \cdot b_{312} - a_{123} \cdot a_{232} \cdot b_{311} \\
+a_{131} \cdot b_{212} \cdot a_{323} - a_{131} \cdot b_{213} \cdot a_{322} - a_{131} \cdot a_{222} \cdot b_{313} + a_{131} \cdot a_{223} \cdot b_{312} - a_{132} \cdot b_{211} \cdot a_{323} \\
+a_{132} \cdot b_{213} \cdot a_{321} + a_{132} \cdot a_{221} \cdot b_{313} - a_{132} \cdot a_{223} \cdot b_{311} + a_{133} \cdot b_{211} \cdot a_{322} - a_{133} \cdot b_{212} \cdot a_{321} \\
-a_{133} \cdot a_{221} \cdot b_{312} + a_{133} \cdot a_{222} \cdot b_{311}\},
\end{gathered}
$$

while,

$$
\begin{gathered}
\det\left(A_{[3\times3\times3]}\right) + det\left(B_{[3\times3\times3]}\right) \\
= \det\left(\begin{array}{ccc|ccc|ccc}
a_{111} + b_{111} & a_{121} & a_{131} & a_{112} + b_{112} & a_{122} & a_{132} & a_{113} + b_{113} & a_{123} & a_{133} \\
a_{211} + b_{211} & a_{221} & a_{231} & a_{212} + b_{212} & a_{222} & a_{232} & a_{213} + b_{213} & a_{223} & a_{233} \\
a_{311} + b_{311} & a_{321} & a_{331} & a_{312} + b_{312} & a_{322} & a_{332} & a_{313} + b_{313} & a_{323} & a_{333}
\end{array}\right) \\
= (a_{111} + b_{111}) \cdot a_{222} \cdot a_{333} - (a_{111} + b_{111}) \cdot a_{232} \cdot a_{323} - (a_{111} + b_{111}) \cdot a_{223} \cdot a_{332} \\
+(a_{111} + b_{111}) \cdot a_{233} \cdot a_{322} - (a_{112} + b_{112}) \cdot a_{221} \cdot a_{333} + (a_{112} + b_{112}) \cdot a_{223} \cdot a_{331} \\
+(a_{112} + b_{112}) \cdot a_{231} \cdot a_{323} - (a_{112} + b_{112}) \cdot a_{233} \cdot a_{321} + (a_{113} + b_{113}) \cdot a_{221} \cdot a_{332} \\
-(a_{113} + b_{113}) \cdot a_{222} \cdot a_{331} - (a_{113} + b_{113}) \cdot a_{231} \cdot a_{322} + (a_{113} + b_{113}) \cdot a_{232} \cdot a_{321} \\
-a_{121} \cdot (a_{212} + b_{212}) \cdot a_{333} + a_{121} \cdot (a_{213} + b_{213}) \cdot a_{332} + a_{121} \cdot a_{232} \cdot (a_{313} + b_{313}) \\
-a_{121} \cdot a_{233} \cdot (a_{312} + b_{312}) + a_{122} \cdot (a_{211} + b_{211}) \cdot a_{333} - a_{122} \cdot (a_{213} + b_{213}) \cdot a_{331} \\
-a_{122} \cdot a_{231} \cdot (a_{313} + b_{313}) + a_{122} \cdot a_{233} \cdot (a_{311} + b_{311}) - a_{123} \cdot (a_{211} + b_{211}) \cdot a_{332} \\
+a_{123} \cdot (a_{212} + b_{212}) \cdot a_{331} + a_{123} \cdot a_{231} \cdot (a_{312} + b_{312}) - a_{123} \cdot a_{232} \cdot (a_{311} + b_{311}) \\
+a_{131} \cdot (a_{212} + b_{212}) \cdot a_{323} - a_{131} \cdot (a_{213} + b_{213}) \cdot a_{322} - a_{131} \cdot a_{222} \cdot (a_{313} + b_{313}) \\
+a_{131} \cdot a_{223} \cdot (a_{312} + b_{312}) - a_{132} \cdot (a_{211} + b_{211}) \cdot a_{323} + a_{132} \cdot (a_{213} + b_{213}) \cdot a_{321} \\
+a_{132} \cdot a_{221} \cdot (a_{313} + b_{313}) - a_{132} \cdot a_{223} \cdot (a_{311} + b_{311}) + a_{133} \cdot (a_{211} + b_{211}) \cdot a_{322} \\
-a_{133} \cdot (a_{212} + b_{212}) \cdot a_{321} - a_{133} \cdot a_{221} \cdot (a_{312} + b_{312}) + a_{133} \cdot a_{222} \cdot (a_{311} + b_{311}) \\
= a_{111} \cdot a_{222} \cdot a_{333} + b_{111} \cdot a_{222} \cdot a_{333} - a_{111} \cdot a_{232} \cdot a_{323} - b_{111} \cdot a_{232} \cdot a_{323} - a_{111} \cdot a_{223} \cdot a_{332} \\
-b_{111} \cdot a_{223} \cdot a_{332} + a_{111} \cdot a_{233} \cdot a_{322} + b_{111} \cdot a_{233} \cdot a_{322} - a_{112} \cdot a_{221} \cdot a_{333} - b_{112} \cdot a_{221} \cdot a_{333} \\
+a_{112} \cdot a_{223} \cdot a_{331} + b_{112} \cdot a_{223} \cdot a_{331} + a_{112} \cdot a_{231} \cdot a_{323} + b_{112} \cdot a_{231} \cdot a_{323} - a_{112} \cdot a_{233} \cdot a_{321} \\
-b_{112} \cdot a_{233} \cdot a_{321} + a_{113} \cdot a_{221} \cdot a_{332} + b_{113} \cdot a_{221} \cdot a_{332} - a_{113} \cdot a_{222} \cdot a_{331} - b_{113} \cdot a_{222} \cdot a_{331} \\
-a_{113} \cdot a_{231} \cdot a_{322} - b_{113} \cdot a_{231} \cdot a_{322} + a_{113} \cdot a_{232} \cdot a_{321} + b_{113} \cdot a_{232} \cdot a_{321} - a_{121} \cdot a_{212} \cdot a_{333} \\
-a_{121} \cdot b_{212} \cdot a_{333} + a_{121} \cdot a_{213} \cdot a_{332} + a_{121} \cdot b_{213} \cdot a_{332} + a_{121} \cdot a_{232} \cdot a_{313} + a_{121} \cdot a_{232} \cdot b_{313} \\
-a_{121} \cdot a_{233} \cdot a_{312} - a_{121} \cdot a_{233} \cdot b_{312} + a_{122} \cdot a_{211} \cdot a_{333} + a_{122} \cdot b_{211} \cdot a_{333} - a_{122} \cdot a_{213} \cdot a_{331} \\
-a_{122} \cdot b_{213} \cdot a_{331} - a_{122} \cdot a_{231} \cdot a_{313} - a_{122} \cdot a_{231} \cdot b_{313} + a_{122} \cdot a_{233} \cdot a_{311} + a_{122} \cdot a_{233} \cdot b_{311} \\
-a_{123} \cdot a_{211} \cdot a_{332} - a_{123} \cdot b_{211} \cdot a_{332} + a_{123} \cdot a_{212} \cdot a_{331} + a_{123} \cdot b_{212} \cdot a_{331} + a_{123} \cdot a_{231} \cdot a_{312} \\
+a_{123} \cdot a_{231} \cdot b_{312} - a_{123} \cdot a_{232} \cdot a_{311} - a_{123} \cdot a_{232} \cdot b_{311} + a_{131} \cdot a_{212} \cdot a_{323} + a_{131} \cdot b_{212} \cdot a_{323} \\
-a_{131} \cdot a_{213} \cdot a_{322} - a_{131} \cdot b_{213} \cdot a_{322} - a_{131} \cdot a_{222} \cdot a_{313} - a_{131} \cdot a_{222} \cdot b_{313} + a_{131} \cdot a_{223} \cdot a_{312} \\
+a_{131} \cdot a_{223} \cdot b_{312} - a_{132} \cdot a_{211} \cdot a_{323} - a_{132} \cdot b_{211} \cdot a_{323} + a_{132} \cdot a_{213} \cdot a_{321} + a_{132} \cdot b_{213} \cdot a_{321} \\
+a_{132} \cdot a_{221} \cdot a_{313} + a_{132} \cdot a_{221} \cdot b_{313} - a_{132} \cdot a_{223} \cdot a_{311} - a_{132} \cdot a_{223} \cdot b_{311} + a_{133} \cdot a_{211} \cdot a_{322} \\
+a_{133} \cdot b_{211} \cdot a_{322} - a_{133} \cdot a_{212} \cdot a_{321} - a_{133} \cdot b_{212} \cdot a_{321} - a_{133} \cdot a_{221} \cdot a_{312} - a_{133} \cdot a_{221} \cdot b_{312}
\end{gathered}
$$

$$+a_{133}\cdot a_{222}\cdot a_{311}+a_{133}\cdot a_{222}\cdot b_{311}.$$

If we compare results of above equations, we can see that we have the same result in both cases.

7. For plan $k=1$: Let A and B be cubic-matries of order 3, where all elements on the plan $k=1$ and $k=2$ are identical in both matrices, then we have:

$$det(A_{[3\times3\times3]})+det(B_{[3\times3\times3]})=det\begin{pmatrix}a_{111}&a_{121}&a_{131}&a_{112}&a_{122}&a_{132}&a_{113}&a_{123}&a_{133}\\a_{211}&a_{221}&a_{231}&a_{212}&a_{222}&a_{232}&a_{213}&a_{223}&a_{233}\\a_{311}&a_{321}&a_{331}&a_{312}&a_{322}&a_{332}&a_{313}&a_{323}&a_{333}\end{pmatrix}$$
$$+det\begin{pmatrix}a_{111}&a_{121}&a_{131}&a_{112}&a_{122}&a_{132}&b_{113}&b_{123}&b_{133}\\a_{211}&a_{221}&a_{231}&a_{212}&a_{222}&a_{232}&b_{213}&b_{223}&b_{233}\\a_{311}&a_{321}&a_{331}&a_{312}&a_{322}&a_{332}&b_{313}&b_{323}&b_{333}\end{pmatrix}$$
$$=\{a_{111}\cdot a_{222}\cdot a_{333}-a_{111}\cdot a_{232}\cdot a_{323}-a_{111}\cdot a_{223}\cdot a_{332}+a_{111}\cdot a_{233}\cdot a_{322}-a_{112}\cdot a_{221}\cdot a_{333}$$
$$+a_{112}\cdot a_{223}\cdot a_{331}+a_{112}\cdot a_{231}\cdot a_{323}-a_{112}\cdot a_{233}\cdot a_{321}+a_{113}\cdot a_{221}\cdot a_{332}-a_{113}\cdot a_{222}\cdot a_{331}$$
$$-a_{113}\cdot a_{231}\cdot a_{322}+a_{113}\cdot a_{232}\cdot a_{321}-a_{121}\cdot a_{212}\cdot a_{333}+a_{121}\cdot a_{213}\cdot a_{332}+a_{121}\cdot a_{232}\cdot a_{313}$$
$$-a_{121}\cdot a_{233}\cdot a_{312}+a_{122}\cdot a_{211}\cdot a_{333}-a_{122}\cdot a_{213}\cdot a_{331}-a_{122}\cdot a_{231}\cdot a_{313}+a_{122}\cdot a_{233}\cdot a_{311}$$
$$-a_{123}\cdot a_{211}\cdot a_{332}+a_{123}\cdot a_{212}\cdot a_{331}+a_{123}\cdot a_{231}\cdot a_{312}-a_{123}\cdot a_{232}\cdot a_{311}+a_{131}\cdot a_{212}\cdot a_{323}$$
$$-a_{131}\cdot a_{213}\cdot a_{322}-a_{131}\cdot a_{222}\cdot a_{313}+a_{131}\cdot a_{223}\cdot a_{312}-a_{132}\cdot a_{211}\cdot a_{323}+a_{132}\cdot a_{213}\cdot a_{321}$$
$$+a_{132}\cdot a_{221}\cdot a_{313}-a_{132}\cdot a_{223}\cdot a_{311}+a_{133}\cdot a_{211}\cdot a_{322}-a_{133}\cdot a_{212}\cdot a_{321}-a_{133}\cdot a_{221}\cdot a_{312}$$
$$+a_{133}\cdot a_{222}\cdot a_{311}\}+\{a_{111}\cdot a_{222}\cdot b_{333}-a_{111}\cdot a_{232}\cdot b_{323}-a_{111}\cdot b_{223}\cdot a_{332}+a_{111}\cdot b_{233}\cdot a_{322}$$
$$-a_{112}\cdot a_{221}\cdot b_{333}+a_{112}\cdot b_{223}\cdot a_{331}+a_{112}\cdot a_{231}\cdot b_{323}-a_{112}\cdot b_{233}\cdot a_{321}+b_{113}\cdot a_{221}\cdot a_{332}$$
$$-b_{113}\cdot a_{222}\cdot a_{331}-b_{113}\cdot a_{231}\cdot a_{322}+b_{113}\cdot a_{232}\cdot a_{321}-a_{121}\cdot a_{212}\cdot b_{333}+a_{121}\cdot b_{213}\cdot a_{332}$$
$$+a_{121}\cdot a_{232}\cdot b_{313}-a_{121}\cdot b_{233}\cdot a_{312}+a_{122}\cdot a_{211}\cdot b_{333}-a_{122}\cdot b_{213}\cdot a_{331}-a_{122}\cdot a_{231}\cdot b_{313}$$
$$+a_{122}\cdot b_{233}\cdot a_{311}-b_{123}\cdot a_{211}\cdot a_{332}+b_{123}\cdot a_{212}\cdot a_{331}+b_{123}\cdot a_{231}\cdot a_{312}-b_{123}\cdot a_{232}\cdot a_{311}$$
$$+a_{131}\cdot a_{212}\cdot b_{323}-a_{131}\cdot b_{213}\cdot a_{322}-a_{131}\cdot a_{222}\cdot b_{313}+a_{131}\cdot b_{223}\cdot a_{312}-a_{132}\cdot a_{211}\cdot b_{323}$$
$$+a_{132}\cdot b_{213}\cdot a_{321}+a_{132}\cdot a_{221}\cdot b_{313}-a_{132}\cdot b_{223}\cdot a_{311}+b_{133}\cdot a_{211}\cdot a_{322}-b_{133}\cdot a_{212}\cdot a_{321}$$
$$-b_{133}\cdot a_{221}\cdot a_{312}+b_{133}\cdot a_{222}\cdot a_{311}\},$$

while,

$$\det(A_{[3\times3\times3]})+det(B_{[3\times3\times3]})$$
$$=\det\begin{pmatrix}a_{111}&a_{121}&a_{131}&a_{112}&a_{122}&a_{132}&a_{113}+b_{113}&a_{123}+b_{123}&a_{133}+b_{133}\\a_{211}&a_{221}&a_{231}&a_{212}&a_{222}&a_{232}&a_{213}+b_{213}&a_{223}+b_{223}&a_{233}+b_{233}\\a_{311}&a_{321}&a_{331}&a_{312}&a_{322}&a_{332}&a_{313}+b_{313}&a_{323}+b_{323}&a_{333}+b_{333}\end{pmatrix}$$
$$=a_{111}\cdot a_{222}\cdot(a_{333}+b_{333})-a_{111}\cdot a_{232}\cdot(a_{323}+b_{323})-a_{111}\cdot(a_{223}+b_{223})\cdot a_{332}$$
$$+a_{111}\cdot(a_{233}+b_{233})\cdot a_{322}-a_{112}\cdot a_{221}\cdot(a_{333}+b_{333})+a_{112}\cdot(a_{223}+b_{223})\cdot a_{331}$$
$$+a_{112}\cdot a_{231}\cdot(a_{323}+b_{323})-a_{112}\cdot(a_{233}+b_{233})\cdot a_{321}+(a_{113}+b_{113})\cdot a_{221}\cdot a_{332}$$
$$-(a_{113}+b_{113})\cdot a_{222}\cdot a_{331}-(a_{113}+b_{113})\cdot a_{231}\cdot a_{322}+(a_{113}+b_{113})\cdot a_{232}\cdot a_{321}$$
$$-a_{121}\cdot a_{212}\cdot(a_{333}+b_{333})+a_{121}\cdot(a_{213}+b_{213})\cdot a_{332}+a_{121}\cdot a_{232}\cdot(a_{313}+b_{313})$$
$$-a_{121}\cdot(a_{233}+b_{233})\cdot a_{312}+a_{122}\cdot a_{211}\cdot(a_{333}+b_{333})-a_{122}\cdot(a_{213}+b_{213})\cdot a_{331}$$
$$-a_{122}\cdot a_{231}\cdot(a_{313}+b_{313})+a_{122}\cdot(a_{233}+b_{233})\cdot a_{311}-(a_{123}+b_{123})\cdot a_{211}\cdot a_{332}$$
$$+(a_{123}+b_{123})\cdot a_{212}\cdot a_{331}+(a_{123}+b_{123})\cdot a_{231}\cdot a_{312}-(a_{123}+b_{123})\cdot a_{232}\cdot a_{311}$$
$$+a_{131}\cdot a_{212}\cdot(a_{323}+b_{323})-a_{131}\cdot(a_{213}+b_{213})\cdot a_{322}-a_{131}\cdot a_{222}\cdot(a_{313}+b_{313})$$
$$+a_{131}\cdot(a_{223}+b_{223})\cdot a_{312}-a_{132}\cdot a_{211}\cdot(a_{323}+b_{323})+a_{132}\cdot(a_{213}+b_{213})\cdot a_{321}$$
$$+a_{132}\cdot a_{221}\cdot(a_{313}+b_{313})-a_{132}\cdot(a_{223}+b_{223})\cdot a_{311}+(a_{133}+b_{133})\cdot a_{211}\cdot a_{322}$$
$$-(a_{133}+b_{133})\cdot a_{212}\cdot a_{321}-(a_{133}+b_{133})\cdot a_{221}\cdot a_{312}+(a_{133}+b_{133})\cdot a_{222}\cdot a_{311}$$
$$=a_{111}\cdot a_{222}\cdot a_{333}+a_{111}\cdot a_{222}\cdot b_{333}-a_{111}\cdot a_{232}\cdot a_{323}-a_{111}\cdot a_{232}\cdot b_{323}-a_{111}\cdot a_{223}\cdot a_{332}$$

$$-a_{111}\cdot b_{223}\cdot a_{332}+a_{111}\cdot a_{233}\cdot a_{322}+a_{111}\cdot b_{233}\cdot a_{322}-a_{112}\cdot a_{221}\cdot a_{333}-a_{112}\cdot a_{221}\cdot b_{333}$$
$$+a_{112}\cdot a_{223}\cdot a_{331}+a_{112}\cdot b_{223}\cdot a_{331}+a_{112}\cdot a_{231}\cdot a_{323}+a_{112}\cdot a_{231}\cdot b_{323}-a_{112}\cdot a_{233}\cdot a_{321}$$
$$-a_{112}\cdot b_{233}\cdot a_{321}+a_{113}\cdot a_{221}\cdot a_{332}+b_{113}\cdot a_{221}\cdot a_{332}-a_{113}\cdot a_{222}\cdot a_{331}-b_{113}\cdot a_{222}\cdot a_{331}$$
$$-a_{113}\cdot a_{231}\cdot a_{322}-b_{113}\cdot a_{231}\cdot a_{322}+a_{113}\cdot a_{232}\cdot a_{321}+b_{113}\cdot a_{232}\cdot a_{321}-a_{121}\cdot a_{212}\cdot a_{333}$$
$$-a_{121}\cdot a_{212}\cdot b_{333}+a_{121}\cdot a_{213}\cdot a_{332}+a_{121}\cdot b_{213}\cdot a_{332}+a_{121}\cdot a_{232}\cdot a_{313}+a_{121}\cdot a_{232}\cdot b_{313}$$
$$-a_{121}\cdot a_{233}\cdot a_{312}-a_{121}\cdot b_{233}\cdot a_{312}+a_{122}\cdot a_{211}\cdot a_{333}+a_{122}\cdot a_{211}\cdot b_{333}-a_{122}\cdot a_{213}\cdot a_{331}$$
$$-a_{122}\cdot b_{213}\cdot a_{331}-a_{122}\cdot a_{231}\cdot a_{313}-a_{122}\cdot a_{231}\cdot b_{313}+a_{122}\cdot a_{233}\cdot a_{311}+a_{122}\cdot b_{233}\cdot a_{311}$$
$$-a_{123}\cdot a_{211}\cdot a_{332}-b_{123}\cdot a_{211}\cdot a_{332}+a_{123}\cdot a_{212}\cdot a_{331}+b_{123}\cdot a_{212}\cdot a_{331}+a_{123}\cdot a_{231}\cdot a_{312}$$
$$+b_{123}\cdot a_{231}\cdot a_{312}-a_{123}\cdot a_{232}\cdot a_{311}-b_{123}\cdot a_{232}\cdot a_{311}+a_{131}\cdot a_{212}\cdot a_{323}+a_{131}\cdot a_{212}\cdot b_{323}$$
$$-a_{131}\cdot a_{213}\cdot a_{322}-a_{131}\cdot b_{213}\cdot a_{322}-a_{131}\cdot a_{222}\cdot a_{313}-a_{131}\cdot a_{222}\cdot b_{313}+a_{131}\cdot a_{223}\cdot a_{312}$$
$$+a_{131}\cdot b_{223}\cdot a_{312}-a_{132}\cdot a_{211}\cdot a_{323}-a_{132}\cdot a_{211}\cdot b_{323}+a_{132}\cdot a_{213}\cdot a_{321}+a_{132}\cdot b_{213}\cdot a_{321}$$
$$+a_{132}\cdot a_{221}\cdot a_{313}+a_{132}\cdot a_{221}\cdot b_{313}-a_{132}\cdot a_{223}\cdot a_{311}-a_{132}\cdot b_{223}\cdot a_{311}+a_{133}\cdot a_{211}\cdot a_{322}$$
$$+b_{133}\cdot a_{211}\cdot a_{322}-a_{133}\cdot a_{212}\cdot a_{321}-b_{133}\cdot a_{212}\cdot a_{321}-a_{133}\cdot a_{221}\cdot a_{312}-b_{133}\cdot a_{221}\cdot a_{312}$$
$$+a_{133}\cdot a_{222}\cdot a_{311}+b_{133}\cdot a_{222}\cdot a_{311}.$$

If we compare results of above equations, we can see that we have the same result in both cases.

8. For plan $k=2$: Let A and B be cubic-matries of order 3, where all elements on the plan $k=1$ and $k=3$ are identical in both matrices, then we have:

$$det\left(A_{[3\times3\times3]}\right)+det\left(B_{[3\times3\times3]}\right)=det\left(\begin{matrix}a_{111}&a_{121}&a_{131}\\a_{211}&a_{221}&a_{231}\\a_{311}&a_{321}&a_{331}\end{matrix}\middle|\begin{matrix}a_{112}&a_{122}&a_{132}\\a_{212}&a_{222}&a_{232}\\a_{312}&a_{322}&a_{332}\end{matrix}\middle|\begin{matrix}a_{113}&a_{123}&a_{133}\\a_{213}&a_{223}&a_{233}\\a_{313}&a_{323}&a_{333}\end{matrix}\right)$$
$$+det\left(\begin{matrix}a_{111}&a_{121}&a_{131}\\a_{211}&a_{221}&a_{231}\\a_{311}&a_{321}&a_{331}\end{matrix}\middle|\begin{matrix}b_{112}&b_{122}&b_{132}\\b_{212}&b_{222}&b_{232}\\b_{312}&b_{322}&b_{332}\end{matrix}\middle|\begin{matrix}a_{113}&a_{123}&a_{133}\\a_{213}&a_{223}&a_{233}\\a_{313}&a_{323}&a_{333}\end{matrix}\right)$$
$$=\{a_{111}\cdot a_{222}\cdot a_{333}-a_{111}\cdot a_{232}\cdot a_{323}-a_{111}\cdot a_{223}\cdot a_{332}+a_{111}\cdot a_{233}\cdot a_{322}-a_{112}\cdot a_{221}\cdot a_{333}$$
$$+a_{112}\cdot a_{223}\cdot a_{331}+a_{112}\cdot a_{231}\cdot a_{323}-a_{112}\cdot a_{233}\cdot a_{321}+a_{113}\cdot a_{221}\cdot a_{332}-a_{113}\cdot a_{222}\cdot a_{331}$$
$$-a_{113}\cdot a_{231}\cdot a_{322}+a_{113}\cdot a_{232}\cdot a_{321}-a_{121}\cdot a_{212}\cdot a_{333}+a_{121}\cdot a_{213}\cdot a_{332}+a_{121}\cdot a_{232}\cdot a_{313}$$
$$-a_{121}\cdot a_{233}\cdot a_{312}+a_{122}\cdot a_{211}\cdot a_{333}-a_{122}\cdot a_{213}\cdot a_{331}-a_{122}\cdot a_{231}\cdot a_{313}+a_{122}\cdot a_{233}\cdot a_{311}$$
$$-a_{123}\cdot a_{211}\cdot a_{332}+a_{123}\cdot a_{212}\cdot a_{331}+a_{123}\cdot a_{231}\cdot a_{312}-a_{123}\cdot a_{232}\cdot a_{311}+a_{131}\cdot a_{212}\cdot a_{323}$$
$$-a_{131}\cdot a_{213}\cdot a_{322}-a_{131}\cdot a_{222}\cdot a_{313}+a_{131}\cdot a_{223}\cdot a_{312}-a_{132}\cdot a_{211}\cdot a_{323}+a_{132}\cdot a_{213}\cdot a_{321}$$
$$+a_{132}\cdot a_{221}\cdot a_{313}-a_{132}\cdot a_{223}\cdot a_{311}+a_{133}\cdot a_{211}\cdot a_{322}-a_{133}\cdot a_{212}\cdot a_{321}-a_{133}\cdot a_{221}\cdot a_{312}$$
$$+a_{133}\cdot a_{222}\cdot a_{311}\}+\{a_{111}\cdot b_{222}\cdot a_{333}-a_{111}\cdot b_{232}\cdot a_{323}-a_{111}\cdot a_{223}\cdot b_{332}+a_{111}\cdot a_{233}\cdot b_{322}$$
$$-b_{112}\cdot a_{221}\cdot a_{333}+b_{112}\cdot a_{223}\cdot a_{331}+b_{112}\cdot a_{231}\cdot a_{323}-b_{112}\cdot a_{233}\cdot a_{321}+a_{113}\cdot a_{221}\cdot b_{332}$$
$$-a_{113}\cdot b_{222}\cdot a_{331}-a_{113}\cdot a_{231}\cdot b_{322}+a_{113}\cdot b_{232}\cdot a_{321}-a_{121}\cdot b_{212}\cdot a_{333}+a_{121}\cdot a_{213}\cdot b_{332}$$
$$+a_{121}\cdot b_{232}\cdot a_{313}-a_{121}\cdot a_{233}\cdot b_{312}+b_{122}\cdot a_{211}\cdot a_{333}-b_{122}\cdot a_{213}\cdot a_{331}-b_{122}\cdot a_{231}\cdot a_{313}$$
$$+b_{122}\cdot a_{233}\cdot a_{311}-a_{123}\cdot a_{211}\cdot b_{332}+a_{123}\cdot b_{212}\cdot a_{331}+a_{123}\cdot a_{231}\cdot b_{312}-a_{123}\cdot b_{232}\cdot a_{311}$$
$$+a_{131}\cdot b_{212}\cdot a_{323}-a_{131}\cdot a_{213}\cdot b_{322}-a_{131}\cdot b_{222}\cdot a_{313}+a_{131}\cdot a_{223}\cdot b_{312}-b_{132}\cdot a_{211}\cdot a_{323}$$
$$+b_{132}\cdot a_{213}\cdot a_{321}+b_{132}\cdot a_{221}\cdot a_{313}-b_{132}\cdot a_{223}\cdot a_{311}+a_{133}\cdot a_{211}\cdot b_{322}-a_{133}\cdot b_{212}\cdot a_{321}$$
$$-a_{133}\cdot a_{221}\cdot b_{312}+a_{133}\cdot b_{222}\cdot a_{311}\},$$

while,

$$\det\left(A_{[3\times3\times3]}\right)+det\left(B_{[3\times3\times3]}\right)$$
$$=\det\left(\begin{matrix}a_{111}&a_{121}&a_{131}\\a_{211}&a_{221}&a_{231}\\a_{311}&a_{321}&a_{331}\end{matrix}\middle|\begin{matrix}a_{112}+b_{112}&a_{122}+b_{122}&a_{132}+b_{132}\\a_{212}+b_{212}&a_{222}+b_{222}&a_{232}+b_{232}\\a_{312}+b_{312}&a_{322}+b_{322}&a_{332}+b_{332}\end{matrix}\middle|\begin{matrix}a_{113}&a_{123}&a_{133}\\a_{213}&a_{223}&a_{233}\\a_{313}&a_{323}&a_{333}\end{matrix}\right)$$

$$= a_{111} \cdot (a_{222} + b_{222}) \cdot a_{333} - a_{111} \cdot (a_{232} + b_{232}) \cdot a_{323} - a_{111} \cdot a_{223} \cdot (a_{332} + b_{332})$$
$$+a_{111} \cdot a_{233} \cdot (a_{322} + b_{322}) - (a_{112} + b_{112}) \cdot a_{221} \cdot a_{333} + (a_{112} + b_{112}) \cdot a_{223} \cdot a_{331}$$
$$+(a_{112} + b_{112}) \cdot a_{231} \cdot a_{323} - (a_{112} + b_{112}) \cdot a_{233} \cdot a_{321} + a_{113} \cdot a_{221} \cdot (a_{332} + b_{332})$$
$$-a_{113} \cdot (a_{222} + b_{222}) \cdot a_{331} - a_{113} \cdot a_{231} \cdot (a_{322} + b_{322}) + a_{113} \cdot (a_{232} + b_{232}) \cdot a_{321}$$
$$-a_{121} \cdot (a_{212} + b_{212}) \cdot a_{333} + a_{121} \cdot a_{213} \cdot (a_{332} + b_{332}) + a_{121} \cdot (a_{232} + b_{232}) \cdot a_{313}$$
$$-a_{121} \cdot a_{233} \cdot (a_{312} + b_{312}) + (a_{122} + b_{122}) \cdot a_{211} \cdot a_{333} - (a_{122} + b_{122}) \cdot a_{213} \cdot a_{331}$$
$$-(a_{122} + b_{122}) \cdot a_{231} \cdot a_{313} + (a_{122} + b_{122}) \cdot a_{233} \cdot a_{311} - a_{123} \cdot a_{211} \cdot (a_{332} + b_{332})$$
$$+a_{123} \cdot (a_{212} + b_{212}) \cdot a_{331} + a_{123} \cdot a_{231} \cdot (a_{312} + b_{312}) - a_{123} \cdot (a_{232} + b_{232}) \cdot a_{311}$$
$$+a_{131} \cdot (a_{212} + b_{212}) \cdot a_{323} - a_{131} \cdot a_{213} \cdot (a_{322} + b_{322}) - a_{131} \cdot (a_{222} + b_{222}) \cdot a_{313}$$
$$+a_{131} \cdot a_{223} \cdot (a_{312} + b_{312}) - (a_{132} + b_{132}) \cdot a_{211} \cdot a_{323} + (a_{132} + b_{132}) \cdot a_{213} \cdot a_{321}$$
$$+(a_{132} + b_{132}) \cdot a_{221} \cdot a_{313} - (a_{132} + b_{132}) \cdot a_{223} \cdot a_{311} + a_{133} \cdot a_{211} \cdot (a_{322} + b_{322})$$
$$-a_{133} \cdot (a_{212} + b_{212}) \cdot a_{321} - a_{133} \cdot a_{221} \cdot (a_{312} + b_{312}) + a_{133} \cdot (a_{222} + b_{222}) \cdot a_{311}$$
$$= a_{111} \cdot a_{222} \cdot a_{333} + a_{111} \cdot b_{222} \cdot a_{333} - a_{111} \cdot a_{232} \cdot a_{323} - a_{111} \cdot b_{232} \cdot a_{323} - a_{111} \cdot a_{223} \cdot a_{332}$$
$$-a_{111} \cdot a_{223} \cdot b_{332} + a_{111} \cdot a_{233} \cdot a_{322} + a_{111} \cdot a_{233} \cdot b_{322} - a_{112} \cdot a_{221} \cdot a_{333} - b_{112} \cdot a_{221} \cdot a_{333}$$
$$+a_{112} \cdot a_{223} \cdot a_{331} + b_{112} \cdot a_{223} \cdot a_{331} + a_{112} \cdot a_{231} \cdot a_{323} + b_{112} \cdot a_{231} \cdot a_{323} - a_{112} \cdot a_{233} \cdot a_{321}$$
$$-b_{112} \cdot a_{233} \cdot a_{321} + a_{113} \cdot a_{221} \cdot a_{332} + a_{113} \cdot a_{221} \cdot b_{332} - a_{113} \cdot a_{222} \cdot a_{331} - a_{113} \cdot b_{222} \cdot a_{331}$$
$$-a_{113} \cdot a_{231} \cdot a_{322} - a_{113} \cdot a_{231} \cdot b_{322} + a_{113} \cdot a_{232} \cdot a_{321} + a_{113} \cdot b_{232} \cdot a_{321} - a_{121} \cdot a_{212} \cdot a_{333}$$
$$-a_{121} \cdot b_{212} \cdot a_{333} + a_{121} \cdot a_{213} \cdot a_{332} + a_{121} \cdot a_{213} \cdot b_{332} + a_{121} \cdot a_{232} \cdot a_{313} + a_{121} \cdot b_{232} \cdot a_{313}$$
$$-a_{121} \cdot a_{233} \cdot a_{312} - a_{121} \cdot a_{233} \cdot b_{312} + a_{122} \cdot a_{211} \cdot a_{333} + b_{122} \cdot a_{211} \cdot a_{333} - a_{122} \cdot a_{213} \cdot a_{331}$$
$$-b_{122} \cdot a_{213} \cdot a_{331} - a_{122} \cdot a_{231} \cdot a_{313} - b_{122} \cdot a_{231} \cdot a_{313} + a_{122} \cdot a_{233} \cdot a_{311} + b_{122} \cdot a_{233} \cdot a_{311}$$
$$-a_{123} \cdot a_{211} \cdot a_{332} - a_{123} \cdot a_{211} \cdot b_{332} + a_{123} \cdot a_{212} \cdot a_{331} + a_{123} \cdot b_{212} \cdot a_{331} + a_{123} \cdot a_{231} \cdot a_{312}$$
$$+a_{123} \cdot a_{231} \cdot b_{312} - a_{123} \cdot a_{232} \cdot a_{311} - a_{123} \cdot b_{232} \cdot a_{311} + a_{131} \cdot a_{212} \cdot a_{323} + a_{131} \cdot b_{212} \cdot a_{323}$$
$$-a_{131} \cdot a_{213} \cdot a_{322} - a_{131} \cdot a_{213} \cdot b_{322} - a_{131} \cdot a_{222} \cdot a_{313} - a_{131} \cdot b_{222} \cdot a_{313} + a_{131} \cdot a_{223} \cdot a_{312}$$
$$+a_{131} \cdot a_{223} \cdot b_{312} - a_{132} \cdot a_{211} \cdot a_{323} - b_{132} \cdot a_{211} \cdot a_{323} + a_{132} \cdot a_{213} \cdot a_{321} + b_{132} \cdot a_{213} \cdot a_{321}$$
$$+a_{132} \cdot a_{221} \cdot a_{313} + b_{132} \cdot a_{221} \cdot a_{313} - a_{132} \cdot a_{223} \cdot a_{311} - b_{132} \cdot a_{223} \cdot a_{311} + a_{133} \cdot a_{211} \cdot a_{322}$$
$$+a_{133} \cdot a_{211} \cdot b_{322} - a_{133} \cdot a_{212} \cdot a_{321} - a_{133} \cdot b_{212} \cdot a_{321} - a_{133} \cdot a_{221} \cdot a_{312} - a_{133} \cdot a_{221} \cdot b_{312}$$
$$+a_{133} \cdot a_{222} \cdot a_{311} + a_{133} \cdot b_{222} \cdot a_{311}.$$

If we compare results of above equations, we can see that we have the same result in both cases.

9. For plan $k = 3$: Let A and B be cubic-matries of order 3, where all elements on the plan $k = 2$ and $k = 3$ are identical in both matrices, then we have:

$$det(A_{[3\times3\times3]}) + det(B_{[3\times3\times3]}) = det\begin{pmatrix} a_{111} & a_{121} & a_{131} & a_{112} & a_{122} & a_{132} & a_{113} & a_{123} & a_{133} \\ a_{211} & a_{221} & a_{231} & a_{212} & a_{222} & a_{232} & a_{213} & a_{223} & a_{233} \\ a_{311} & a_{321} & a_{331} & a_{312} & a_{322} & a_{332} & a_{313} & a_{323} & a_{333} \end{pmatrix}$$
$$+det\begin{pmatrix} b_{111} & b_{121} & b_{131} & a_{112} & a_{122} & a_{132} & a_{113} & a_{123} & a_{133} \\ b_{211} & b_{221} & b_{231} & a_{212} & a_{222} & a_{232} & a_{213} & a_{223} & a_{233} \\ b_{311} & b_{321} & b_{331} & a_{312} & a_{322} & a_{332} & a_{313} & a_{323} & a_{333} \end{pmatrix}$$
$$= \{a_{111} \cdot a_{222} \cdot a_{333} - a_{111} \cdot a_{232} \cdot a_{323} - a_{111} \cdot a_{223} \cdot a_{332} + a_{111} \cdot a_{233} \cdot a_{322} - a_{112} \cdot a_{221} \cdot a_{333}$$
$$+a_{112} \cdot a_{223} \cdot a_{331} + a_{112} \cdot a_{231} \cdot a_{323} - a_{112} \cdot a_{233} \cdot a_{321} + a_{113} \cdot a_{221} \cdot a_{332} - a_{113} \cdot a_{222} \cdot a_{331}$$
$$-a_{113} \cdot a_{231} \cdot a_{322} + a_{113} \cdot a_{232} \cdot a_{321} - a_{121} \cdot a_{212} \cdot a_{333} + a_{121} \cdot a_{213} \cdot a_{332} + a_{121} \cdot a_{232} \cdot a_{313}$$
$$-a_{121} \cdot a_{233} \cdot a_{312} + a_{122} \cdot a_{211} \cdot a_{333} - a_{122} \cdot a_{213} \cdot a_{331} - a_{122} \cdot a_{231} \cdot a_{313} + a_{122} \cdot a_{233} \cdot a_{311}$$
$$-a_{123} \cdot a_{211} \cdot a_{332} + a_{123} \cdot a_{212} \cdot a_{331} + a_{123} \cdot a_{231} \cdot a_{312} - a_{123} \cdot a_{232} \cdot a_{311} + a_{131} \cdot a_{212} \cdot a_{323}$$
$$-a_{131} \cdot a_{213} \cdot a_{322} - a_{131} \cdot a_{222} \cdot a_{313} + a_{131} \cdot a_{223} \cdot a_{312} - a_{132} \cdot a_{211} \cdot a_{323} + a_{132} \cdot a_{213} \cdot a_{321}$$
$$+a_{132} \cdot a_{221} \cdot a_{313} - a_{132} \cdot a_{223} \cdot a_{311} + a_{133} \cdot a_{211} \cdot a_{322} - a_{133} \cdot a_{212} \cdot a_{321} - a_{133} \cdot a_{221} \cdot a_{312}$$
$$+a_{133} \cdot a_{222} \cdot a_{311}\} + \{b_{111} \cdot a_{222} \cdot a_{333} - b_{111} \cdot a_{232} \cdot a_{323} - b_{111} \cdot a_{223} \cdot a_{332} + b_{111} \cdot a_{233} \cdot a_{322}$$

$$-a_{112}\cdot b_{221}\cdot a_{333}+a_{112}\cdot a_{223}\cdot b_{331}+a_{112}\cdot b_{231}\cdot a_{323}-a_{112}\cdot a_{233}\cdot b_{321}+a_{113}\cdot b_{221}\cdot a_{332}$$
$$-a_{113}\cdot a_{222}\cdot b_{331}-a_{113}\cdot b_{231}\cdot a_{322}+a_{113}\cdot a_{232}\cdot b_{321}-b_{121}\cdot a_{212}\cdot a_{333}+b_{121}\cdot a_{213}\cdot a_{332}$$
$$+b_{121}\cdot a_{232}\cdot a_{313}-b_{121}\cdot a_{233}\cdot a_{312}+a_{122}\cdot b_{211}\cdot a_{333}-a_{122}\cdot a_{213}\cdot b_{331}-a_{122}\cdot b_{231}\cdot a_{313}$$
$$+a_{122}\cdot a_{233}\cdot b_{311}-a_{123}\cdot b_{211}\cdot a_{332}+a_{123}\cdot a_{212}\cdot b_{331}+a_{123}\cdot b_{231}\cdot a_{312}-a_{123}\cdot a_{232}\cdot b_{311}$$
$$+b_{131}\cdot a_{212}\cdot a_{323}-b_{131}\cdot a_{213}\cdot a_{322}-b_{131}\cdot a_{222}\cdot a_{313}+b_{131}\cdot a_{223}\cdot a_{312}-a_{132}\cdot b_{211}\cdot a_{323}$$
$$+a_{132}\cdot a_{213}\cdot b_{321}+a_{132}\cdot b_{221}\cdot a_{313}-a_{132}\cdot a_{223}\cdot b_{311}+a_{133}\cdot b_{211}\cdot a_{322}-a_{133}\cdot a_{212}\cdot b_{321}$$
$$-a_{133}\cdot b_{221}\cdot a_{312}+a_{133}\cdot a_{222}\cdot b_{311}\},$$

while,

$$\det\left(A_{[3\times3\times3]}\right)+det\left(B_{[3\times3\times3]}\right)$$
$$=\det\left(\begin{matrix}a_{111}+b_{111} & a_{121}+b_{121} & a_{131}+b_{131}\\ a_{211}+b_{211} & a_{221}+b_{221} & a_{231}+b_{231}\\ a_{311}+b_{311} & a_{321}+b_{321} & a_{331}+b_{331}\end{matrix}\left|\begin{matrix}a_{112} & a_{122} & a_{132}\\ a_{212} & a_{222} & a_{232}\\ a_{312} & a_{322} & a_{332}\end{matrix}\right|\begin{matrix}a_{113} & a_{123} & a_{133}\\ a_{213} & a_{223} & a_{233}\\ a_{313} & a_{323} & a_{333}\end{matrix}\right)$$
$$=(a_{111}+b_{111})\cdot a_{222}\cdot a_{333}-(a_{111}+b_{111})\cdot a_{232}\cdot a_{323}-(a_{111}+b_{111})\cdot a_{223}\cdot a_{332}$$
$$+(a_{111}+b_{111})\cdot a_{233}\cdot a_{322}-a_{112}\cdot(a_{221}+b_{221})\cdot a_{333}+a_{112}\cdot a_{223}\cdot(a_{331}+b_{331})$$
$$+a_{112}\cdot(a_{231}+b_{231})\cdot a_{323}-a_{112}\cdot a_{233}\cdot(a_{321}+b_{321})+a_{113}\cdot(a_{221}+b_{221})\cdot a_{332}$$
$$-a_{113}\cdot a_{222}\cdot(a_{331}+b_{331})-a_{113}\cdot(a_{231}+b_{231})\cdot a_{322}+a_{113}\cdot a_{232}\cdot(a_{321}+b_{321})$$
$$-(a_{121}+b_{121})\cdot a_{212}\cdot a_{333}+(a_{121}+b_{121})\cdot a_{213}\cdot a_{332}+(a_{121}+b_{121})\cdot a_{232}\cdot a_{313}$$
$$-(a_{121}+b_{121})\cdot a_{233}\cdot a_{312}+a_{122}\cdot(a_{211}+b_{211})\cdot a_{333}-a_{122}\cdot a_{213}\cdot(a_{331}+b_{331})$$
$$-a_{122}\cdot(a_{231}+b_{231})\cdot a_{313}+a_{122}\cdot a_{233}\cdot(a_{311}+b_{311})-a_{123}\cdot(a_{211}+b_{211})\cdot a_{332}$$
$$+a_{123}\cdot a_{212}\cdot(a_{331}+b_{331})+a_{123}\cdot(a_{231}+b_{231})\cdot a_{312}-a_{123}\cdot a_{232}\cdot(a_{311}+b_{311})$$
$$+(a_{131}+b_{131})\cdot a_{212}\cdot a_{323}-(a_{131}+b_{131})\cdot a_{213}\cdot a_{322}-(a_{131}+b_{131})\cdot a_{222}\cdot a_{313}$$
$$+(a_{131}+b_{131})\cdot a_{223}\cdot a_{312}-a_{132}\cdot(a_{211}+b_{211})\cdot a_{323}+a_{132}\cdot a_{213}\cdot(a_{321}+b_{321})$$
$$+a_{132}\cdot(a_{221}+b_{221})\cdot a_{313}-a_{132}\cdot a_{223}\cdot(a_{311}+b_{311})+a_{133}\cdot(a_{211}+b_{211})\cdot a_{322}$$
$$-a_{133}\cdot a_{212}\cdot(a_{321}+b_{321})-a_{133}\cdot(a_{221}+b_{221})\cdot a_{312}+a_{133}\cdot a_{222}\cdot(a_{311}+b_{311})$$
$$=a_{111}\cdot a_{222}\cdot a_{333}+b_{111}\cdot a_{222}\cdot a_{333}-a_{111}\cdot a_{232}\cdot a_{323}-b_{111}\cdot a_{232}\cdot a_{323}-a_{111}\cdot a_{223}\cdot a_{332}$$
$$-b_{111}\cdot a_{223}\cdot a_{332}+a_{111}\cdot a_{233}\cdot a_{322}+b_{111}\cdot a_{233}\cdot a_{322}-a_{112}\cdot a_{221}\cdot a_{333}-a_{112}\cdot b_{221}\cdot a_{333}$$
$$+a_{112}\cdot a_{223}\cdot a_{331}+a_{112}\cdot a_{223}\cdot b_{331}+a_{112}\cdot a_{231}\cdot a_{323}+a_{112}\cdot b_{231}\cdot a_{323}-a_{112}\cdot a_{233}\cdot a_{321}$$
$$-a_{112}\cdot a_{233}\cdot b_{321}+a_{113}\cdot a_{221}\cdot a_{332}+a_{113}\cdot b_{221}\cdot a_{332}-a_{113}\cdot a_{222}\cdot a_{331}-a_{113}\cdot a_{222}\cdot b_{331}$$
$$-a_{113}\cdot a_{231}\cdot a_{322}-a_{113}\cdot a_{231}\cdot a_{322}+a_{113}\cdot a_{232}\cdot a_{321}+a_{113}\cdot a_{232}\cdot b_{321}-a_{121}\cdot a_{212}\cdot a_{333}$$
$$-b_{121}\cdot a_{212}\cdot a_{333}+a_{121}\cdot a_{213}\cdot a_{332}+b_{121}\cdot a_{213}\cdot a_{332}+a_{121}\cdot a_{232}\cdot a_{313}+b_{121}\cdot a_{232}\cdot a_{313}$$
$$-a_{121}\cdot a_{233}\cdot a_{312}-b_{121}\cdot a_{233}\cdot a_{312}+a_{122}\cdot a_{211}\cdot a_{333}+a_{122}\cdot b_{211}\cdot a_{333}-a_{122}\cdot a_{213}\cdot a_{331}$$
$$-a_{122}\cdot a_{213}\cdot b_{331}-a_{122}\cdot a_{231}\cdot a_{313}-a_{122}\cdot b_{231}\cdot a_{313}+a_{122}\cdot a_{233}\cdot a_{311}+a_{122}\cdot a_{233}\cdot b_{311}$$
$$-a_{123}\cdot a_{211}\cdot a_{332}-a_{123}\cdot b_{211}\cdot a_{332}+a_{123}\cdot a_{212}\cdot a_{331}+a_{123}\cdot a_{212}\cdot b_{331}+a_{123}\cdot a_{231}\cdot a_{312}$$
$$+a_{123}\cdot b_{231}\cdot a_{312}-a_{123}\cdot a_{232}\cdot a_{311}-a_{123}\cdot a_{232}\cdot b_{311}+a_{131}\cdot a_{212}\cdot a_{323}+b_{131}\cdot a_{212}\cdot a_{323}$$
$$-a_{131}\cdot a_{213}\cdot a_{322}-b_{131}\cdot a_{213}\cdot a_{322}-a_{131}\cdot a_{222}\cdot a_{313}-b_{131}\cdot a_{222}\cdot a_{313}+a_{131}\cdot a_{223}\cdot a_{312}$$
$$+b_{131}\cdot a_{223}\cdot a_{312}-a_{132}\cdot a_{211}\cdot a_{323}-a_{132}\cdot b_{211}\cdot a_{323}+a_{132}\cdot a_{213}\cdot a_{321}+a_{132}\cdot a_{213}\cdot b_{321}$$
$$+a_{132}\cdot a_{221}\cdot a_{313}+a_{132}\cdot b_{221}\cdot a_{313}-a_{132}\cdot a_{223}\cdot a_{311}-a_{132}\cdot a_{223}\cdot b_{311}+a_{133}\cdot a_{211}\cdot a_{322}$$
$$+a_{133}\cdot b_{211}\cdot a_{322}-a_{133}\cdot a_{212}\cdot a_{321}-a_{133}\cdot a_{212}\cdot b_{321}-a_{133}\cdot a_{221}\cdot a_{312}-a_{133}\cdot b_{221}\cdot a_{312}$$
$$+a_{133}\cdot a_{222}\cdot a_{311}+a_{133}\cdot a_{222}\cdot b_{311}.$$

If we compare results of above equations, we can see that we have the same result in both cases.
The proof is complete.

Theorem 2:

Let it be the cubic matrix B, which is formed by multiplying a plane (in the indices j and k, it does not work in the index i) of the matrix A by a scalar α, and addition another plane, then we have:

$$det(A) = det(B)$$

Proof:

Case 1: The cubic-matrix A of order 2, (and B has order 2), we will proof the case 1 for each "vertical page" and "vertical layer", as following:

$$det(A_{[2\times2\times2]}) = \left(\begin{matrix} a_{111} & a_{121} \\ a_{211} & a_{221} \end{matrix}\middle| \begin{matrix} a_{112} & a_{122} \\ a_{212} & a_{222} \end{matrix}\right) = a_{111} \cdot a_{222} - a_{112} \cdot a_{221} - a_{121} \cdot a_{212} + a_{122} \cdot a_{211}.$$

1. For plan $j = 1$: Let us add first vertical page to second vertical page while multiplying by a scalar α.

$$det(B_{[2\times2\times2]}) = \left(\begin{matrix} a_{111} & a_{121} + \alpha \cdot a_{111} \\ a_{211} & a_{221} + \alpha \cdot a_{211} \end{matrix}\middle| \begin{matrix} a_{112} & a_{122} + \alpha \cdot a_{112} \\ a_{212} & a_{222} + \alpha \cdot a_{212} \end{matrix}\right) = a_{111} \cdot (a_{222} + \alpha \cdot a_{212})$$
$$-(a_{122} + \alpha \cdot a_{112}) \cdot a_{221} - (a_{121} + \alpha \cdot a_{111}) \cdot a_{212} + a_{122} \cdot (a_{221} + \alpha \cdot a_{211}).$$

After expanding further, we get the following result:

$$det(B_{[2\times2\times2]}) = a_{111} \cdot a_{222} - a_{112} \cdot a_{221} - a_{121} \cdot a_{212} + a_{122} \cdot a_{211}$$

If we compare results of above equations, we can see that we have the same result in both cases.

2. For plan $j = 2$: Let us add first vertical page to first vertical page while multiplying by a scalar α.

$$det(B_{[2\times2\times2]}) = \left(\begin{matrix} a_{111} + \alpha \cdot a_{121} & a_{121} \\ a_{211} + \alpha \cdot a_{211} & a_{221} \end{matrix}\middle| \begin{matrix} a_{112} + \alpha \cdot a_{122} & a_{122} \\ a_{212} + \alpha \cdot a_{222} & a_{222} \end{matrix}\right) = (a_{111} + \alpha \cdot a_{121}) \cdot a_{222}$$
$$-(a_{112} + \alpha \cdot a_{122}) \cdot a_{221} - a_{121} \cdot (a_{212} + \alpha \cdot a_{222}) + a_{122} \cdot (a_{211} + \alpha \cdot a_{221})$$

After expanding further, we get the following result:

$$det(B_{[2\times2\times2]}) = a_{111} \cdot a_{222} - a_{112} \cdot a_{221} - a_{121} \cdot a_{212} + a_{122} \cdot a_{211}$$

If we compare results of above equations, we can see that we have the same result in both cases.

3. For plan $k = 1$: Let us add first vertical page to second vertical layer while multiplying by a scalar α.

$$det(B_{[2\times2\times2]}) = \left(\begin{matrix} a_{111} & a_{121} \\ a_{211} & a_{221} \end{matrix}\middle| \begin{matrix} a_{112} + \alpha \cdot a_{111} & a_{122} + \alpha \cdot a_{121} \\ a_{212} + \alpha \cdot a_{211} & a_{222} + \alpha \cdot a_{221} \end{matrix}\right) = a_{111} \cdot (a_{222} + \alpha \cdot a_{221})$$
$$-(a_{112} + \alpha \cdot a_{111}) \cdot a_{221} - a_{121} \cdot (a_{212} + \alpha \cdot a_{211}) + (a_{122} + \alpha \cdot a_{121}) \cdot a_{211}$$

After expanding further, we get the following result:

$$det(B_{[2\times2\times2]}) = a_{111} \cdot a_{222} - a_{112} \cdot a_{221} - a_{121} \cdot a_{212} + a_{122} \cdot a_{211}$$

If we compare results of above equations, we can see that we have the same result in both cases.

4. For plan $k = 2$: Let us add first vertical page to first vertical layer while multiplying by a scalar α.

$$det(B_{[2\times2\times2]}) = \left(\begin{matrix} a_{111} + \alpha \cdot a_{112} & a_{121} + \alpha \cdot a_{122} \\ a_{211} + \alpha \cdot a_{212} & a_{221} + \alpha \cdot a_{222} \end{matrix}\middle| \begin{matrix} a_{112} & a_{122} \\ a_{212} & a_{222} \end{matrix}\right) = (a_{111} + \alpha \cdot a_{112}) \cdot a_{222}$$
$$-a_{112} \cdot (a_{221} + \alpha \cdot a_{222}) - (a_{121} + \alpha \cdot a_{122}) \cdot a_{212} + a_{122} \cdot (a_{211} + \alpha \cdot a_{212})$$

After expanding further, we get the following result:

$$det(B_{[2\times2\times2]}) = a_{111} \cdot a_{222} - a_{112} \cdot a_{221} - a_{121} \cdot a_{212} + a_{122} \cdot a_{211}$$

If we compare results of above equations, we can see that we have the same result in both cases.

Case 2: The cubic-matrix A of order 3, (and B has order 3), we will proof the case 1 for each "vertical page" and "vertical layer", as following:

$$det\left(A_{[3\times3\times3]}\right) = det\left(\begin{matrix} a_{111} & a_{121} & a_{131} \\ a_{211} & a_{221} & a_{231} \\ a_{311} & a_{321} & a_{331} \end{matrix}\middle| \begin{matrix} a_{112} & a_{122} & a_{132} \\ a_{212} & a_{222} & a_{232} \\ a_{312} & a_{322} & a_{332} \end{matrix}\middle| \begin{matrix} a_{113} & a_{123} & a_{133} \\ a_{213} & a_{223} & a_{233} \\ a_{313} & a_{323} & a_{333} \end{matrix}\right)$$

$$\begin{gathered} = a_{111}\cdot a_{222}\cdot a_{333} - a_{111}\cdot a_{232}\cdot a_{323} - a_{111}\cdot a_{223}\cdot a_{332} + a_{111}\cdot a_{233}\cdot a_{322} - a_{112}\cdot a_{221}\cdot a_{333} \\ + a_{112}\cdot a_{223}\cdot a_{331} + a_{112}\cdot a_{231}\cdot a_{323} - a_{112}\cdot a_{233}\cdot a_{321} + a_{113}\cdot a_{221}\cdot a_{332} - a_{113}\cdot a_{222}\cdot a_{331} \\ - a_{113}\cdot a_{231}\cdot a_{322} + a_{113}\cdot a_{232}\cdot a_{321} - a_{121}\cdot a_{212}\cdot a_{333} + a_{121}\cdot a_{213}\cdot a_{332} + a_{121}\cdot a_{232}\cdot a_{313} \\ - a_{121}\cdot a_{233}\cdot a_{312} + a_{122}\cdot a_{211}\cdot a_{333} - a_{122}\cdot a_{213}\cdot a_{331} - a_{122}\cdot a_{231}\cdot a_{313} + a_{122}\cdot a_{233}\cdot a_{311} \\ - a_{123}\cdot a_{211}\cdot a_{332} + a_{123}\cdot a_{212}\cdot a_{331} + a_{123}\cdot a_{231}\cdot a_{312} - a_{123}\cdot a_{232}\cdot a_{311} + a_{131}\cdot a_{212}\cdot a_{323} \\ - a_{131}\cdot a_{213}\cdot a_{322} - a_{131}\cdot a_{222}\cdot a_{313} + a_{131}\cdot a_{223}\cdot a_{312} - a_{132}\cdot a_{211}\cdot a_{323} + a_{132}\cdot a_{213}\cdot a_{321} \\ + a_{132}\cdot a_{221}\cdot a_{313} - a_{132}\cdot a_{223}\cdot a_{311} + a_{133}\cdot a_{211}\cdot a_{322} - a_{133}\cdot a_{212}\cdot a_{321} - a_{133}\cdot a_{221}\cdot a_{312} \\ + a_{133}\cdot a_{222}\cdot a_{311} \end{gathered}$$

1. For plan $j = 1$: Let us add first vertical page to second vertical page while multiplying by a scalar α.

$$det\left(B_{[3\times3\times3]}\right)$$

$$= det\left(\begin{matrix} a_{111} & a_{121}+\alpha\cdot a_{111} & a_{131} \\ a_{211} & a_{221}+\alpha\cdot a_{211} & a_{231} \\ a_{311} & a_{321}+\alpha\cdot a_{311} & a_{331} \end{matrix}\middle| \begin{matrix} a_{112} & a_{122}+\alpha\cdot a_{112} & a_{132} \\ a_{212} & a_{222}+\alpha\cdot a_{212} & a_{232} \\ a_{312} & a_{322}+\alpha\cdot a_{312} & a_{332} \end{matrix}\middle| \begin{matrix} a_{113} & a_{123}+\alpha\cdot a_{113} & a_{133} \\ a_{213} & a_{223}+\alpha\cdot a_{213} & a_{233} \\ a_{313} & a_{323}+\alpha\cdot a_{313} & a_{333} \end{matrix}\right)$$

$$\begin{gathered} = a_{111}\cdot(a_{222}+\alpha\cdot a_{212})\cdot a_{333} - a_{111}\cdot a_{232}\cdot(a_{323}+\alpha\cdot a_{313}) - a_{111}\cdot(a_{223}+\alpha\cdot a_{213})\cdot a_{332} \\ + a_{111}\cdot a_{233}\cdot(a_{322}+\alpha\cdot a_{312}) - a_{112}\cdot(a_{221}+\alpha\cdot a_{211})\cdot a_{333} + a_{112}\cdot(a_{223}+\alpha\cdot a_{213})\cdot a_{331} \\ + a_{112}\cdot a_{231}\cdot(a_{323}+\alpha\cdot a_{313}) - a_{112}\cdot a_{233}\cdot(a_{321}+\alpha\cdot a_{311}) + a_{113}\cdot(a_{221}+\alpha\cdot a_{211})\cdot a_{332} \\ - a_{113}\cdot(a_{222}+\alpha\cdot a_{212})\cdot a_{331} - a_{113}\cdot a_{231}\cdot(a_{322}+\alpha\cdot a_{312}) + a_{113}\cdot a_{232}\cdot(a_{321}+\alpha\cdot a_{311}) \\ -(a_{121}+\alpha\cdot a_{111})\cdot a_{212}\cdot a_{333} + (a_{121}+\alpha\cdot a_{111})\cdot a_{213}\cdot a_{332} + (a_{121}+\alpha\cdot a_{111})\cdot a_{232}\cdot a_{313} \\ -(a_{121}+\alpha\cdot a_{111})\cdot a_{233}\cdot a_{312} + (a_{122}+\alpha\cdot a_{112})\cdot a_{211}\cdot a_{333} - (a_{122}+\alpha\cdot a_{112})\cdot a_{213}\cdot a_{331} \\ -(a_{122}+\alpha\cdot a_{112})\cdot a_{231}\cdot a_{313} + (a_{122}+\alpha\cdot a_{112})\cdot a_{233}\cdot a_{311} - (a_{123}+\alpha\cdot a_{113})\cdot a_{211}\cdot a_{332} \\ +(a_{123}+\alpha\cdot a_{113})\cdot a_{212}\cdot a_{331} + (a_{123}+\alpha\cdot a_{113})\cdot a_{231}\cdot a_{312} - (a_{123}+\alpha\cdot a_{113})\cdot a_{232}\cdot a_{311} \\ + a_{131}\cdot a_{212}\cdot(a_{323}+\alpha\cdot a_{313}) - a_{131}\cdot a_{213}\cdot(a_{322}+\alpha\cdot a_{312}) - a_{131}\cdot(a_{222}+\alpha\cdot a_{212})\cdot a_{313} \\ + a_{131}\cdot(a_{223}+\alpha\cdot a_{213})\cdot a_{312} - a_{132}\cdot a_{211}\cdot(a_{323}+\alpha\cdot a_{313}) + a_{132}\cdot a_{213}\cdot(a_{321}+\alpha\cdot a_{311}) \\ + a_{132}\cdot(a_{221}+\alpha\cdot a_{211})\cdot a_{313} - a_{132}\cdot(a_{223}+\alpha\cdot a_{213})\cdot a_{311} + a_{133}\cdot a_{211}\cdot(a_{322}+\alpha\cdot a_{312}) \\ - a_{133}\cdot a_{212}\cdot(a_{321}+\alpha\cdot a_{311}) - a_{133}\cdot(a_{221}+\alpha\cdot a_{211})\cdot a_{312} + a_{133}\cdot(a_{222}+\alpha\cdot a_{212})\cdot a_{311}. \end{gathered}$$

After expanding further, we get the following result:

$$\begin{gathered} det\left(B_{[3\times3\times3]}\right) = a_{111}\cdot a_{222}\cdot a_{333} - a_{111}\cdot a_{232}\cdot a_{323} - a_{111}\cdot a_{223}\cdot a_{332} + a_{111}\cdot a_{233}\cdot a_{322} \\ - a_{112}\cdot a_{221}\cdot a_{333} + a_{112}\cdot a_{223}\cdot a_{331} + a_{112}\cdot a_{231}\cdot a_{323} - a_{112}\cdot a_{233}\cdot a_{321} + a_{113}\cdot a_{221}\cdot a_{332} \\ - a_{113}\cdot a_{222}\cdot a_{331} - a_{113}\cdot a_{231}\cdot a_{322} + a_{113}\cdot a_{232}\cdot a_{321} - a_{121}\cdot a_{212}\cdot a_{333} + a_{121}\cdot a_{213}\cdot a_{332} \\ + a_{121}\cdot a_{232}\cdot a_{313} - a_{121}\cdot a_{233}\cdot a_{312} + a_{122}\cdot a_{211}\cdot a_{333} - a_{122}\cdot a_{213}\cdot a_{331} - a_{122}\cdot a_{231}\cdot a_{313} \\ + a_{122}\cdot a_{233}\cdot a_{311} - a_{123}\cdot a_{211}\cdot a_{332} + a_{123}\cdot a_{212}\cdot a_{331} + a_{123}\cdot a_{231}\cdot a_{312} - a_{123}\cdot a_{232}\cdot a_{311} \\ + a_{131}\cdot a_{212}\cdot a_{323} - a_{131}\cdot a_{213}\cdot a_{322} - a_{131}\cdot a_{222}\cdot a_{313} + a_{131}\cdot a_{223}\cdot a_{312} - a_{132}\cdot a_{211}\cdot a_{323} \\ + a_{132}\cdot a_{213}\cdot a_{321} + a_{132}\cdot a_{221}\cdot a_{313} - a_{132}\cdot a_{223}\cdot a_{311} + a_{133}\cdot a_{211}\cdot a_{322} - a_{133}\cdot a_{212}\cdot a_{321} \\ - a_{133}\cdot a_{221}\cdot a_{312} + a_{133}\cdot a_{222}\cdot a_{311}. \end{gathered}$$

If we compare results of above equations, we can see that we have the same result in both cases.

2. For plan $j = 2$: Let us add first vertical page to third vertical page while multiplying by a scalar α.

$$det(B_{[3\times3\times3]})$$
$$= det\left(\begin{matrix} a_{111} & a_{121} & a_{131}+\alpha\cdot a_{111} \\ a_{211} & a_{221} & a_{231}+\alpha\cdot a_{211} \\ a_{311} & a_{321} & a_{331}+\alpha\cdot a_{311}\end{matrix}\middle| \begin{matrix} a_{112} & a_{122} & a_{132}+\alpha\cdot a_{112} \\ a_{212} & a_{222} & a_{232}+\alpha\cdot a_{212} \\ a_{312} & a_{322} & a_{332}+\alpha\cdot a_{312}\end{matrix}\middle| \begin{matrix} a_{113} & a_{123} & a_{133}+\alpha\cdot a_{113} \\ a_{213} & a_{223} & a_{233}+\alpha\cdot a_{213} \\ a_{313} & a_{323} & a_{333}+\alpha\cdot a_{313}\end{matrix}\right)$$
$$= a_{111}\cdot a_{222}\cdot(a_{333}+\alpha\cdot a_{313}) - a_{111}\cdot(a_{232}+\alpha\cdot a_{212})\cdot a_{323} - a_{111}\cdot a_{223}\cdot(a_{332}+\alpha\cdot a_{312})$$
$$+a_{111}\cdot(a_{233}+\alpha\cdot a_{213})\cdot a_{322} - a_{112}\cdot a_{221}\cdot(a_{333}+\alpha\cdot a_{313}) + a_{112}\cdot a_{223}\cdot(a_{331}+\alpha\cdot a_{311})$$
$$+a_{112}\cdot(a_{231}+\alpha\cdot a_{211})\cdot a_{323} - a_{112}\cdot(a_{233}+\alpha\cdot a_{213})\cdot a_{321} + a_{113}\cdot a_{221}\cdot(a_{332}+\alpha\cdot a_{312})$$
$$-a_{113}\cdot a_{222}\cdot(a_{331}+\alpha\cdot a_{311}) - a_{113}\cdot(a_{231}+\alpha\cdot a_{211})\cdot a_{322} + a_{113}\cdot(a_{232}+\alpha\cdot a_{212})\cdot a_{321}$$
$$-a_{121}\cdot a_{212}\cdot(a_{333}+\alpha\cdot a_{313}) + a_{121}\cdot a_{213}\cdot(a_{332}+\alpha\cdot a_{312}) + a_{121}\cdot(a_{232}+\alpha\cdot a_{212})\cdot a_{313}$$
$$-a_{121}\cdot(a_{233}+\alpha\cdot a_{213})\cdot a_{312} + a_{122}\cdot a_{211}\cdot(a_{333}+\alpha\cdot a_{313}) - a_{122}\cdot a_{213}\cdot(a_{331}+\alpha\cdot a_{311})$$
$$-a_{122}\cdot(a_{231}+\alpha\cdot a_{211})\cdot a_{313} + a_{122}\cdot(a_{233}+\alpha\cdot a_{213})\cdot a_{311} - a_{123}\cdot a_{211}\cdot(a_{332}+\alpha\cdot a_{312})$$
$$+a_{123}\cdot a_{212}\cdot(a_{331}+\alpha\cdot a_{311}) + a_{123}\cdot(a_{231}+\alpha\cdot a_{211})\cdot a_{312} - a_{123}\cdot(a_{232}+\alpha\cdot a_{212})\cdot a_{311}$$
$$+(a_{131}+\alpha\cdot a_{111})\cdot a_{212}\cdot a_{323} - (a_{131}+\alpha\cdot a_{111})\cdot a_{213}\cdot a_{322} - (a_{131}+\alpha\cdot a_{111})\cdot a_{222}\cdot a_{313}$$
$$+(a_{131}+\alpha\cdot a_{111})\cdot a_{223}\cdot a_{312} - (a_{132}+\alpha\cdot a_{112})\cdot a_{211}\cdot a_{323} + (a_{132}+\alpha\cdot a_{112})\cdot a_{213}\cdot a_{321}$$
$$+(a_{132}+\alpha\cdot a_{112})\cdot a_{221}\cdot a_{313} - (a_{132}+\alpha\cdot a_{112})\cdot a_{223}\cdot a_{311} + (a_{133}+\alpha\cdot a_{113})\cdot a_{211}\cdot a_{322}$$
$$-(a_{133}+\alpha\cdot a_{113})\cdot a_{212}\cdot a_{321} - (a_{133}+\alpha\cdot a_{113})\cdot a_{221}\cdot a_{312} + (a_{133}+\alpha\cdot a_{113})\cdot a_{222}\cdot a_{311}.$$

After expanding further, we get the following result:

$$det(B_{[3\times3\times3]}) = a_{111}\cdot a_{222}\cdot a_{333} - a_{111}\cdot a_{232}\cdot a_{323} - a_{111}\cdot a_{223}\cdot a_{332} + a_{111}\cdot a_{233}\cdot a_{322}$$
$$-a_{112}\cdot a_{221}\cdot a_{333} + a_{112}\cdot a_{223}\cdot a_{331} + a_{112}\cdot a_{231}\cdot a_{323} - a_{112}\cdot a_{233}\cdot a_{321} + a_{113}\cdot a_{221}\cdot a_{332}$$
$$-a_{113}\cdot a_{222}\cdot a_{331} - a_{113}\cdot a_{231}\cdot a_{322} + a_{113}\cdot a_{232}\cdot a_{321} - a_{121}\cdot a_{212}\cdot a_{333} + a_{121}\cdot a_{213}\cdot a_{332}$$
$$+a_{121}\cdot a_{232}\cdot a_{313} - a_{121}\cdot a_{233}\cdot a_{312} + a_{122}\cdot a_{211}\cdot a_{333} - a_{122}\cdot a_{213}\cdot a_{331} - a_{122}\cdot a_{231}\cdot a_{313}$$
$$+a_{122}\cdot a_{233}\cdot a_{311} - a_{123}\cdot a_{211}\cdot a_{332} + a_{123}\cdot a_{212}\cdot a_{331} + a_{123}\cdot a_{231}\cdot a_{312} - a_{123}\cdot a_{232}\cdot a_{311}$$
$$+a_{131}\cdot a_{212}\cdot a_{323} - a_{131}\cdot a_{213}\cdot a_{322} - a_{131}\cdot a_{222}\cdot a_{313} + a_{131}\cdot a_{223}\cdot a_{312} - a_{132}\cdot a_{211}\cdot a_{323}$$
$$+a_{132}\cdot a_{213}\cdot a_{321} + a_{132}\cdot a_{221}\cdot a_{313} - a_{132}\cdot a_{223}\cdot a_{311} + a_{133}\cdot a_{211}\cdot a_{322} - a_{133}\cdot a_{212}\cdot a_{321}$$
$$-a_{133}\cdot a_{221}\cdot a_{312} + a_{133}\cdot a_{222}\cdot a_{311}.$$

3. For plan $j = 3$: Let us add second vertical page to third vertical page while multiplying by a scalar α.

$$det(B_{[3\times3\times3]})$$
$$= det\left(\begin{matrix} a_{111} & a_{121} & a_{131}+\alpha\cdot a_{121} \\ a_{211} & a_{221} & a_{231}+\alpha\cdot a_{221} \\ a_{311} & a_{321} & a_{331}+\alpha\cdot a_{321}\end{matrix}\middle| \begin{matrix} a_{112} & a_{122} & a_{132}+\alpha\cdot a_{122} \\ a_{212} & a_{222} & a_{232}+\alpha\cdot a_{222} \\ a_{312} & a_{322} & a_{332}+\alpha\cdot a_{322}\end{matrix}\middle| \begin{matrix} a_{113} & a_{123} & a_{133}+\alpha\cdot a_{123} \\ a_{213} & a_{223} & a_{233}+\alpha\cdot a_{223} \\ a_{313} & a_{323} & a_{333}+\alpha\cdot a_{323}\end{matrix}\right)$$
$$= a_{111}\cdot a_{222}\cdot(a_{333}+\alpha\cdot a_{323}) - a_{111}\cdot(a_{232}+\alpha\cdot a_{222})\cdot a_{323} - a_{111}\cdot a_{223}\cdot(a_{332}+\alpha\cdot a_{312})$$
$$+a_{111}\cdot(a_{233}+\alpha\cdot a_{223})\cdot a_{322} - a_{112}\cdot a_{221}\cdot(a_{333}+\alpha\cdot a_{323}) + a_{112}\cdot a_{223}\cdot(a_{331}+\alpha\cdot a_{321})$$
$$+a_{112}\cdot(a_{231}+\alpha\cdot a_{211})\cdot a_{323} - a_{112}\cdot(a_{233}+\alpha\cdot a_{223})\cdot a_{321} + a_{113}\cdot a_{221}\cdot(a_{332}+\alpha\cdot a_{322})$$
$$-a_{113}\cdot a_{222}\cdot(a_{331}+\alpha\cdot a_{321}) - a_{113}\cdot(a_{231}+\alpha\cdot a_{211})\cdot a_{322} + a_{113}\cdot(a_{232}+\alpha\cdot a_{222})\cdot a_{321}$$
$$-a_{121}\cdot a_{212}\cdot(a_{333}+\alpha\cdot a_{323}) + a_{121}\cdot a_{213}\cdot(a_{332}+\alpha\cdot a_{322}) + a_{121}\cdot(a_{232}+\alpha\cdot a_{212})\cdot a_{313}$$
$$-a_{121}\cdot(a_{233}+\alpha\cdot a_{223})\cdot a_{312} + a_{122}\cdot a_{211}\cdot(a_{333}+\alpha\cdot a_{323}) - a_{122}\cdot a_{213}\cdot(a_{331}+\alpha\cdot a_{321})$$
$$-a_{122}\cdot(a_{231}+\alpha\cdot a_{211})\cdot a_{313} + a_{122}\cdot(a_{233}+\alpha\cdot a_{223})\cdot a_{311} - a_{123}\cdot a_{211}\cdot(a_{332}+\alpha\cdot a_{322})$$
$$+a_{123}\cdot a_{212}\cdot(a_{331}+\alpha\cdot a_{321}) + a_{123}\cdot(a_{231}+\alpha\cdot a_{211})\cdot a_{312} - a_{123}\cdot(a_{232}+\alpha\cdot a_{222})\cdot a_{311}$$
$$+(a_{131}+\alpha\cdot a_{121})\cdot a_{212}\cdot a_{323} - (a_{131}+\alpha\cdot a_{121})\cdot a_{213}\cdot a_{322} - (a_{131}+\alpha\cdot a_{121})\cdot a_{222}\cdot a_{313}$$
$$+(a_{131}+\alpha\cdot a_{121})\cdot a_{223}\cdot a_{312} - (a_{132}+\alpha\cdot a_{122})\cdot a_{211}\cdot a_{323} + (a_{132}+\alpha\cdot a_{122})\cdot a_{213}\cdot a_{321}$$

$$+(a_{132}+\alpha\cdot a_{122})\cdot a_{221}\cdot a_{313}-(a_{132}+\alpha\cdot a_{122})\cdot a_{223}\cdot a_{311}+(a_{133}+\alpha\cdot a_{123})\cdot a_{211}\cdot a_{322}$$
$$-(a_{133}+\alpha\cdot a_{123})\cdot a_{212}\cdot a_{321}-(a_{133}+\alpha\cdot a_{123})\cdot a_{221}\cdot a_{312}+(a_{133}+\alpha\cdot a_{123})\cdot a_{222}\cdot a_{311}.$$

After expanding further, we get the following result:

$$det(B_{[3\times3\times3]})=a_{111}\cdot a_{222}\cdot a_{333}-a_{111}\cdot a_{232}\cdot a_{323}-a_{111}\cdot a_{223}\cdot a_{332}+a_{111}\cdot a_{233}\cdot a_{322}$$
$$-a_{112}\cdot a_{221}\cdot a_{333}+a_{112}\cdot a_{223}\cdot a_{331}+a_{112}\cdot a_{231}\cdot a_{323}-a_{112}\cdot a_{233}\cdot a_{321}+a_{113}\cdot a_{221}\cdot a_{332}$$
$$-a_{113}\cdot a_{222}\cdot a_{331}-a_{113}\cdot a_{231}\cdot a_{322}+a_{113}\cdot a_{232}\cdot a_{321}-a_{121}\cdot a_{212}\cdot a_{333}+a_{121}\cdot a_{213}\cdot a_{332}$$
$$+a_{121}\cdot a_{232}\cdot a_{313}-a_{121}\cdot a_{233}\cdot a_{312}+a_{122}\cdot a_{211}\cdot a_{333}-a_{122}\cdot a_{213}\cdot a_{331}-a_{122}\cdot a_{231}\cdot a_{313}$$
$$+a_{122}\cdot a_{233}\cdot a_{311}-a_{123}\cdot a_{211}\cdot a_{332}+a_{123}\cdot a_{212}\cdot a_{331}+a_{123}\cdot a_{231}\cdot a_{312}-a_{123}\cdot a_{232}\cdot a_{311}$$
$$+a_{131}\cdot a_{212}\cdot a_{323}-a_{131}\cdot a_{213}\cdot a_{322}-a_{131}\cdot a_{222}\cdot a_{313}+a_{131}\cdot a_{223}\cdot a_{312}-a_{132}\cdot a_{211}\cdot a_{323}$$
$$+a_{132}\cdot a_{213}\cdot a_{321}+a_{132}\cdot a_{221}\cdot a_{313}-a_{132}\cdot a_{223}\cdot a_{311}+a_{133}\cdot a_{211}\cdot a_{322}-a_{133}\cdot a_{212}\cdot a_{321}$$
$$-a_{133}\cdot a_{221}\cdot a_{312}+a_{133}\cdot a_{222}\cdot a_{311}.$$

If we compare results of above equations, we can see that we have the same result in both cases.

4. For plan $k=1$: Let us add first vertical layer to second vertical layer while multiplying by a scalar α.

$$det(B_{[3\times3\times3]})$$
$$=det\left(\begin{array}{ccc|ccc|ccc} a_{111} & a_{121} & a_{131} & a_{112}+\alpha\cdot a_{111} & a_{122}+\alpha\cdot a_{121} & a_{132}+\alpha\cdot a_{131} & a_{113} & a_{123} & a_{133}\\ a_{211} & a_{221} & a_{231} & a_{212}+\alpha\cdot a_{211} & a_{222}+\alpha\cdot a_{221} & a_{232}+\alpha\cdot a_{231} & a_{213} & a_{223} & a_{233}\\ a_{311} & a_{321} & a_{331} & a_{312}+\alpha\cdot a_{311} & a_{322}+\alpha\cdot a_{321} & a_{332}+\alpha\cdot a_{331} & a_{313} & a_{323} & a_{333}\end{array}\right)$$
$$=a_{111}\cdot(a_{222}+\alpha\cdot a_{221})\cdot a_{333}-a_{111}\cdot(a_{232}+\alpha\cdot a_{231})\cdot a_{323}-a_{111}\cdot a_{223}\cdot(a_{332}+\alpha\cdot a_{331})$$
$$+a_{111}\cdot a_{233}\cdot(a_{322}+\alpha\cdot a_{321})-(a_{112}+\alpha\cdot a_{111})\cdot a_{221}\cdot a_{333}+(a_{112}+\alpha\cdot a_{111})\cdot a_{223}\cdot a_{331}$$
$$+(a_{112}+\alpha\cdot a_{111})\cdot a_{231}\cdot a_{323}-(a_{112}+\alpha\cdot a_{111})\cdot a_{233}\cdot a_{321}+a_{113}\cdot a_{221}\cdot(a_{332}+\alpha\cdot a_{331})$$
$$-a_{113}\cdot(a_{222}+\alpha\cdot a_{221})\cdot a_{331}-a_{113}\cdot a_{231}\cdot(a_{322}+\alpha\cdot a_{321})+a_{113}\cdot(a_{232}+\alpha\cdot a_{231})\cdot a_{321}$$
$$-a_{121}\cdot(a_{212}+\alpha\cdot a_{211})\cdot a_{333}+a_{121}\cdot a_{213}\cdot(a_{332}+\alpha\cdot a_{331})+a_{121}\cdot(a_{232}+\alpha\cdot a_{231})\cdot a_{313}$$
$$-a_{121}\cdot a_{233}\cdot(a_{312}+\alpha\cdot a_{311})+(a_{122}+\alpha\cdot a_{121})\cdot a_{211}\cdot a_{333}-(a_{122}+\alpha\cdot a_{121})\cdot a_{213}\cdot a_{331}$$
$$-(a_{122}+\alpha\cdot a_{121})\cdot a_{231}\cdot a_{313}+(a_{122}+\alpha\cdot a_{121})\cdot a_{233}\cdot a_{311}-a_{123}\cdot a_{211}\cdot(a_{332}+\alpha\cdot a_{331})$$
$$+a_{123}\cdot(a_{212}+\alpha\cdot a_{211})\cdot a_{331}+a_{123}\cdot a_{231}\cdot(a_{312}+\alpha\cdot a_{311})-a_{123}\cdot(a_{232}+\alpha\cdot a_{231})\cdot a_{311}$$
$$+a_{131}\cdot(a_{212}+\alpha\cdot a_{211})\cdot a_{323}-a_{131}\cdot a_{213}\cdot(a_{322}+\alpha\cdot a_{321})-a_{131}\cdot(a_{222}+\alpha\cdot a_{221})\cdot a_{313}$$
$$+a_{131}\cdot a_{223}\cdot(a_{312}+\alpha\cdot a_{311})-(a_{132}+\alpha\cdot a_{131})\cdot a_{211}\cdot a_{323}+(a_{132}+\alpha\cdot a_{131})\cdot a_{213}\cdot a_{321}$$
$$+(a_{132}+\alpha\cdot a_{131})\cdot a_{221}\cdot a_{313}-(a_{132}+\alpha\cdot a_{131})\cdot a_{223}\cdot a_{311}+a_{133}\cdot a_{211}\cdot(a_{322}+\alpha\cdot a_{321})$$
$$-a_{133}\cdot(a_{212}+\alpha\cdot a_{211})\cdot a_{321}-a_{133}\cdot a_{221}\cdot(a_{312}+\alpha\cdot a_{311})+a_{133}\cdot(a_{222}+\alpha\cdot a_{221})\cdot a_{311}.$$

After expanding further, we get the following result:

$$det(B_{[3\times3\times3]})=a_{111}\cdot a_{222}\cdot a_{333}-a_{111}\cdot a_{232}\cdot a_{323}-a_{111}\cdot a_{223}\cdot a_{332}+a_{111}\cdot a_{233}\cdot a_{322}$$
$$-a_{112}\cdot a_{221}\cdot a_{333}+a_{112}\cdot a_{223}\cdot a_{331}+a_{112}\cdot a_{231}\cdot a_{323}-a_{112}\cdot a_{233}\cdot a_{321}+a_{113}\cdot a_{221}\cdot a_{332}$$
$$-a_{113}\cdot a_{222}\cdot a_{331}-a_{113}\cdot a_{231}\cdot a_{322}+a_{113}\cdot a_{232}\cdot a_{321}-a_{121}\cdot a_{212}\cdot a_{333}+a_{121}\cdot a_{213}\cdot a_{332}$$
$$+a_{121}\cdot a_{232}\cdot a_{313}-a_{121}\cdot a_{233}\cdot a_{312}+a_{122}\cdot a_{211}\cdot a_{333}-a_{122}\cdot a_{213}\cdot a_{331}-a_{122}\cdot a_{231}\cdot a_{313}$$
$$+a_{122}\cdot a_{233}\cdot a_{311}-a_{123}\cdot a_{211}\cdot a_{332}+a_{123}\cdot a_{212}\cdot a_{331}+a_{123}\cdot a_{231}\cdot a_{312}-a_{123}\cdot a_{232}\cdot a_{311}$$
$$+a_{131}\cdot a_{212}\cdot a_{323}-a_{131}\cdot a_{213}\cdot a_{322}-a_{131}\cdot a_{222}\cdot a_{313}+a_{131}\cdot a_{223}\cdot a_{312}-a_{132}\cdot a_{211}\cdot a_{323}$$
$$+a_{132}\cdot a_{213}\cdot a_{321}+a_{132}\cdot a_{221}\cdot a_{313}-a_{132}\cdot a_{223}\cdot a_{311}+a_{133}\cdot a_{211}\cdot a_{322}-a_{133}\cdot a_{212}\cdot a_{321}$$
$$-a_{133}\cdot a_{221}\cdot a_{312}+a_{133}\cdot a_{222}\cdot a_{311}.$$

If we compare results of above equations, we can see that we have the same result in both cases.

5. For plan $k = 2$: Let us add first vertical layer to third vertical layer while multiplying by a scalar α.

$$det(B_{[3\times3\times3]})$$
$$= det\begin{pmatrix} a_{111} & a_{121} & a_{131} & a_{112} & a_{122} & a_{132} & a_{113}+\alpha\cdot a_{111} & a_{123}+\alpha\cdot a_{121} & a_{133}+\alpha\cdot a_{131} \\ a_{211} & a_{221} & a_{231} & a_{212} & a_{222} & a_{232} & a_{213}+\alpha\cdot a_{211} & a_{223}+\alpha\cdot a_{221} & a_{233}+\alpha\cdot a_{231} \\ a_{311} & a_{321} & a_{331} & a_{312} & a_{322} & a_{332} & a_{313}+\alpha\cdot a_{311} & a_{323}+\alpha\cdot a_{321} & a_{333}+\alpha\cdot a_{331} \end{pmatrix}$$
$$= a_{111}\cdot a_{222}\cdot(a_{333}+\alpha\cdot a_{331}) - a_{111}\cdot a_{232}\cdot(a_{323}+\alpha\cdot a_{321}) - a_{111}\cdot(a_{223}+\alpha\cdot a_{221})\cdot a_{332}$$
$$+a_{111}\cdot(a_{233}+\alpha\cdot a_{231})\cdot a_{322} - a_{112}\cdot a_{221}\cdot(a_{333}+\alpha\cdot a_{331}) + a_{112}\cdot(a_{223}+\alpha\cdot a_{221})\cdot a_{331}$$
$$+a_{112}\cdot a_{231}\cdot(a_{323}+\alpha\cdot a_{321}) - a_{112}\cdot(a_{233}+\alpha\cdot a_{231})\cdot a_{321} + (a_{113}+\alpha\cdot a_{111})\cdot a_{221}\cdot a_{332}$$
$$-(a_{113}+\alpha\cdot a_{111})\cdot a_{222}\cdot a_{331} - (a_{113}+\alpha\cdot a_{111})\cdot a_{231}\cdot a_{322} + (a_{113}+\alpha\cdot a_{111})\cdot a_{232}\cdot a_{321}$$
$$-a_{121}\cdot a_{212}\cdot(a_{333}+\alpha\cdot a_{331}) + a_{121}\cdot(a_{213}+\alpha\cdot a_{211})\cdot a_{332} + a_{121}\cdot a_{232}\cdot(a_{313}+\alpha\cdot a_{311})$$
$$-a_{121}\cdot(a_{233}+\alpha\cdot a_{231})\cdot a_{312} + a_{122}\cdot a_{211}\cdot(a_{333}+\alpha\cdot a_{331}) - a_{122}\cdot(a_{213}+\alpha\cdot a_{211})\cdot a_{331}$$
$$-a_{122}\cdot a_{231}\cdot(a_{313}+\alpha\cdot a_{311}) + a_{122}\cdot(a_{233}+\alpha\cdot a_{231})\cdot a_{311} - (a_{123}+\alpha\cdot a_{121})\cdot a_{211}\cdot a_{332}$$
$$+(a_{123}+\alpha\cdot a_{121})\cdot a_{212}\cdot a_{331} + (a_{123}+\alpha\cdot a_{121})\cdot a_{231}\cdot a_{312} - (a_{123}+\alpha\cdot a_{121})\cdot a_{232}\cdot a_{311}$$
$$+a_{131}\cdot a_{212}\cdot(a_{323}+\alpha\cdot a_{321}) - a_{131}\cdot(a_{213}+\alpha\cdot a_{211})\cdot a_{322} - a_{131}\cdot a_{222}\cdot(a_{313}+\alpha\cdot a_{311})$$
$$+a_{131}\cdot(a_{223}+\alpha\cdot a_{221})\cdot a_{312} - a_{132}\cdot a_{211}\cdot(a_{323}+\alpha\cdot a_{321}) + a_{132}\cdot(a_{213}+\alpha\cdot a_{211})\cdot a_{321}$$
$$+a_{132}\cdot a_{221}\cdot(a_{313}+\alpha\cdot a_{311}) - a_{132}\cdot(a_{223}+\alpha\cdot a_{221})\cdot a_{311} + (a_{133}+\alpha\cdot a_{131})\cdot a_{211}\cdot a_{322}$$
$$-(a_{133}+\alpha\cdot a_{131})\cdot a_{212}\cdot a_{321} - (a_{133}+\alpha\cdot a_{131})\cdot a_{221}\cdot a_{312} + (a_{133}+\alpha\cdot a_{131})\cdot a_{222}\cdot a_{311}.$$

After expanding further, we get the following result:

$$det(B_{[3\times3\times3]}) = a_{111}\cdot a_{222}\cdot a_{333} - a_{111}\cdot a_{232}\cdot a_{323} - a_{111}\cdot a_{223}\cdot a_{332} + a_{111}\cdot a_{233}\cdot a_{322}$$
$$-a_{112}\cdot a_{221}\cdot a_{333} + a_{112}\cdot a_{223}\cdot a_{331} + a_{112}\cdot a_{231}\cdot a_{323} - a_{112}\cdot a_{233}\cdot a_{321} + a_{113}\cdot a_{221}\cdot a_{332}$$
$$-a_{113}\cdot a_{222}\cdot a_{331} - a_{113}\cdot a_{231}\cdot a_{322} + a_{113}\cdot a_{232}\cdot a_{321} - a_{121}\cdot a_{212}\cdot a_{333} + a_{121}\cdot a_{213}\cdot a_{332}$$
$$+a_{121}\cdot a_{232}\cdot a_{313} - a_{121}\cdot a_{233}\cdot a_{312} + a_{122}\cdot a_{211}\cdot a_{333} - a_{122}\cdot a_{213}\cdot a_{331} - a_{122}\cdot a_{231}\cdot a_{313}$$
$$+a_{122}\cdot a_{233}\cdot a_{311} - a_{123}\cdot a_{211}\cdot a_{332} + a_{123}\cdot a_{212}\cdot a_{331} + a_{123}\cdot a_{231}\cdot a_{312} - a_{123}\cdot a_{232}\cdot a_{311}$$
$$+a_{131}\cdot a_{212}\cdot a_{323} - a_{131}\cdot a_{213}\cdot a_{322} - a_{131}\cdot a_{222}\cdot a_{313} + a_{131}\cdot a_{223}\cdot a_{312} - a_{132}\cdot a_{211}\cdot a_{323}$$
$$+a_{132}\cdot a_{213}\cdot a_{321} + a_{132}\cdot a_{221}\cdot a_{313} - a_{132}\cdot a_{223}\cdot a_{311} + a_{133}\cdot a_{211}\cdot a_{322} - a_{133}\cdot a_{212}\cdot a_{321}$$
$$-a_{133}\cdot a_{221}\cdot a_{312} + a_{133}\cdot a_{222}\cdot a_{311}.$$

If we compare results of above equations, we can see that we have the same result in both cases.

6. For plan $k = 3$: Let us add first vertical layer to third vertical layer while multiplying by a scalar α.

$$det(B_{[3\times3\times3]})$$
$$= det\begin{pmatrix} a_{111} & a_{121} & a_{131} & a_{112} & a_{122} & a_{132} & a_{113}+\alpha\cdot a_{112} & a_{123}+\alpha\cdot a_{122} & a_{133}+\alpha\cdot a_{132} \\ a_{211} & a_{221} & a_{231} & a_{212} & a_{222} & a_{232} & a_{213}+\alpha\cdot a_{212} & a_{223}+\alpha\cdot a_{222} & a_{233}+\alpha\cdot a_{232} \\ a_{311} & a_{321} & a_{331} & a_{312} & a_{322} & a_{332} & a_{313}+\alpha\cdot a_{312} & a_{323}+\alpha\cdot a_{322} & a_{333}+\alpha\cdot a_{332} \end{pmatrix}$$
$$= a_{111}\cdot a_{222}\cdot(a_{333}+\alpha\cdot a_{332}) - a_{111}\cdot a_{232}\cdot(a_{323}+\alpha\cdot a_{322}) - a_{111}\cdot(a_{223}+\alpha\cdot a_{222})\cdot a_{332}$$
$$+a_{111}\cdot(a_{233}+\alpha\cdot a_{232})\cdot a_{322} - a_{112}\cdot a_{221}\cdot(a_{333}+\alpha\cdot a_{332}) + a_{112}\cdot(a_{223}+\alpha\cdot a_{222})\cdot a_{331}$$
$$+a_{112}\cdot a_{231}\cdot(a_{323}+\alpha\cdot a_{322}) - a_{112}\cdot(a_{233}+\alpha\cdot a_{232})\cdot a_{321} + (a_{113}+\alpha\cdot a_{112})\cdot a_{221}\cdot a_{332}$$
$$-(a_{113}+\alpha\cdot a_{112})\cdot a_{222}\cdot a_{331} - (a_{113}+\alpha\cdot a_{112})\cdot a_{231}\cdot a_{322} + (a_{113}+\alpha\cdot a_{112})\cdot a_{232}\cdot a_{321}$$
$$-a_{121}\cdot a_{212}\cdot(a_{333}+\alpha\cdot a_{332}) + a_{121}\cdot(a_{213}+\alpha\cdot a_{212})\cdot a_{332} + a_{121}\cdot a_{232}\cdot(a_{313}+\alpha\cdot a_{312})$$
$$-a_{121}\cdot(a_{233}+\alpha\cdot a_{232})\cdot a_{312} + a_{122}\cdot a_{211}\cdot(a_{333}+\alpha\cdot a_{332}) - a_{122}\cdot(a_{213}+\alpha\cdot a_{212})\cdot a_{331}$$
$$-a_{122}\cdot a_{231}\cdot(a_{313}+\alpha\cdot a_{312}) + a_{122}\cdot(a_{233}+\alpha\cdot a_{232})\cdot a_{311} - (a_{123}+\alpha\cdot a_{122})\cdot a_{211}\cdot a_{332}$$
$$+(a_{123}+\alpha\cdot a_{122})\cdot a_{212}\cdot a_{331} + (a_{123}+\alpha\cdot a_{122})\cdot a_{231}\cdot a_{312} - (a_{123}+\alpha\cdot a_{122})\cdot a_{232}\cdot a_{311}$$

$$+a_{131}\cdot a_{212}\cdot(a_{323}+\alpha\cdot a_{322})-a_{131}\cdot(a_{213}+\alpha\cdot a_{212})\cdot a_{322}-a_{131}\cdot a_{222}\cdot(a_{313}+\alpha\cdot a_{312})\backslash$$
$$+a_{131}\cdot(a_{223}+\alpha\cdot a_{222})\cdot a_{312}-a_{132}\cdot a_{211}\cdot(a_{323}+\alpha\cdot a_{322})+a_{132}\cdot(a_{213}+\alpha\cdot a_{212})\cdot a_{321}$$
$$+a_{132}\cdot a_{221}\cdot(a_{313}+\alpha\cdot a_{312})-a_{132}\cdot(a_{223}+\alpha\cdot a_{222})\cdot a_{311}+(a_{133}+\alpha\cdot a_{132})\cdot a_{211}\cdot a_{322}$$
$$-(a_{133}+\alpha\cdot a_{132})\cdot a_{212}\cdot a_{321}-(a_{133}+\alpha\cdot a_{132})\cdot a_{221}\cdot a_{312}+(a_{133}+\alpha\cdot a_{132})\cdot a_{222}\cdot a_{311}.$$

After expanding further, we get the following result:

$$det(B_{[3\times3\times3]})=a_{111}\cdot a_{222}\cdot a_{333}-a_{111}\cdot a_{232}\cdot a_{323}-a_{111}\cdot a_{223}\cdot a_{332}+a_{111}\cdot a_{233}\cdot a_{322}$$
$$-a_{112}\cdot a_{221}\cdot a_{333}+a_{112}\cdot a_{223}\cdot a_{331}+a_{112}\cdot a_{231}\cdot a_{323}-a_{112}\cdot a_{233}\cdot a_{321}+a_{113}\cdot a_{221}\cdot a_{332}$$
$$-a_{113}\cdot a_{222}\cdot a_{331}-a_{113}\cdot a_{231}\cdot a_{322}+a_{113}\cdot a_{232}\cdot a_{321}-a_{121}\cdot a_{212}\cdot a_{333}+a_{121}\cdot a_{213}\cdot a_{332}$$
$$+a_{121}\cdot a_{232}\cdot a_{313}-a_{121}\cdot a_{233}\cdot a_{312}+a_{122}\cdot a_{211}\cdot a_{333}-a_{122}\cdot a_{213}\cdot a_{331}-a_{122}\cdot a_{231}\cdot a_{313}$$
$$+a_{122}\cdot a_{233}\cdot a_{311}-a_{123}\cdot a_{211}\cdot a_{332}+a_{123}\cdot a_{212}\cdot a_{331}+a_{123}\cdot a_{231}\cdot a_{312}-a_{123}\cdot a_{232}\cdot a_{311}$$
$$+a_{131}\cdot a_{212}\cdot a_{323}-a_{131}\cdot a_{213}\cdot a_{322}-a_{131}\cdot a_{222}\cdot a_{313}+a_{131}\cdot a_{223}\cdot a_{312}-a_{132}\cdot a_{211}\cdot a_{323}$$
$$+a_{132}\cdot a_{213}\cdot a_{321}+a_{132}\cdot a_{221}\cdot a_{313}-a_{132}\cdot a_{223}\cdot a_{311}+a_{133}\cdot a_{211}\cdot a_{322}-a_{133}\cdot a_{212}\cdot a_{321}$$
$$-a_{133}\cdot a_{221}\cdot a_{312}+a_{133}\cdot a_{222}\cdot a_{311}.$$

If we compare results of above equations, we can see that we have the same result in both cases.

Remark: This theorem does not hold for plan "Horizontal Layers".

Theorem 3:
Suppose that A is 3D Determinant with two identical "Vertical Pages" or two identical "Vertical Layers".

Then $|A|=0$.

Proof:

Case 1: The cubic-matrix A of order 2 with two identical "Vertical Pages" or two identical "Vertical Layers", we will proof the case 1, as following:
1. For two identical "Vertical Pages":

$$det(A_{[2\times2\times2]})=\left(\begin{matrix}a_{111} & a_{111}\\ a_{211} & a_{211}\end{matrix}\middle|\begin{matrix}a_{112} & a_{112}\\ a_{212} & a_{212}\end{matrix}\right)=a_{111}\cdot a_{212}-a_{112}\cdot a_{211}-a_{111}\cdot a_{212}+a_{112}\cdot a_{211}=0.$$

2. For two identical "Vertical Layers":

$$det(A_{[2\times2\times2]})=\left(\begin{matrix}a_{111} & a_{121}\\ a_{211} & a_{221}\end{matrix}\middle|\begin{matrix}a_{111} & a_{121}\\ a_{211} & a_{221}\end{matrix}\right)=a_{111}\cdot a_{221}-a_{111}\cdot a_{221}-a_{121}\cdot a_{211}+a_{121}\cdot a_{211}=0.$$

Case 2: The cubic-matrix A of order 3 with two identical "Vertical Pages" or two identical "Vertical Layers", we will proof the case 1, as following:

1. For two identical "Vertical Pages", first "Vertical Page" identical to second "Vertical Page":

$$det(A_{[3\times3\times3]})=det\left(\begin{matrix}a_{111} & a_{111} & a_{131}\\ a_{211} & a_{211} & a_{231}\\ a_{311} & a_{311} & a_{331}\end{matrix}\middle|\begin{matrix}a_{112} & a_{112} & a_{132}\\ a_{212} & a_{212} & a_{232}\\ a_{312} & a_{312} & a_{332}\end{matrix}\middle|\begin{matrix}a_{113} & a_{113} & a_{133}\\ a_{213} & a_{213} & a_{233}\\ a_{313} & a_{313} & a_{333}\end{matrix}\right)$$
$$=a_{111}\cdot a_{212}\cdot a_{333}-a_{111}\cdot a_{232}\cdot a_{313}-a_{111}\cdot a_{213}\cdot a_{332}+a_{111}\cdot a_{233}\cdot a_{312}-a_{112}\cdot a_{211}\cdot a_{333}$$
$$+a_{112}\cdot a_{213}\cdot a_{331}+a_{112}\cdot a_{231}\cdot a_{313}-a_{112}\cdot a_{233}\cdot a_{311}+a_{113}\cdot a_{211}\cdot a_{332}-a_{113}\cdot a_{212}\cdot a_{331}$$
$$-a_{113}\cdot a_{231}\cdot a_{312}+a_{113}\cdot a_{232}\cdot a_{311}-a_{111}\cdot a_{212}\cdot a_{333}+a_{111}\cdot a_{213}\cdot a_{332}+a_{111}\cdot a_{232}\cdot a_{313}$$
$$-a_{111}\cdot a_{233}\cdot a_{312}+a_{112}\cdot a_{211}\cdot a_{333}-a_{112}\cdot a_{213}\cdot a_{331}-a_{112}\cdot a_{231}\cdot a_{313}+a_{112}\cdot a_{233}\cdot a_{311}$$
$$-a_{113}\cdot a_{211}\cdot a_{332}+a_{113}\cdot a_{212}\cdot a_{331}+a_{113}\cdot a_{231}\cdot a_{312}-a_{113}\cdot a_{232}\cdot a_{311}+a_{131}\cdot a_{212}\cdot a_{313}$$
$$-a_{131}\cdot a_{213}\cdot a_{312}-a_{131}\cdot a_{212}\cdot a_{313}+a_{131}\cdot a_{213}\cdot a_{312}-a_{132}\cdot a_{211}\cdot a_{313}+a_{132}\cdot a_{213}\cdot a_{311}$$

$$+a_{132} \cdot a_{211} \cdot a_{313} - a_{132} \cdot a_{213} \cdot a_{311} + a_{133} \cdot a_{211} \cdot a_{312} - a_{133} \cdot a_{212} \cdot a_{311} - a_{133} \cdot a_{211} \cdot a_{312}$$
$$+a_{133} \cdot a_{212} \cdot a_{311} = 0.$$

2. For two identical "Vertical Pages", first "Vertical Page" identical to third "Vertical Page":

$$det\left(A_{[3\times3\times3]}\right) = det\left(\begin{array}{ccc|ccc|ccc} a_{111} & a_{121} & a_{111} & a_{112} & a_{122} & a_{112} & a_{113} & a_{123} & a_{113} \\ a_{211} & a_{221} & a_{211} & a_{212} & a_{222} & a_{212} & a_{213} & a_{223} & a_{213} \\ a_{311} & a_{321} & a_{311} & a_{312} & a_{322} & a_{312} & a_{313} & a_{323} & a_{313} \end{array}\right)$$
$$= a_{111} \cdot a_{222} \cdot a_{313} - a_{111} \cdot a_{212} \cdot a_{323} - a_{111} \cdot a_{223} \cdot a_{312} + a_{111} \cdot a_{213} \cdot a_{322} - a_{112} \cdot a_{221} \cdot a_{313}$$
$$+a_{112} \cdot a_{223} \cdot a_{311} + a_{112} \cdot a_{211} \cdot a_{323} - a_{112} \cdot a_{213} \cdot a_{321} + a_{113} \cdot a_{221} \cdot a_{312} - a_{113} \cdot a_{222} \cdot a_{311}$$
$$-a_{113} \cdot a_{231} \cdot a_{322} + a_{113} \cdot a_{212} \cdot a_{321} - a_{121} \cdot a_{212} \cdot a_{313} + a_{121} \cdot a_{213} \cdot a_{312} + a_{121} \cdot a_{212} \cdot a_{313}$$
$$-a_{121} \cdot a_{213} \cdot a_{312} + a_{122} \cdot a_{211} \cdot a_{313} - a_{122} \cdot a_{213} \cdot a_{311} - a_{122} \cdot a_{211} \cdot a_{313} + a_{122} \cdot a_{213} \cdot a_{311}$$
$$-a_{123} \cdot a_{211} \cdot a_{312} + a_{123} \cdot a_{212} \cdot a_{311} + a_{123} \cdot a_{211} \cdot a_{312} - a_{123} \cdot a_{212} \cdot a_{311} + a_{111} \cdot a_{212} \cdot a_{323}$$
$$-a_{111} \cdot a_{213} \cdot a_{322} - a_{111} \cdot a_{222} \cdot a_{313} + a_{111} \cdot a_{223} \cdot a_{312} - a_{112} \cdot a_{211} \cdot a_{323} + a_{112} \cdot a_{213} \cdot a_{321}$$
$$+a_{112} \cdot a_{221} \cdot a_{313} - a_{112} \cdot a_{223} \cdot a_{311} + a_{113} \cdot a_{211} \cdot a_{322} - a_{113} \cdot a_{212} \cdot a_{321} - a_{113} \cdot a_{221} \cdot a_{312}$$
$$+a_{113} \cdot a_{222} \cdot a_{311} = 0.$$

3. For two identical "Vertical Pages", second "Vertical Page" identical to third "Vertical Page":

$$det\left(A_{[3\times3\times3]}\right) = det\left(\begin{array}{ccc|ccc|ccc} a_{111} & a_{121} & a_{121} & a_{112} & a_{122} & a_{122} & a_{113} & a_{123} & a_{123} \\ a_{211} & a_{221} & a_{221} & a_{212} & a_{222} & a_{222} & a_{213} & a_{223} & a_{223} \\ a_{311} & a_{321} & a_{321} & a_{312} & a_{322} & a_{322} & a_{313} & a_{323} & a_{323} \end{array}\right)$$
$$= a_{111} \cdot a_{222} \cdot a_{323} - a_{111} \cdot a_{222} \cdot a_{323} - a_{111} \cdot a_{223} \cdot a_{322} + a_{111} \cdot a_{223} \cdot a_{322} - a_{112} \cdot a_{221} \cdot a_{323}$$
$$+a_{112} \cdot a_{223} \cdot a_{321} + a_{112} \cdot a_{221} \cdot a_{323} - a_{112} \cdot a_{223} \cdot a_{321} + a_{113} \cdot a_{221} \cdot a_{322} - a_{113} \cdot a_{222} \cdot a_{321}$$
$$-a_{113} \cdot a_{221} \cdot a_{322} + a_{113} \cdot a_{222} \cdot a_{321} - a_{121} \cdot a_{212} \cdot a_{323} + a_{121} \cdot a_{213} \cdot a_{322} + a_{121} \cdot a_{222} \cdot a_{313}$$
$$-a_{121} \cdot a_{223} \cdot a_{312} + a_{122} \cdot a_{211} \cdot a_{323} - a_{122} \cdot a_{213} \cdot a_{321} - a_{122} \cdot a_{221} \cdot a_{313} + a_{122} \cdot a_{223} \cdot a_{311}$$
$$-a_{123} \cdot a_{211} \cdot a_{322} + a_{123} \cdot a_{212} \cdot a_{321} + a_{123} \cdot a_{221} \cdot a_{312} - a_{123} \cdot a_{222} \cdot a_{311} + a_{121} \cdot a_{212} \cdot a_{323}$$
$$-a_{121} \cdot a_{213} \cdot a_{322} - a_{121} \cdot a_{222} \cdot a_{313} + a_{121} \cdot a_{223} \cdot a_{312} - a_{122} \cdot a_{211} \cdot a_{323} + a_{122} \cdot a_{213} \cdot a_{321}$$
$$+a_{122} \cdot a_{221} \cdot a_{313} - a_{122} \cdot a_{223} \cdot a_{311} + a_{123} \cdot a_{211} \cdot a_{322} - a_{123} \cdot a_{212} \cdot a_{321} - a_{123} \cdot a_{221} \cdot a_{312}$$
$$+a_{123} \cdot a_{222} \cdot a_{311} = 0.$$

4. For two identical "Vertical Layers", first "Vertical Layer" identical to second "Vertical Layer":

$$det\left(A_{[3\times3\times3]}\right) = det\left(\begin{array}{ccc|ccc|ccc} a_{111} & a_{121} & a_{131} & a_{111} & a_{121} & a_{131} & a_{113} & a_{123} & a_{133} \\ a_{211} & a_{221} & a_{231} & a_{211} & a_{221} & a_{231} & a_{213} & a_{223} & a_{233} \\ a_{311} & a_{321} & a_{331} & a_{311} & a_{321} & a_{331} & a_{313} & a_{323} & a_{333} \end{array}\right)$$
$$= a_{111} \cdot a_{221} \cdot a_{333} - a_{111} \cdot a_{231} \cdot a_{323} - a_{111} \cdot a_{223} \cdot a_{331} + a_{111} \cdot a_{233} \cdot a_{321} - a_{111} \cdot a_{221} \cdot a_{333}$$
$$+a_{111} \cdot a_{223} \cdot a_{331} + a_{111} \cdot a_{231} \cdot a_{323} - a_{111} \cdot a_{233} \cdot a_{321} + a_{113} \cdot a_{221} \cdot a_{331} - a_{113} \cdot a_{221} \cdot a_{331}$$
$$-a_{113} \cdot a_{231} \cdot a_{321} + a_{113} \cdot a_{231} \cdot a_{321} - a_{121} \cdot a_{211} \cdot a_{333} + a_{121} \cdot a_{213} \cdot a_{331} + a_{121} \cdot a_{231} \cdot a_{313}$$
$$-a_{121} \cdot a_{233} \cdot a_{311} + a_{121} \cdot a_{211} \cdot a_{333} - a_{121} \cdot a_{213} \cdot a_{331} - a_{121} \cdot a_{231} \cdot a_{313} + a_{121} \cdot a_{233} \cdot a_{311}$$
$$-a_{123} \cdot a_{211} \cdot a_{331} + a_{123} \cdot a_{211} \cdot a_{331} + a_{123} \cdot a_{231} \cdot a_{311} - a_{123} \cdot a_{231} \cdot a_{311} + a_{131} \cdot a_{211} \cdot a_{323}$$
$$-a_{131} \cdot a_{213} \cdot a_{321} - a_{131} \cdot a_{221} \cdot a_{313} + a_{131} \cdot a_{223} \cdot a_{311} - a_{131} \cdot a_{211} \cdot a_{323} + a_{131} \cdot a_{213} \cdot a_{321}$$
$$+a_{131} \cdot a_{221} \cdot a_{313} - a_{131} \cdot a_{223} \cdot a_{311} + a_{133} \cdot a_{211} \cdot a_{321} - a_{133} \cdot a_{211} \cdot a_{321} - a_{133} \cdot a_{221} \cdot a_{311}$$
$$+a_{133} \cdot a_{221} \cdot a_{311} = 0.$$

5. For two identical "Vertical Layers", first "Vertical Layer" identical to third "Vertical Layer":

$$det\left(A_{[3\times3\times3]}\right) = det\left(\begin{array}{ccc|ccc|ccc} a_{111} & a_{121} & a_{131} & a_{112} & a_{122} & a_{132} & a_{111} & a_{121} & a_{131} \\ a_{211} & a_{221} & a_{231} & a_{212} & a_{222} & a_{232} & a_{211} & a_{221} & a_{231} \\ a_{311} & a_{321} & a_{331} & a_{312} & a_{322} & a_{332} & a_{311} & a_{321} & a_{331} \end{array}\right)$$
$$= a_{111} \cdot a_{222} \cdot a_{331} - a_{111} \cdot a_{232} \cdot a_{321} - a_{111} \cdot a_{221} \cdot a_{332} + a_{111} \cdot a_{231} \cdot a_{322} - a_{112} \cdot a_{221} \cdot a_{331}$$

$$+a_{112}\cdot a_{221}\cdot a_{331}+a_{112}\cdot a_{231}\cdot a_{321}-a_{112}\cdot a_{231}\cdot a_{321}+a_{111}\cdot a_{221}\cdot a_{332}-a_{111}\cdot a_{222}\cdot a_{331}$$
$$-a_{111}\cdot a_{231}\cdot a_{322}+a_{111}\cdot a_{232}\cdot a_{321}-a_{121}\cdot a_{212}\cdot a_{331}+a_{121}\cdot a_{211}\cdot a_{332}+a_{121}\cdot a_{232}\cdot a_{311}$$
$$-a_{121}\cdot a_{231}\cdot a_{312}+a_{122}\cdot a_{211}\cdot a_{331}-a_{122}\cdot a_{211}\cdot a_{331}-a_{122}\cdot a_{231}\cdot a_{311}+a_{122}\cdot a_{231}\cdot a_{311}$$
$$-a_{121}\cdot a_{211}\cdot a_{332}+a_{121}\cdot a_{212}\cdot a_{331}+a_{121}\cdot a_{231}\cdot a_{312}-a_{121}\cdot a_{232}\cdot a_{311}+a_{131}\cdot a_{212}\cdot a_{321}$$
$$-a_{131}\cdot a_{211}\cdot a_{322}-a_{131}\cdot a_{222}\cdot a_{311}+a_{131}\cdot a_{221}\cdot a_{312}-a_{132}\cdot a_{211}\cdot a_{321}+a_{132}\cdot a_{211}\cdot a_{321}$$
$$+a_{132}\cdot a_{221}\cdot a_{311}-a_{132}\cdot a_{221}\cdot a_{311}+a_{131}\cdot a_{211}\cdot a_{322}-a_{131}\cdot a_{212}\cdot a_{321}-a_{131}\cdot a_{221}\cdot a_{312}$$
$$+a_{131}\cdot a_{222}\cdot a_{311}=0.$$

6. For two identical "Vertical Layers", second "Vertical Layer" identical to third "Vertical Layer":

$$det\left(A_{[3\times3\times3]}\right)=det\left(\begin{array}{ccc|ccc|ccc} a_{111} & a_{121} & a_{131} & a_{112} & a_{122} & a_{132} & a_{112} & a_{122} & a_{132} \\ a_{211} & a_{221} & a_{231} & a_{212} & a_{222} & a_{232} & a_{212} & a_{222} & a_{232} \\ a_{311} & a_{321} & a_{331} & a_{312} & a_{322} & a_{332} & a_{312} & a_{322} & a_{332} \end{array}\right)$$
$$=a_{111}\cdot a_{222}\cdot a_{332}-a_{111}\cdot a_{232}\cdot a_{322}-a_{111}\cdot a_{222}\cdot a_{332}+a_{111}\cdot a_{232}\cdot a_{322}-a_{112}\cdot a_{221}\cdot a_{332}$$
$$+a_{112}\cdot a_{222}\cdot a_{331}+a_{112}\cdot a_{231}\cdot a_{322}-a_{112}\cdot a_{232}\cdot a_{321}+a_{112}\cdot a_{221}\cdot a_{332}-a_{112}\cdot a_{222}\cdot a_{331}$$
$$-a_{112}\cdot a_{231}\cdot a_{322}+a_{112}\cdot a_{232}\cdot a_{321}-a_{121}\cdot a_{212}\cdot a_{332}+a_{121}\cdot a_{212}\cdot a_{332}+a_{121}\cdot a_{232}\cdot a_{312}$$
$$-a_{121}\cdot a_{232}\cdot a_{312}+a_{122}\cdot a_{211}\cdot a_{332}-a_{122}\cdot a_{212}\cdot a_{331}-a_{122}\cdot a_{231}\cdot a_{312}+a_{122}\cdot a_{232}\cdot a_{311}$$
$$-a_{122}\cdot a_{211}\cdot a_{332}+a_{122}\cdot a_{212}\cdot a_{331}+a_{122}\cdot a_{231}\cdot a_{312}-a_{122}\cdot a_{232}\cdot a_{311}+a_{131}\cdot a_{212}\cdot a_{322}$$
$$-a_{131}\cdot a_{212}\cdot a_{322}-a_{131}\cdot a_{222}\cdot a_{312}+a_{131}\cdot a_{222}\cdot a_{312}-a_{132}\cdot a_{211}\cdot a_{322}+a_{132}\cdot a_{212}\cdot a_{321}$$
$$+a_{132}\cdot a_{221}\cdot a_{312}-a_{132}\cdot a_{222}\cdot a_{311}+a_{132}\cdot a_{211}\cdot a_{322}-a_{132}\cdot a_{212}\cdot a_{321}-a_{132}\cdot a_{221}\cdot a_{312}$$
$$+a_{132}\cdot a_{222}\cdot a_{311}=0.$$

Scientific Ethics Declaration

The authors declare that the scientific ethical and legal responsibility of this article published in EPSTEM journal belongs to the authors.

Acknowledgements or Notes

* This article was presented as an oral presentation at the International Conference on Technology, Engineering and Science (www.icontes.net) held in Antalya/Turkey on November 16-19, 2023.

References

Amiri, A., Fathy, M., & Bayat, M. (2010). Generalization of some determinantal identities for non-square matrices based on Radic's definition. *TWMS J. Pure Appl. Math, 1*(2), 163-175.

Artin, M. (1991). *Algebra*. Upper Saddle River: Prentice Hall

Bretscher, O. (2005) *Linear algebra with applications* (3rd ed.). Upper Saddle River: Prentice Hall,

Filipi, K., Zaka, O., & Jusufi, A. (2019). The construction of a corp in the set of points in a line of desargues affine plane. *Matematicki Bilten, 43*(1), 27-46.

Lang, S. (1987) *Linear algebra.* Springer.

Makarewicz, A., Pikuta, P., & Szałkowski, D. (2014). Properties of the determinant of a rectangular matrix. Annales Universitatis *Mariae Curie-Skłodowska, sectio A–Mathematica, 68(*1), 31-41.

Milne-Thomson, L. M. (1941). Determinant expansions. *The Mathematical Gazette, 25*(265), 130-135.

Peters, J. F., & Zaka, O. (2023). Dyck fundamental group on Arcwise-connected polygon cycles. *Afrika Matematika, 34*(2), 31.

Poole, D. (2005) *Linear algebra. A modern introduction* (4th ed. ,pp. 265–267). Cengage Learning.

Radic, M. (1966). A definition of the determinant of a rectangular matrix. *Glas. Mat, 1*(21), 17-22.

Radić, M. (2005). About a determinant of rectangular matrix and its geometric interpretation. *Beiträge zur Algebra und Geometrie, 46*(2), 321-349.

Rose, E. H. (2002) *Linear algebra. A pure mathematical approach* (pp. 57-60). Springer

Salihu, A. (2018). A modern modification of Gjonbalaj-Salihu cornice determinant, transformation to semi-diagonal determinant. *International Journal of Mathematics and Computer Science, 13,* 133-138.
Salihu, A., & Marevci, F. (2019a). Determinants order decrease/increase for k orders, interpretation with computer algorithms and comparison. *Computer Science, 9*(2), 501-518.
Salihu, A., & Marevci, F. (2021). Chio's-like method for calculating the rectangular (non-square) determinants: Computer algorithm interpretation and comparison. *European Journal of Pure and Applied Mathematics, 14*(2), 431-450.
Salihu, A., & Zaka, O. (2023a). Some 3D-determinant properties for calculating of cubic-matrix of order 2 and order 3. *Preprints*
Salihu, A., Jusufi, A., & Salihu, F. (2019b). Comparison of computer execution time of cornice determinant calculation. *International Journal of Mathematics and Computer Science, 14,* 9-16.
Salihu, A., Snopçe, H., Ajdari, J., & Luma, A. (2022b). Generalization of Dodgson's condensation method for calculating determinant of rectangular matrices. In *2022 International Conference on Electrical, Computer and Energy Technologies (ICECET)* (pp. 1-6). IEEE.
Salihu, A., Snopce, H., Luma, A., & Ajdari, J. (2022a). Optimization of Dodgson's condensation method for rectangular determinant calculations. *Advanced Mathematical Models & Applications, 7*(3), 264-274.
Salihu, A., Snopce, H., Luma, A., & Ajdari, J. (2022c). Time complexity analysis for Cullis. Radic and Dodgson's generalized/modified method for rectangular determinants calculations. *International Jwenal of Computers and Their Applications, 29*(4), 245-254.
Salihu, A., Snopce, H., Luma, A., & Ajdari, J. (2023b). Comparison of time complexity growth for different methods/algorithms for rectangular determinant calculations. In *2023 International Conference on Recent Trends in Electronics and Communication (ICRTEC)* (pp. 1-6). IEEE.
Schneide, H., & Barker, G.P. (1973) *Matrices and linear algebra* (2nd.ed.). Dover Books on Mathematics
Zaka, O. (2016b). *Contribution to reports of some algebraic structures with affine plane geometry and applications.* Polytechnic University of Tirana, Tirana, Albania.
Zaka, O. (2017). 3D matrix ring with a "common" multiplication. *Open Access Library Journal, 4(*5), 1-11.
Zaka, O. (2018a). A description of collineations-groups of an affine plane. *Libertas Mathematica (new series), 37*(2), 81-96.
Zaka, O. (2018b). Three vertex and parallelograms in the affine plane: Similarity and addition abelian groups of similarly n-vertexes in the Desargues affine plane. *Mathematical Modelling and Applications, 3(*1), 9-15.
Zaka, O. (2019a). Dilations of line in itself as the automorphism of the skew-field constructed over in the same line in Desargues affine plane. *Applied Mathematical Sciences, 13*(5), 231-237.
Zaka, O. (2019d). The general linear group of degree n for 3D matrices GL(n; n; p; F). *Libertas Mathematica, New Series. 39*(1), 13–30.
Zaka, O., & Filipi, K. (2016a). The transform of a line of Desargues affine plane in an additive group of its points. *Journal of Current Research, 8*(7), 34983-34990.
Zaka, O., & Mohammed, M. A. (2020a). Skew-field of trace-preserving endomorphisms, of translation group in affine plane. *Proyecciones (Antofagasta), 39*(4), 835-850.
Zaka, O., & Mohammed, M. A. (2020b). The endomorphisms algebra of translations group and associative unitary ring of trace-preserving endomorphisms in affine plane. *Proyecciones (Antofagasta), 39*(4), 821-834.
Zaka, O., & Peters, J. F. (2019b). Isomorphic-dilations of the skew-fields constructed over parallel lines in the desargues affine plane. *Balkan Journal of Geom. Applications, 25,* 141-157.
Zaka, O., & Peters, J. F. (2019c). Ordered line and skew-fields in the Desargues affine plane. *Balkan Journal of Geom. Appl.. 26,* 141-156.
Zaka, O., & Salihu, A. (2023). Laplace method for calculate the determinant of cubic-matrix of order 2 and order 3. *Preprints*

Author Information

Armend Salihu
Universum International College
Prishtine, Kosovo
Contact e-mail: *ar.salihu@gmail.com*

Orgest Zaka
Agricultural University of Tirana
Tirana, Albania

To cite this article:

Salihu, A. & Zaka, O. (2023). Some 3D-determinant properties for calculating of cubic-matrix of order 2 and order 3. *The Eurasia Proceedings of Science, Technology, Engineering & Mathematics (EPSTEM), 26,* 554-577.