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Multilayered Semi-Elliptical Arch Structures: A Damping Analysis

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Abstract: Arch structures are used in various load-bearing applications in up-to-date engineering. In some cases these structures are made of multilayered materials with elastic-plastic behaviour. In many applications, the arch structures are subjected to cyclic external loading. This circumstance necessitates developing analyses of damping energy. Such analyses can be very useful for evaluating the effects of a variety of factors and parameters on the damping energy. The present paper is concerned with analyzing of damping energy in a semi-elliptical multilayered arch structure under cyclic bending. The left foot of the arch is clamped. Besides, the arch right foot is restrained by a rotational spring. An approach for determining of the changes in curvature and the coordinates of neutral axis in the portions of the arch is developed. Then damping energy in the arch is derived by integrating the damping energy density in the layers and summation for the arch portions. A parametric investigation of effects of the strength of the rotational spring, ratio of the ellipse semi axes and loading on damping energy is carried-out.

Keywords: Damping energy, Semi-elliptical arch, Elastic-plastic behaviour, Bending, Parametric investigation

Introduction

The application of arch structural members in various load-bearing structures is closely connected with search of new constructive solutions, usage of efficient engineering materials and creation of new methods for analyzing their mechanical response to various loads and external influences. One of the important directions for modernization of arch structures and enhancement of their safety, reliability and efficiency is the usage of multilayered materials.

In fact, the modern multilayered materials are advanced engineering materials capable of withstanding heavy loadings in extreme environment (Kim et al., 1999; Rzhanitsyn, 1986; Tokova et al., 2016). Application of layers of different materials in a multilayered system allows for design of efficient load-bearing multilayered structures and equipments distinguished for relatively low deadweight, high strength and stiffness, perfect response to various influences (temperature, humidity, etc.) and excellent mechanical behaviour (Kaul, 2014; Lloyd & Molina-Aldareguia, 2003; Rizov, 2018; Yu et al., 2003). Thus, the usage of multilayered materials in various areas of up-to-date engineering has intensified in recent decades (Rizov, 2021; Sy-Ngoc Nguyen et al., 2015; Sy-Ngoc Nguyen et al., 2020). The progress in the usage of multilayered materials in modern arch structures requires more theoretical knowledge elucidating the mechanical behaviour of these structures under various loading conditions.

In the present paper, the energy damping in multilayered arch structures of elastic-plastic behaviour under cyclic bending is analyzed theoretically by applying the apparatus of engineering mechanics (it should be underlined that the previous damping analyses usually consider beam structural members (Dowling, 2007; Rizov, 2021). In particular, an arch whose left foot is rigidly fixed while the right foot is restrained by a rotational spring is considered here. The arch has a semi-elliptical shape of small initial curvature. The damping energy in the arch is derived. A parametric investigation that elucidates the influence of the strength of the rotational spring, ratio of the ellipse semi axes and load magnitude on the damping energy is performed.

Deriving of Damping Energy

The load-bearing structure, Q_1Q_3, shown in Fig. 1 represents a semi-elliptical arch. The arch is built-up by n longitudinal layers (the cross-section of the arch is shown in Fig. 2).

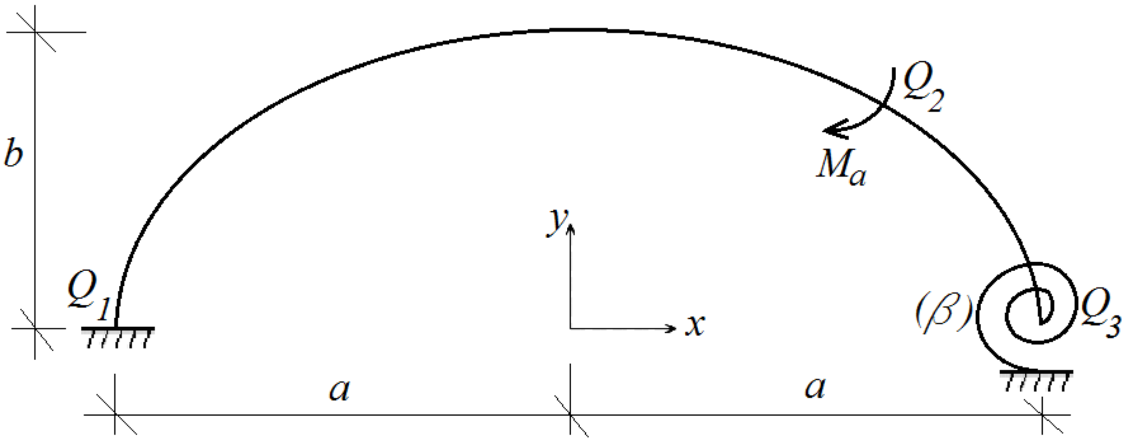

Figure 1. Semi-elliptical arch structure.

The left foot of the arch is clamped (Fig. 1). The arch is restrained also by a rotational spring in the right foot. The strength of the rotational spring is denoted by β. The arch is under a cyclic bending moment, M_a, about zero mean applied in section, Q_2, as shown in Fig. 1. The arch geometry is given by (Fig. 1)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \tag{1}$$

where

$$-a \le x \le a, \tag{2}$$

$$0 \le y \le b. \tag{3}$$

In formulas (1), (2) and (3), a and b are the horizontal and vertical semi axes of the ellipse, respectively.

The arch has elastic-plastic mechanical behaviour treated by the following constitutive law (Dowling, 2007):

$$\varepsilon_a = \frac{\sigma_{ai}}{E_i} + \left(\frac{\sigma_{ai}}{H_i}\right)^{\frac{1}{m_i}}, \tag{4}$$

where the subscript, i, refers to the *i*-th layer of the arch, ε_a and σ_{ai} are the half ranges of the strain and stress, respectively, E_i is the modulus of elasticity, H_i and m_i are material parameters. Deriving of the damping energy, ΔU, in the arch is given by

$$\Delta U = \Delta U_{Q1Q2} + \Delta U_{Q2Q3}, \tag{5}$$

where ΔU_{Q1Q2} and ΔU_{Q2Q3} are the damping energies in arch portions, Q_1Q_2 and Q_2Q_3, respectively (Fig. 1).

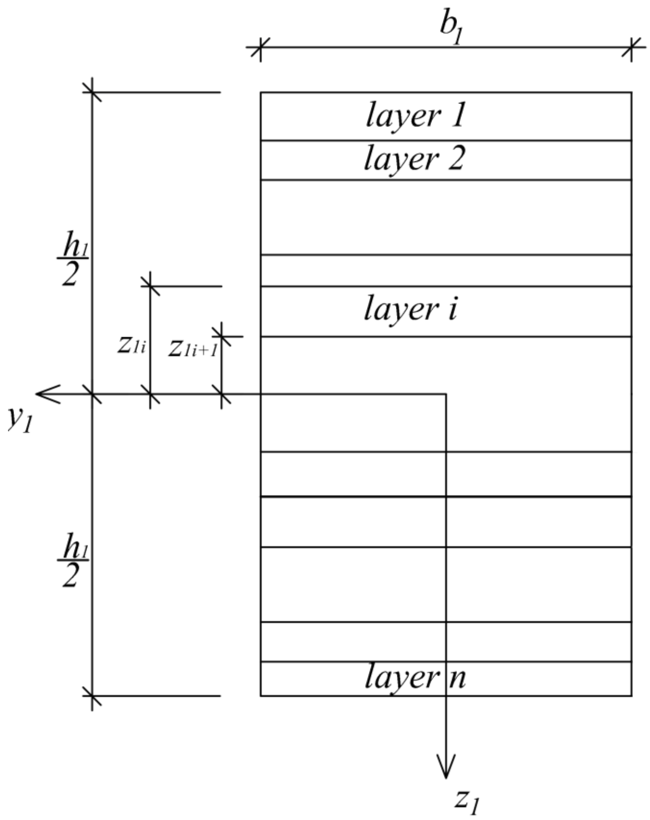

Figure 2. Cross-section of the arch structure

Formula (6) is applied for obtaining of ΔU_{Q1Q2} , i.e.

$$\Delta U_{Q1Q2} = s_{Q1Q2} b_1 \sum_{i=1}^{i=n} \int_{z_{1i}}^{z_{1i+1}} \Delta u_{Q1Q2i} dz_1 , \qquad (6)$$

where s_{Q1Q2} is the length of portion, Q_1Q_2 , of the arch, b_1 is the width (Fig. 2), z_{1i} and z_{1i+1} are the coordinates of the upper and lower surface of the *i*-th layer (Fig. 2), Δu_{Q2Q3i} is the damping energy density, z_1 is the vertical centric axis. Formula (7) is used for calculating of Δu_{Q2Q3i} (Dowling, 2007).

$$\Delta u_{Q1Q2i} = \frac{4(1-m_i)\sigma_{ai}^{1+\frac{1}{m_i}}}{(1+m_i)(H_i)^{\frac{1}{m_i}}} , \qquad (7)$$

where

$$i = 1, 2, \ ..., n . \qquad (8)$$

In order to calculate s_{Q1Q2} , we apply formula (9), i.e.

$$s_{Q1Q2} = \int_{-a}^{x_{Q2}} \sqrt{1 + y'^2} dx , \qquad (9)$$

where x_{Q2} is abscissa of Q_2 . The quantities, y and y' , can be derived from (1).

The distribution of ε_a along the arch thickness portion, Q_1Q_2 , is defined by (10), i.e.

$$\varepsilon_a = \kappa(z_1 - z_{1nn}), \tag{10}$$

where

$$-\frac{h_1}{2} \le z_1 \le \frac{h_1}{2}. \tag{11}$$

In formulas (10) and (11), κ is the change in curvature, z_{1nn} is the neutral axis coordinate, h_1 is the thickness. The changes in curvatures and neutral axis coordinates in portions, Q_1Q_2 and Q_2Q_3 , of the arch are determined from equations (12) – (16).

$$\varphi_{Q3} = \kappa\, s_{Q1Q2} + \kappa_{Q2Q3} s_{Q2Q3}, \tag{12}$$

$$N_{aQ1Q2} = b_1 \sum_{i=1}^{i=n} \int_{z_{1i}}^{z_{1i+1}} \sigma_a dz_1, \tag{13}$$

$$M_{aQ1Q2} = b_1 \sum_{i=1}^{i=n} \int_{z_{1i}}^{z_{1i+1}} \sigma_a z_1 dz_1, \tag{14}$$

$$N_{aQ2Q3} = b_1 \sum_{i=1}^{i=n} \int_{z_{1i}}^{z_{1i+1}} \sigma_{aQ2Q3} dz_1, \tag{15}$$

$$M_{aQ2Q3} = b_1 \sum_{i=1}^{i=n} \int_{z_{1i}}^{z_{1i+1}} \sigma_{aQ2Q3} z_1 dz_1, \tag{16}$$

where

$$s_{Q2Q3} = \int_{x_{Q2}}^{a} \sqrt{1 + y'^2}\, dx, \tag{17}$$

$$N_{aQ1Q2} = 0, \tag{18}$$

$$N_{aQ2Q3} = 0, \tag{19}$$

$$M_{aQ1Q2} = M_{aQ3} - M_a, \tag{20}$$

$$M_{aQ3} = \varphi_{Q3} \beta, \tag{21}$$

$$M_{aQ2Q3} = M_{aQ3}. \tag{22}$$

In formulas (12) – (22), φ_{Q3} is the angle of rotation of section, Q_3, of the arch (actually, formula (12) is obtained by using the integrals of Maxwell-Mohr), κ_{Q2Q3} and s_{Q2Q3} are the change in the curvature and the length of portion, Q_2Q_3 , of the arch, N_{aQ1Q2} and N_{aQ2Q3} are the axial forces in the two portions of the arch (here, $N_{aQ1Q2} = 0$ and $N_{aQ2Q3} = 0$), M_{aQ1Q2} and M_{aQ2Q3} are the bending moments in the two arch portions, M_{aQ3} is the bending moment in section, Q_3, of the arch. The MatLab is applied for solving equations (12) – (16). The damping energy in arch portion, Q_2Q_3, is given by (23)

$$\Delta U_{Q2Q3} = s_{Q2Q3} b_1 \sum_{i=1}^{i=n} \int_{z_{1i}}^{z_{1i+1}} \Delta u_{Q2Q3i} dz_1 , \tag{23}$$

where

$$\Delta u_{Q2Q3i} = \frac{4(1-m_i)\sigma_{aQ2Q3i}^{1+\frac{1}{m_i}}}{(1+m_i)(H_i)^{\frac{1}{m_i}}} . \tag{24}$$

Here, σ_{aQ2Q3i} is the stress in the *i*-th layer.

Formulas (6) and (23) are substituted in (5) to obtain the damping energy in the arch structure (the integrals are solved by the MatLab).

Numerical Results

Numerical results indicating the effects of various parameters (strength of the rotational spring, ratio of the ellipse semi axes, load magnitude, etc.) on the damping energy in the multilayered semi-elliptical arch structure are derived by using the following data: $a = 0.450$ m, $b = 0.300$ m, $b_1 = 0.005$ m, $h_1 = 0.008$ m, $n = 5$ and $M_a = 4$ Nm.

First, the influence of the strength of the rotational spring, β, on the damping energy is analyzed. Figure 3 shows the damping energy variation with increasing of the value of β. One can see that the damping energy reduces when β increases (Fig. 3). The increase of E_2 / E_1 ratio (here, subscripts, 1 and 2, refer to layers 1 and 2 of the arch structure, respectively) also causes a reduction of the damping energy as illustrated in Fig. 3.

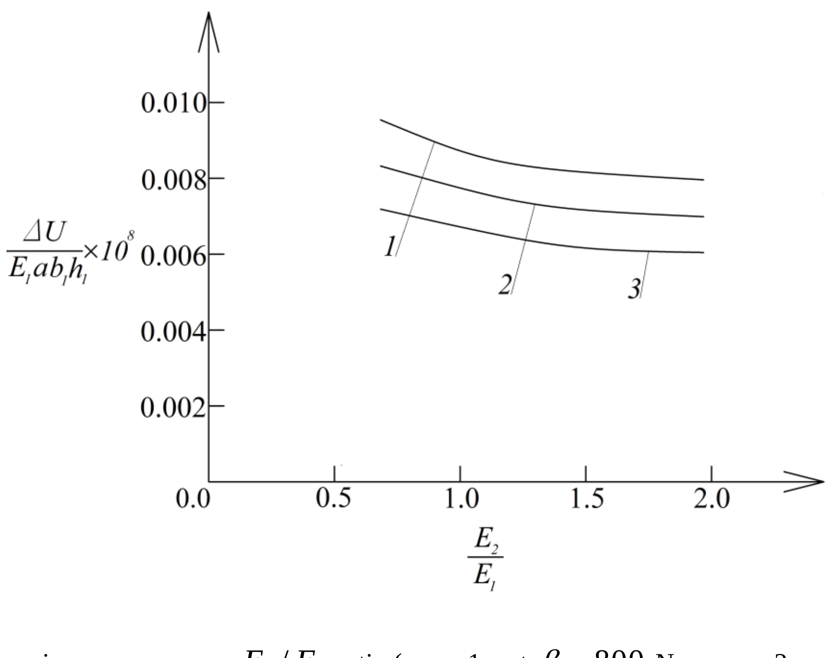

Figure 3. The damping energy versus E_1 / E_2 ratio (curve 1 – at $\beta = 800$ Nm, curve 2 – at $\beta = 1600$ Nm and curve 3 – at $\beta = 2400$ Nm).

The influence of the geometry of the arch structure on the damping energy is analyzed too. The geometry is presented by the ratio of the ellipse semi axes, a/b. The influence of a/b and H_2/H_1 ratios is shown in Fig. 4. It can be observed that increase of the ratio of the ellipse semi axes leads to growth of the damping energy (Fig. 4). The increase of H_2/H_1 ratio causes a substantial reduction of the damping energy as indicated by the curves in Fig. 4. Besides, comparison of Fig. 3 and Fig. 4 reveals that the reduction of the damping energy due to increase of H_2/H_1 ratio is stronger than this caused by increase of E_2/E_1 ratio. Finally, the effects of the bending moment value, M_a, and the ratio of the thickness and width of the arch cross-section, h_1/b_1, on the damping energy are evaluated.

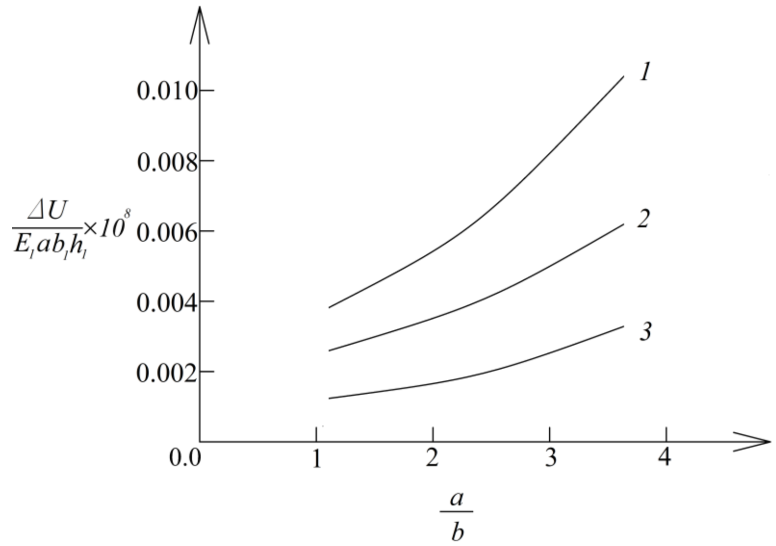

Figure 4. The damping energy versus a/b ratio (curve 1 – at $H_1/H_2 = 0.5$, curve 2 – at $H_1/H_2 = 1.0$ and curve 3 – at $H_1/H_2 = 2.0$).

These effects are illustrated by the curves shown in Fig. 5

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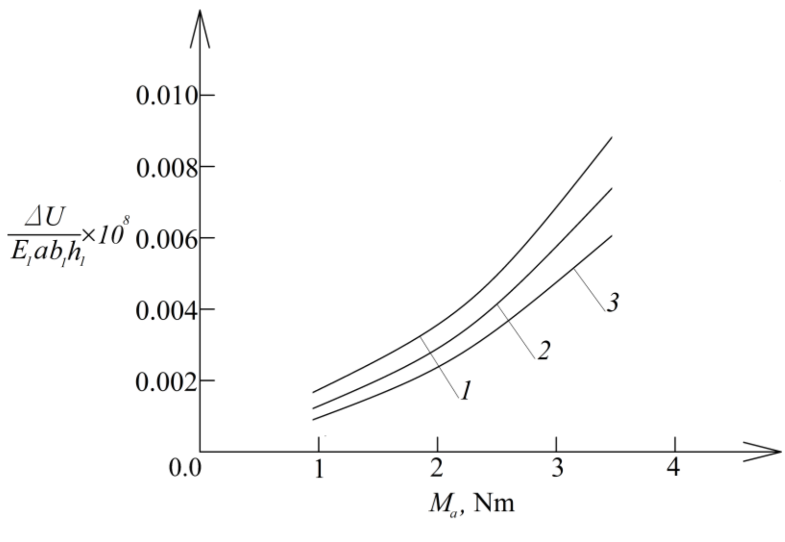

Figure 5. The damping energy versus M_a (curve 1 – at $h_1/b_1 = 1.4$, curve 2 – at $h_1/b_1 = 1.6$ and curve 3 – at $h_1/b_1 = 1.8$).

It can be seen in Fig. 5 that when M_a increases, the behaviour of the damping energy is characterized by a quick growth. The increase of h_1 / b_1 ratio leads to reduction of the damping energy (Fig. 5).

Conclusion

A theoretical investigation of the damping energy in a multilayered semi-elliptical arch structure is performed. The investigation yielded the following findings:

1) the damping energy reduces when the strength of the rotational spring increases;

2) increase of E_2 / E_1 ratio causes a reduction of the damping energy;

3) increase of the ratio of the ellipse semi axes leads to growth of the damping energy;

4) increase of H_2 / H_1 ratio causes a substantial reduction of the damping energy;

5) increase of h_1 / b_1 ratio leads also to reduction of the damping energy.

Recommendations

The analysis developed in the present paper can be recommended for application in engineering practice as a tool for investigation of the damping energy in multilayered semi-elliptical arch structures.

Scientific Ethics Declaration

The author declares that the scientific ethical and legal responsibility of this article published in EPSTEM journal belongs to the author.

Acknowledgements or Notes

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