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Flexural Torsional Buckling of a Beam from the Dee Bridge of 1846 - Analytical and Finite Element Analyses

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Abstract: The Industrial Revolution in Victorian England was a period marked by rapid development of the machine industry, accompanied by new technologies in metallurgy and a need for transport between economic areas that was fast, cheap and convenient. This is a period when the railway transport arose and in a short time acquired an image familiar to us to this day. Passage of train sets across rivers and hard-to-reach areas requires transport facilities. In this period, an extremely large number of bridges and aqueducts were built, which have different constructive solutions, different technologies are applied and they are made of different materials, depending on the preference of their creator. This paper examines the collapse of one of the beams of the Dee Bridge on 24 May 1847. The analysis carried out shows the irrationality of the cross-sectional dimensions and types of girder supports witch generate large compressive forces in the beam, leading to flexural torsional buckling.

Keywords: Beam, Off-center supported beam, Flexural torsional buckling, Critical force, Finite element method

Introduction

The Industrial Revolution in Victorian England created the conditions for rapid development of railway transport and its accompanying facilities. At this stage in the development of engineering sciences, there is still a lack of knowledge regarding structures, materials, dynamic effects (Walley, 2020). An extraordinary number of bridges and aqueducts was built during this period (Petrov, 2020; Sutherland, 2009; Bill, 2014). Often used design solutions are innovative and without the necessary calculations. They have been verify, with experimental tests that often did not account for all the factors that could compromise the structures. Minor accidents in the structural elements were commonplace at the time. Investigations into the causes of destruction are a leading factor in the accumulation of experience by their creators. In some cases, however, knowledge has not yet reached that level to provide a logical and indisputable explanation of events. Some accidents continue to arouse interest and specialists seek an explication through new knowledge and technology in structural studies. One such modern computer analysis in Martin and MacLeod (1995) of wind loading on the Tay Rail Bridge explains the reasons for its collapse in 1879. In Martin and MacLeod (2004), the suggestion that the collapse of the same bridge resulted from material fatigue failure caused by dynamic load it was rejected. The paper shows that the evidence to support this is weak.

In the 1840s and 1850s, cast iron replaced stone, brick and wood as a building material. It is cheaper than wrought iron, which is the other alternative to the known old materials, and makes it possible to bridge relatively large openings – up to about 20 meters at the time. Detailed technical information and illustrations, of modern materials, presented against the background of one of the great catastrophes of the mid-19th century - the fall of the Dee Bridge in May 1847 was been made in Lewis (2007). We read about the accumulated knowledge about the materials, the mechanical response of the beams, experimental studies of the dynamic properties of the materials in Walley (2020) and Mylius (2005).

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The cast iron and wrought iron facilities preserved to this day are reconstructed after the assessment of their dynamic load and familiarization with the construction materials from which they are made (Pękalski & Rabiega, 2011; Parke et al., 2005; Taylor et al., 2009).

Statement of the Problem Under Consideration

In November 1846, the Dee Bridge was open on the London-Holyhead (Wales) line, just outside the city of Chester (Petroski, 1994; Clark, 2009; Thorpe, 2015). The project's chief engineer, Robert Stephenson, chose a cast iron girder design. One of the girders is damage and the bridge is close until a new one is cast. After its reopening, about six months later, on 24 May 1847, a local train was crossing the final span when one of the girders failed suddenly, sending most of the train crashing into the river below. Five lives were lost. The accident cause a national furore, and Stephenson came close to being accuse of manslaughter for the design. An unprecedented investigation began, accompanied by a lengthy process of gathering eyewitness accounts and additional "expert" testimony from military and civilian engineers.

The Dee Bridge consists of three 98-foot spans set between masonry piers. Each section is made up of four cast iron beams (two for each line in both directions) and each is made by bolting together three shorter lengths of iron I-sections. Each beam is reinforce with wrought iron trusses (Figure 1). The rails was been laid on wooden sleepers supported by oak beams resting on the lower flanges of the beams (Figure 2). The bridge is located in a horizontal curve with a radius of 7000ft = 2133.60m (Figure 3).

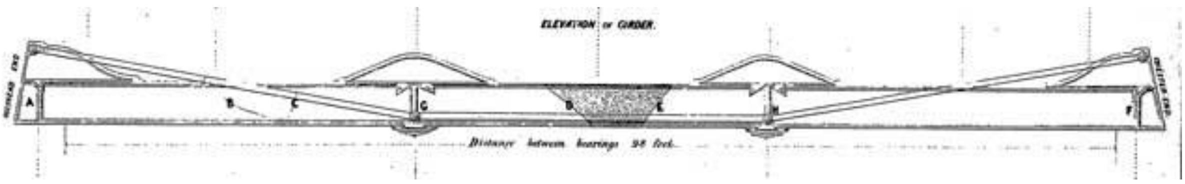

Figure 1. Elevation of one of the girders over the river dee (Taylor, 2013)

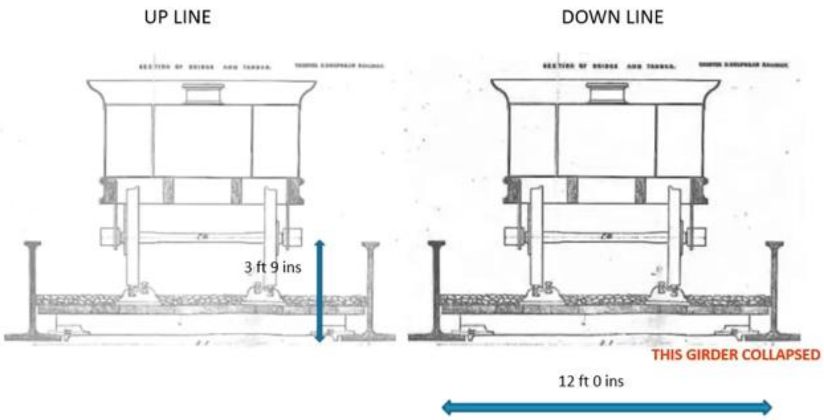

Figure 2. Section of failed bridge (Hayward, 2022)

The River Dee Bridge was first open to local freight traffic on 4 November 1846, but a small fracture was discover near the joint between two girders. Stevenson concluded that this was the result of a casting defect. Next comes strengthening of the bridge, casting and replacement of the damaged part. Painters working on the structure at the time noticed large deformations of several to two inches (5 cm) on both tracks. On faster moving, the deflection on the train is 3 1/2 (9cm) to 4 inches (10cm). However, neither Stevenson nor his team is been informed for this discovery (Lewis & Gagg, 2004; Taylor, 2013). At the time of innovative ideas, contracts could be based on wrong assumptions and prove difficult to enforce. Once a project has gained momentum, those working on it may fear that any attempt to draw attention to risks or defects will be perceive as disloyal (Hawker, 2007).

On the morning of the disaster, six trains passed the bridge without incident, and Stevenson inspected the facility that day (Taylor, 2013). The danger of sparks and cinders from passing trains falling on the oak sleepers and setting them on fire is real. Stevenson subsequently ordered five inches (12.5cm) of ash ballast to be place over the timber. The ash promises fire protection, but the material adds to each of the three spans an additional static (or "dead") load estimated at eighteen tons. The additional dead load of one beam would be about nine tons (Lewis & Gagg, 2004). The ballast was been laid just before the ill-fated train left the Chester station at 6:15 p.m. The train weigh was about sixty tons, and was traveling at about thirty miles an hour (30 mph = 48,28 km/h) toward the bridge (Lewis & Gagg, 2004). The train left Chester station 10 minutes late (Hayward, 2022).

Figure 3. The old bridge is on the left. On the right is the newly built bridge (Hayward, 2022)

Eyewitnesses described a crash for two or three seconds before the four carriages tumbled down into the river "in a row". Others speak of a tremendous crash and a large piece of the beam falling from the middle support. Still others claim that when the train reached the furthest section, they saw an open crack in the middle of the beam. “It opened from the bottom of the beam". The machinist testified that as the 60-ton locomotive crossed the third span of the bridge, part of the structure, began to collapse and he felt the bridge give way beneath him.

Captain Simmons from the Royal Engineers and James Walker are investigating the incident. They check the accuracy of Stevenson's original design and calculations. They perform tests on the surviving beams and measure their deformation under static and dynamic loads. They use a 48-ton locomotive who is moving along the beams at about 20 miles per hour (20 mph = 32,19 km/h). The train according to Lewis and Gagg, 2004 gives a distributed load of twenty-four tons on each beam. According to Hayward, 2014; Hayward, 2011, initially the load on the bridges, at that time, was taken as the average weight per unit length, ignoring the axle concentration and using large safety factors to compensate for the lack of any dynamic tolerance. Here, however, Stevenson has bet on a strangely low safety factor - 1,5. Simmons measured the sag of the joists, and it was 2.36 inches (6cm) at their center. The upper flange moved inward about half an inch (1.27cm), the lower flange moved outward due to the asymmetric load on the beam (Figure 4) (Taylor, 2013). Also from Lewis & Gagg, 2004 we learn that in their joint paper Walker and Simmons report that wrought iron chain gives little or no reinforcement to cast iron beams because the ends of the chain is connected to the cast iron itself (Figure 1). The beam was test in the foundry for deformation under a static load of twenty-five tons and two and a half inches (6,35cm) of sag was measure. However, the foundry test does not replicate the working conditions of the beam because the load is not on the flange. Walker and Simmons examined both types of ironwork, and reported that the cast iron was sound and contained no voids or holes (especially in the fracture surfaces). Likewise, wrought iron was of good quality. However, they believe that the vibrations caused by the trains can weaken the materials.

At that time, it was already clear that dynamic impact affects wear ability, but knowledge was still lacking. Until the end of the nineteenth century, there is no way to study this problem, and this gave reason to explain incomprehensible processes at the time with the unstudied dynamic influence (Walley, 2020). According to Stevenson's theory, the dynamic interaction of materials and machinery played a role in the collapse. He negates a manufacturing defect in a cast iron beam or an inherent brittleness of the metal. Simmons and Walker describe a "general" condition, where the breaking weight of any cast iron beam may be reduce by repetitive vibration, thus opening up a realm of possibilities.

The term "breaking weight" comes from the research of Eaton Hodgkinson, a British engineer. In 1830, the Manchester Literary and Philosophical Society published his Optimized Sections, which greatly influenced engineering practice in the 19th century. Hodgkinson derived an empirical formula for the maximum concentrated force F that a simple beam of span L, sectional height d, and bottom girdle area “A” can support, namely:

$$F = \frac{26Ad}{L} \tag{1}$$

For our beam, this means force: $F = 60,65\,t$

Results and Discussion - Modern Analysis

The beam is $L = 98\,ft = 29,87m$ long and $h = 45in = 114,3cm$ high.

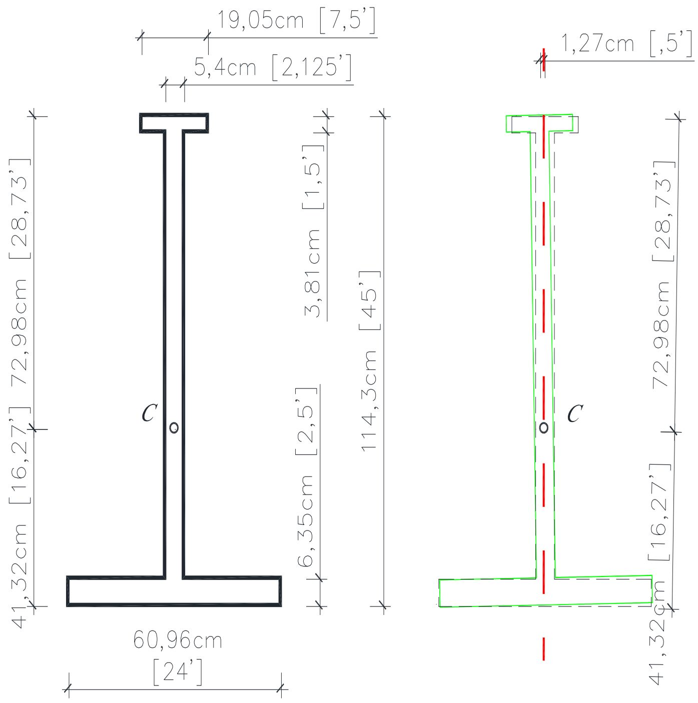

Figure 4. Cross section of the beam (Clark, 2009)

The reasons for the failure of the beam are three main underestimated factors:

1. Off-center supports and the larger normal stresses.
2. The inertial force arising when the train moves in a curve.
3. Flexural torsional buckling.

Off-Center Supports

The beam is support along its bottom edge, not at the center of gravity of the end sections. Such off-center support leads to a particularly stressed and deformed state of the beam (Doicheva & Mladenov, 2009; Doicheva, 2011). The center-supported beam, loaded with vertical loads, only will not generate axial internal force, while off-center supported beam, such are available (Figure 5). Often they are large, even in some cases exceeding the vertical loads that caused them (Mladenov et al., 2012; Mladenov & Doicheva, 2011). Depending on the position of the supports along the height of the end section, the axial forces are either tensile or compressive. Supports located along the bottom edge of the beam lead to the occurrence of compressive horizontal support reactions, and accordingly, compressive axial forces, who can endanger the stability of the beam.

The example below will show how the internal forces vary depending on the different locations of the support in a beam. The load is taken as a distributed load q=30kN/m'. The geometric characteristics of cross section of the beam, were determined with AutoCAD 2022 and they are: area of the cross section $A = 1021,77cm^2$ and moments of inertia $I_y = 1603643,62cm^4$; $I_z = 123434,25cm^4$. Calculations were perform in the MATLAB R2017b environment, with a finite element (FEM) program, for off-center supports, reported in Doicheva, 2022.

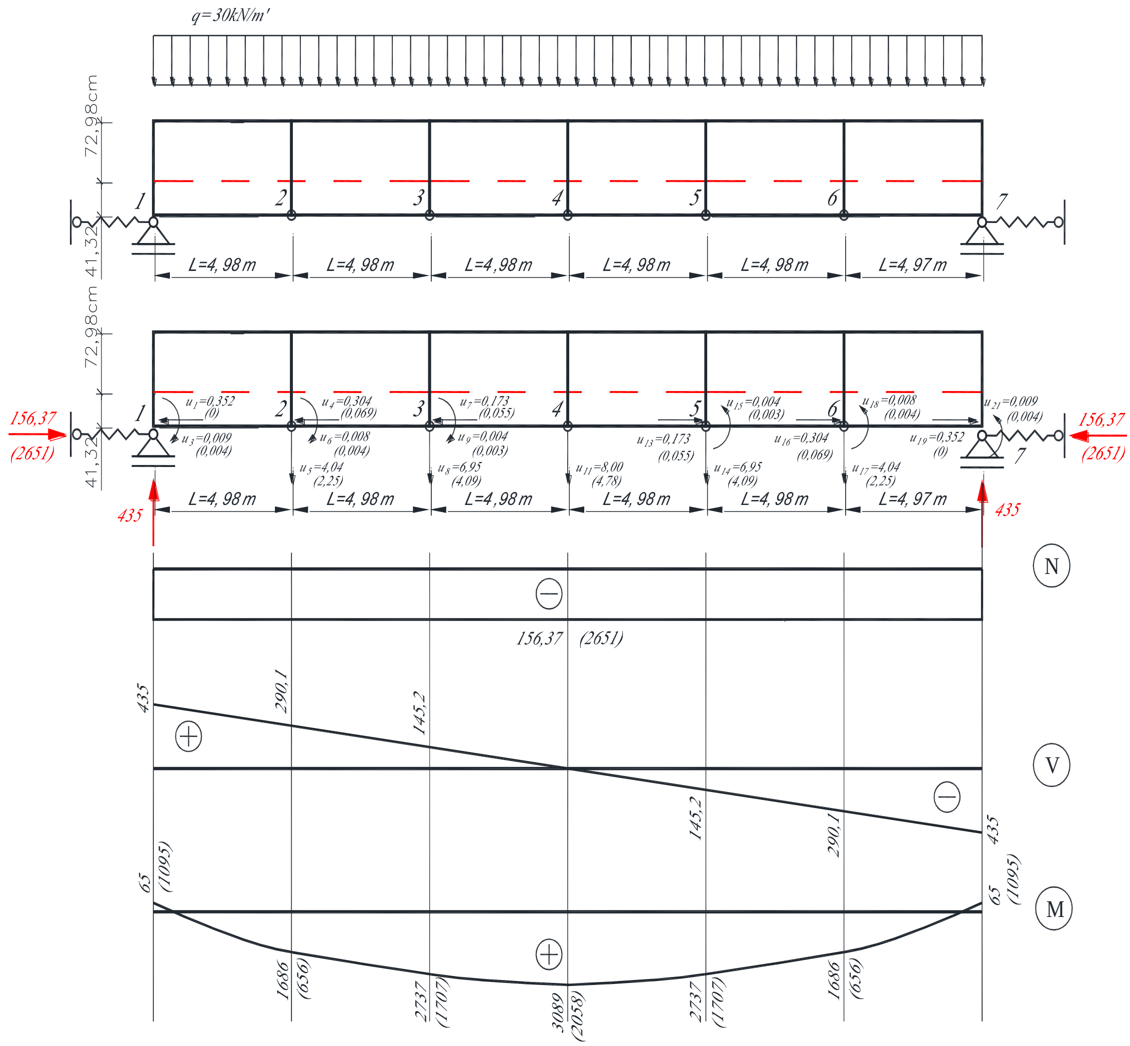

Figure 5. Off-center supported beam

Limiting the movement of the bottom support, i.e. increasing the stiffness of a linear spring, leads to an increase in the horizontal support reaction in it and an increase in the axial force. The values in parentheses in Figure 5 show the internal forces at fixed supports. How will it affect the tense state, the controversial reinforcing chain – Figure 6. The following two things may be noted.

The Reduced Magnitudes of the Vertical Displacements

The reduced magnitudes of the vertical displacements, for the node in the middle u=8cm, and when applying the chain u=5.39cm, which is about 33% less, but the reduced displacements come at the expense of greatly increased axial forces.

The Growth of Axial Forces

They will grow even faster, because the bending of the beam leads to tension in the lower threads of the beam and pressure in the upper ones (Figure 7). The points along the bottom edge of the beam where the chain is attached move down faster than the attachment points along the upper edge of the beam. Therefore, the chain will stretch and put additional load on the upper strands in a direction coinciding with the direction of pressure. This will accelerate the deformations and displacements of the chain attachment nodes at points A and B, and the axial compressive forces will increase. Also if in the case without chain Nmax= 156.37kN, then when applying the chain Nmax= 1942.9kN, which is 12.5 times greater compressive axial force. The correction in the normal stresses is:

$$\sigma_x = \frac{N}{A} = \frac{1942,9}{1021,77} = 1,90 kN/cm^2 \text{ (compression)} \qquad (2)$$

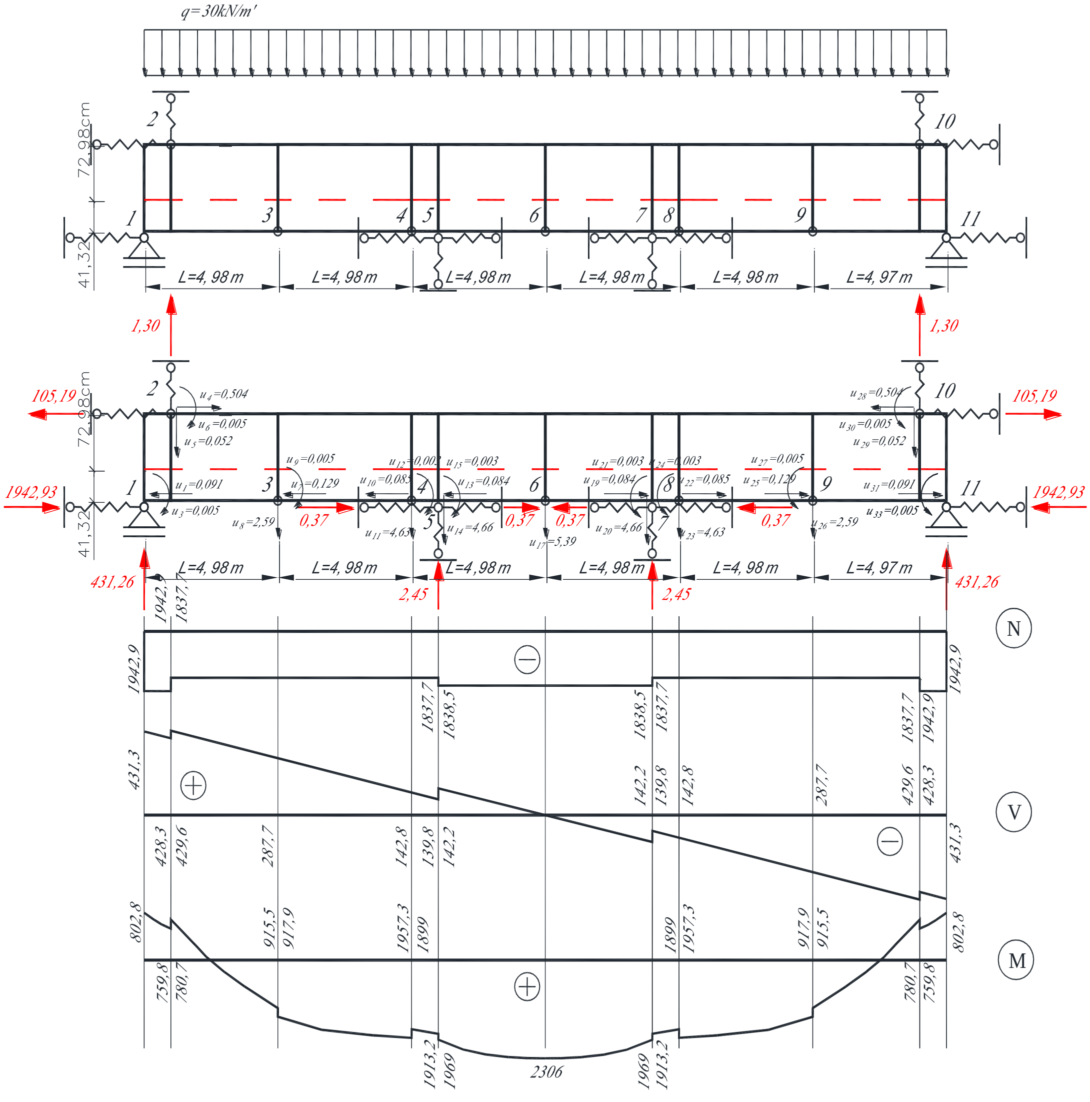

Figure 6. Off-center supported beam with reinforcing chain

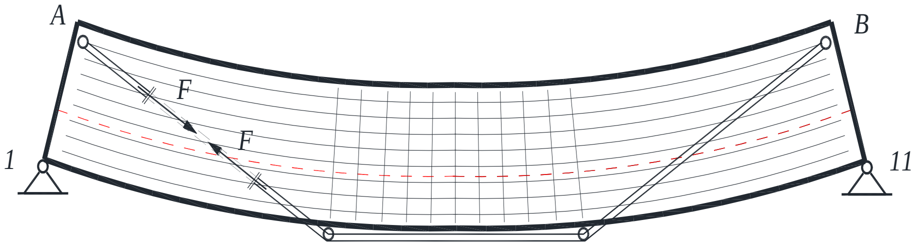

Figure 7. Beam bending

The conclusion that has to be drawn is that, the chain is completely unacceptable with the designed support, on the beam itself. This conclusion was also reach by Simmons and Walker, and it is report in their paper, that wrought iron chain provides little or no reinforcement to cast iron beams, because the ends of the chain are tie to the beam itself (Lewis & Gagg 2004). We can only guess, but it is logical to assume that Robert Stephenson sought to reduce the displacement values of the beam deflection. The displacements are easy to see, but he was hardly aware of the large axial compressive forces occurring. How the internal forces change with the additional slag load is shown in a Figure 8.

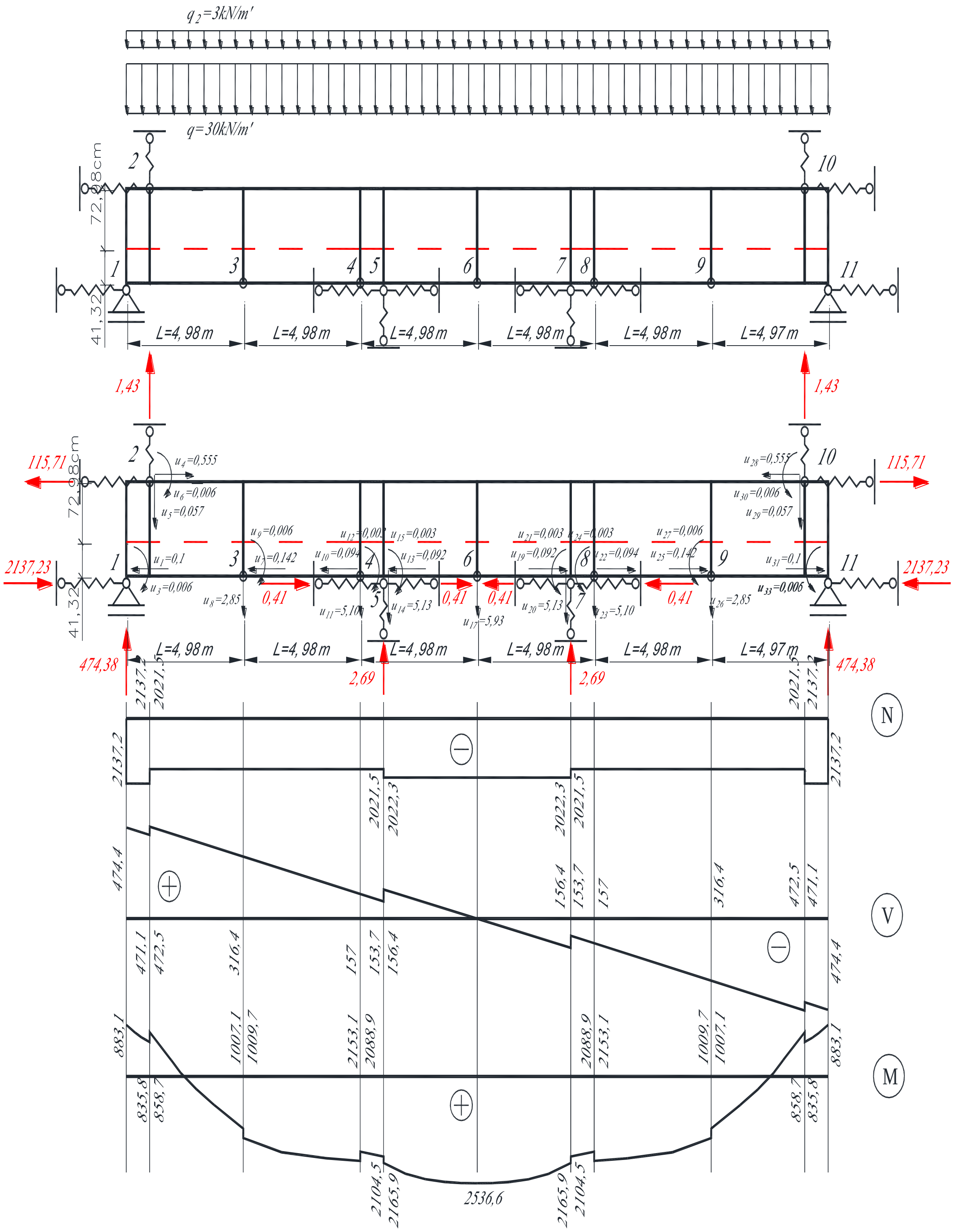

Figure 8. Off-center supported beam with reinforcing chain and additional dead load

The Inertial Force Arising When the Train Moves in a Curve

The bridge is on a curve of radius 7000ft = 2133.60m and the train is moving at 30 mph = 48.28 km/h. It causes the normal acceleration, $a_n = \frac{v^2}{R} = \frac{48280^2}{(60.60)^2 .2133,6} = 0,0843 m/s^2$ and the inertial force for a locomotive weighing 60 tons is $\Phi = M.a_n = 60000.0,0843 = 5057,9N = 5,1kN$.

Flexural Torsional Buckling

The displacement of the load from the vertical axis of symmetry of the beam cross-section was not been taken into account in the calculations. The presence of such a large axial force raises questions about Euler buckling load, just by bending about the vertical z-axis. For the failed beam, F_{cr}^{Euler} will be:

$$F_{cr}^{Euler} = \frac{\pi^2 EI_z}{L^2} = 1911,6kN \tag{3}$$

where:

modulus of elasticity - $E = 1,4.10^4 kN/cm^2$; moments of inertia - $I_z = 123434,25cm^4$, beam length - $L = 29,87m$

In Mladenov & Doicheva 2009 and Mladenov & Doicheva 2008, the flexural-torsional buckling of an off-center supported beam was consider (Figure 9). It was establish that, it is significantly influence by the position of the support devices, along the height of the beam and by the level of load action.

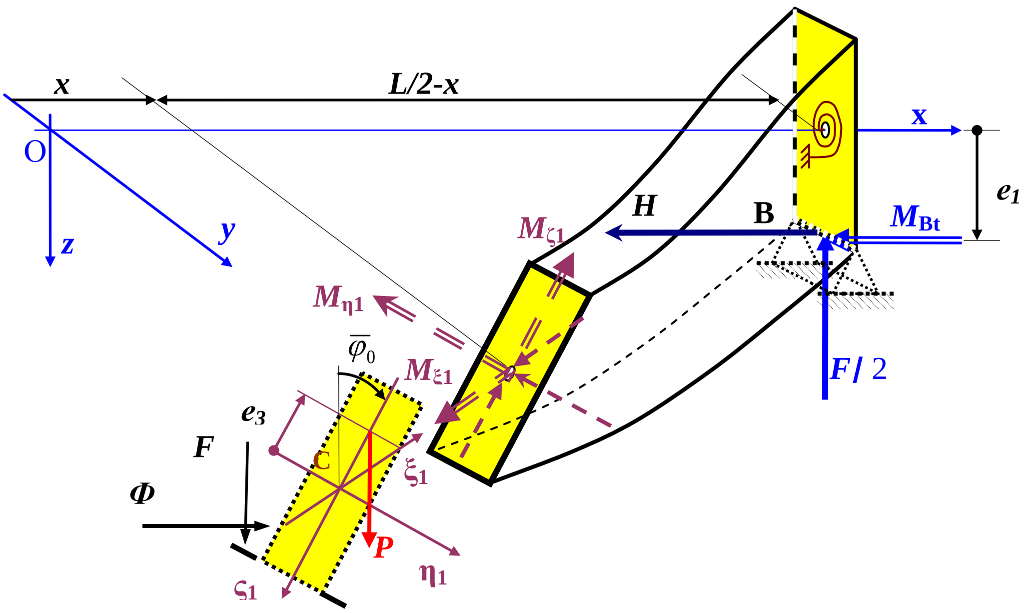

Figure 9. Part of the beam after loss of stability

Supports in the axis of the beam and force acting on the axis of the beam (Mladenov & Doicheva 2009):

$$F_{cr} = \frac{16,97\sqrt{GI_t EI_z}}{L^2} = 557,904kN \tag{4}$$

where:

modulus of elasticity - $E = 1,4.10^4 kN/cm^2$; modulus of angular deformation – $G = 0,441.10^4 kN/cm^2$; torsional moment of inertia - $I_t = 11290cm^4$, calculated by SSRC 1998; moments of inertia -

$I_z = 123434,25 cm^4$, beam length - $L = 29,87 m$, eccentricity of supports - $e_1 = 0 cm$; eccentricity of force - $e_3 = 0 cm$.

Comparing the critical force for off-center support with that of Euler, we have:

$$\frac{F_{cr}^{Euler}}{F_{cr}} = \frac{1911,6}{557,904} = 3,426 \tag{5}$$

Supports are along the bottom edge of the beam and force acting on the axis of the beam (Mladenov & Doicheva 2009):

$$F_{cr} = \frac{10,5938\sqrt{GI_t EI_z}}{L^2} = 348,282 kN \tag{6}$$

where:

eccentricity of supports - $e_1 = 41,32 cm$; eccentricity of force - $e_3 = 0 cm$.

Supports are along the bottom edge of the beam and force acting on the bottom edge of the beam (Mladenov & Doicheva 2009):

$$F_{cr} = \frac{10,772842\sqrt{GI_t EI_z}}{L^2} = 354,167 kN \tag{7}$$

where:

eccentricity of supports - $e_1 = 41,32 cm$; eccentricity of force - $e_3 = -41,32 cm$.

Studies of flexural torsional buckling show, that the critical forces for the beam, under consideration are small and easily achievable from the applied loads. The location of the force at the lower edge of the beam has a stabilizing effect (Rossia et al., 2020; Ranjithkumar & Punitha Kumar, 2022; Mladenov & Doicheva, 2009). This is also evident from the result of formula (7). The critical force is greater than the case of force on the axis of the beam - formula (6). If the load is off the vertical axis of the beam, on the inside of the curve (Figure 9), as it is with our beam, then this will cause an additional torsional effect of the beam.

There is a complex 3D stressed and deformed state of an off-center supported beam, in a horizontal curve with forces acting on the bottom flange of the beam, and horizontal inertial forces acting at the base of the profile stem, which is made of cast iron. The load bends the beam. An inappropriately chosen construction of a beam "reinforced" with a chain and supported on the bottom edge, lead to the appearance of significant compressive horizontal forces. The tall but thin stem of the beam is compress by the axial forces. The stem of the cross-section to try to bend about the z-axis, and to rotate about the x-axis, however, here the inertial forces appear, and the non-centric load located on the flange of the beam intervenes. A load displaced from the vertical axis of the cross-section leads to additional torsion of the beam. Inertial force presses the slender stem, which is under compressional stress and breaking it. We have conditions for a local stability loss. Cast iron is a brittle material, with no plastic deformations, and this accounts for its instantaneous destruction.

The destruction of the girder of the Dee Bridge is of interest with the interweaving of many factors, each of which bears blame for the fatal end. Analyzes of the causes of structure failures can help develop knowledge about equipment. This is how research on off-center supporting and flexural torsional buckling has developed over the past few decades. Engineers must become familiar with the causes of accidents that occur, in order to train and prevent the observed errors.

Conclusion

The search for answers to a major accident that occurred on the Dee River Bridge continues to this day. However, a mechanical analysis shows that, the adopted constructive solution is doomed to failure. The large axial forces caused by the improper support of the beam, with wrought iron chains, led to a tragic accident. The application of additional fire protection material only accelerated the collapse of one of the beams. This analysis would not be possible without the modern knowledge of off-center supports and flexural torsional buckling.

Recommendations

A survey was made of the girder of the bridge over the River Dee, which collapsed on 24 May 1847. The large axial forces occurring in the beam, as a result of the off-center supports are shown. The critical forces were calculated for the different positions of the supports, and the load on the beam. The critical force is small and achievable for the load under consideration.

Scientific Ethics Declaration

The authors declare that the scientific ethical and legal responsibility of this article published in EPSTEM journal belongs to the authors.

Acknowledgements or Notes

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