The Eurasia Proceedings of Science, Technology, Engineering & Mathematics (EPSTEM)
ISSN: 2602-3199

The Eurasia Proceedings of Science, Technology, Engineering & Mathematics (EPSTEM), 2024

Volume 29, Pages 69-75

ICRETS 2024: International Conference on Research in Engineering, Technology and Science

Longitudinal Fracture Analysis with Taking into Account the Elongation Speed of Inhomogeneous Rod Subjected to Centric Tension

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Abstract: The present theoretical study deals with longitudinal fracture of a structural component representing inhomogeneous rod that is loaded in centric tension at its lower end. The upper end of the rod is clamped. The rod is continuously inhomogeneous in radial direction. Besides, the rod exhibits non-linear elastic mechanical behaviour. The parameters involved in the non-linear constitutive law used when analyzing the longitudinal fracture behaviour are smooth functions of the running radius of the rod cross-section. The basic aim of the paper is to analyze the longitudinal fracture with taking into account the influence of the rod elongation speed. The case when the elongation varies with time at a constant speed is treated in detail. The necessary equations for deriving the parameters of the stressed and strained state of the rod are formulated. The problem for determination of strain energy release rate (SEER) for the longitudinal crack in the rod is solved with considering the effect of the elongation speed.

Keywords: Elongation speed, Centric tension, Rod, Fracture

Introduction

Many components of various engineering constructions, machines and mechanisms in fact represent cylindrical rods subjected to centric tension (Darkov & Shapiro, 1989; Gay et al., 2003; Kissiov, 1997). It is obvious that performing detailed studies of behaviour of such rods under different mechanical loadings is very useful and important (Fanani et al., 2021; Gururaja Udupa et al., 2014; Reyne, 1994; Rizov, 2005). Such studies generate valuable information that can be incorporated in sophisticated theoretical models which are meant for carrying-out accurate analyses of specific aspects of the rod mechanical performance under various loads and influences. One of the factors of great influence on the rod performance is the fracture (Rizov, 2018; Rizov, 2019; Rizov & Altenbach, 2020; Tilbrook et al., 2005).

The purpose of the present theoretical paper is to carry-out an analytical study of longitudinal fracture in a rod loaded in centric tension. The study accounts for the speed of the rod elongation (actually, this is the most important point of the study). The rod under consideration is non-linear elastic. Besides, the rod is made of a material that is continuously inhomogeneous along the radius of the rod cross-section (it should be noted here that continuously inhomogeneous engineering materials have become a very attractive alternative of the homogeneous materials in a variety of applications in different sectors of engineering in the recent decades (Nikbakht et al., 2019; Nagaral et al., 2019; Radhika et al., 2020)). However, this kind of material inhomogeneity (in radial direction) is a premise for development of longitudinal cracks in the form of circular cylindrical surface. There is such a longitudinal crack is the rod under consideration here. The SERR for this crack is obtained with taking into account of the rod elongation speed. It is studied how the SERR varies under the influence of some basic factors.

Theoretical Model

The present study is focussed on the inhomogeneous rod in Fig. 1.

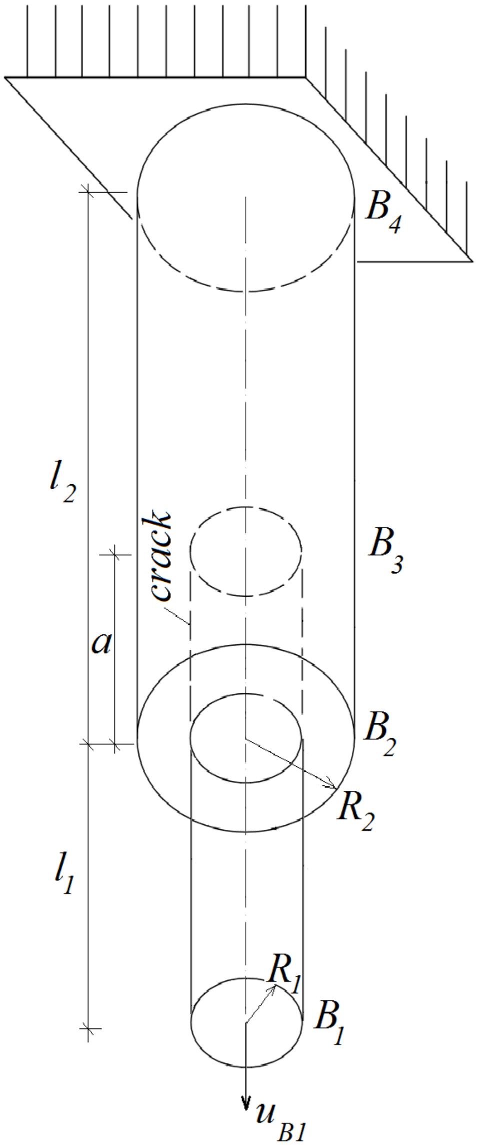

Figure 1. Rod with a longitudinal crack.

This rod is under elongation, u_{B1}, that increases with time, t, at a constant speed, v_{B1}, i.e.

$$u_{B1} = v_{B1} t . \tag{1}$$

The non-linear elastic behaviour of the rod with taking into account the elongation speed is treated by the constitutive law (2) (Lukash, 1997).

$$\sigma = P\left[1-\left(1-\frac{\varepsilon}{Q}\right)^{m}\right]\left(1+D\frac{\varepsilon}{\dot{\varepsilon}_0}\right), \tag{2}$$

where σ, ε and $\dot{\varepsilon}$ are the stress, strain and the speed of the strain, P, Q, D, m and $\dot{\varepsilon}_0$ are material parameters.

The continuous change of P, Q, D and m along the radius, R_1, of the rod cross-section is given by

$$P = P_C + \frac{P_P - P_C}{R_1^b} R^b , \tag{3}$$

$$Q = Q_C + \frac{Q_P - Q_C}{R_1^f} R^f , \tag{4}$$

$$D = D_C + \frac{D_P - D_C}{R_1^g} R^g , \tag{5}$$

$$m = m_C + \frac{m_P - m_C}{R_1^h} R^h , \tag{6}$$

$$0 \le R \le R_1 , \tag{7}$$

where the subscripts, C and P, refer to the centre and the surface of the rod, respectively, b, f, g and h are parameters. Formulas (3) – (6) hold for part, B_1B_2, of the rod (Fig. 1). The same formulas can be applied for presenting the continuous change of P, Q, D and m along rod cross-section radius in part, B_2B_3, of the rod (the only correction that has to be done is to replace R_1 with R_2). It should be specified here that P, Q, D and m change along the rod cross-section radius since the rod is continuously inhomogeneous.
The mechanical response of the rod in Fig. 1 is analyzed via formulas (8) – (13).
The strain, ε, of part, B_1B_3, of the rod is given by

$$\varepsilon = v_\varepsilon t , \tag{8}$$

where v_ε is the speed.
In part, B_3B_4, of the rod, the strain, ε_{B3B4}, has speed, $v_{\varepsilon B3B4}$. Therefore,

$$\varepsilon_{B3B4} = v_{\varepsilon B3B4} t . \tag{9}$$

Equation (10) relates the strains with the rod elongation.

$$\varepsilon(l_1 + a) + \varepsilon_{B3B4}(l_2 - a) = u_{B1} . \tag{10}$$

From Eq. (10), one determines

$$\varepsilon_{B3B4} = \frac{v_{B1} - v_\varepsilon (l_1 + a)}{l_2 - a} , \tag{11}$$

where l_1, l_2 and a are shown in Fig. 1. (a is the crack length). The longitudinal crack represents cylindrical surface with radius, R_1. This crack is located in part, B_3B_4, of the rod (Fig. 1). The speed, v_ε, is obtained from equilibrium equation (12).

$$N_{B1B3} = N_{B3B4} , \tag{12}$$

where N_{B1B3} and N_{B3B4} are the axial forces in rod parts, B_1B_3 and B_3B_4, respectively.
Equation (12) is expressed via the stresses, σ and σ_{B3B4}.

$$\iint_{(A_1)} \sigma dA = \iint_{(A_2)} \sigma_{B3B4} dA, \tag{13}$$

where A_1 and A_2 are the cross-sections in parts, B_1B_3 and B_3B_4, of the rod, σ is determined by (2), σ_{B3B4} is obtained by replacing of ε with ε_{B3B4} in (2). Equation (13) is used to derive v_ε by the MatLab.

The final step in the current study is to derive the SERR, G, for the longitudinal crack in the rod. Formula (14) is used for this purpose.

$$G = \frac{dU^*}{2\pi R_1 da}, \tag{14}$$

where U^* is the complementary strain energy in the rod.

Formula (15) is used for calculating the complementary strain energy.

$$U^* = (l_1 + a)\iint_{(A_1)} u_{01}^* dA + (l_2 - a)\iint_{(A_2)} u_{02}^* dA, \tag{16}$$

where u_{01}^* and u_{02}^* are the complementary strain energy densities. They are derived by formulas (17) and (18), respectively.

$$u_{01}^* = \sigma\varepsilon - \int_0^{\varepsilon} \sigma d\varepsilon, \tag{17}$$

$$u_{02}^* = \sigma_{B3B4}\varepsilon_{B3B4} - \int_0^{\varepsilon_{B3B4}} \sigma_{B3B4} d\varepsilon_{B3B4}. \tag{18}$$

The SERR found by (14) is confirmed by analyzing the energy balance via formula (19).

$$G = \frac{1}{2\pi R_1}\left(N_{B1B3}\frac{\partial u_{B1}}{\partial a} - \frac{\partial U}{\partial a} \right), \tag{19}$$

where U is the strain energy in the rod (U is found by replacing of the complementary strain energy densities with the strain energy densities in formula (16)).

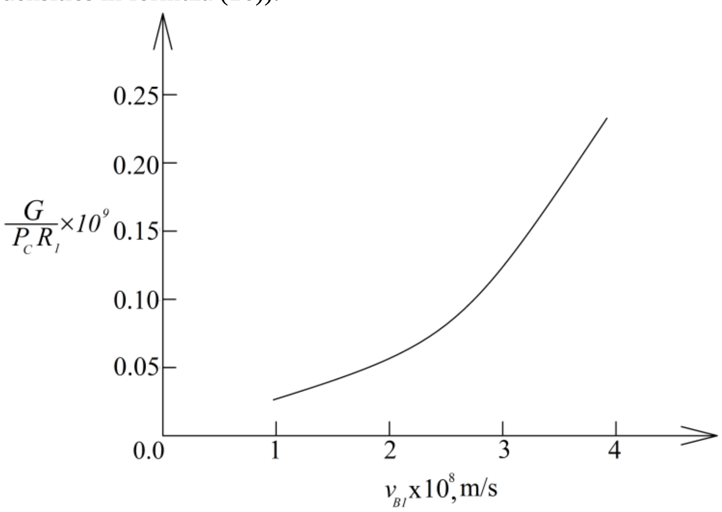

Figure 2. The non-dimensional SERR versus v_{B1}.

Numerical Results

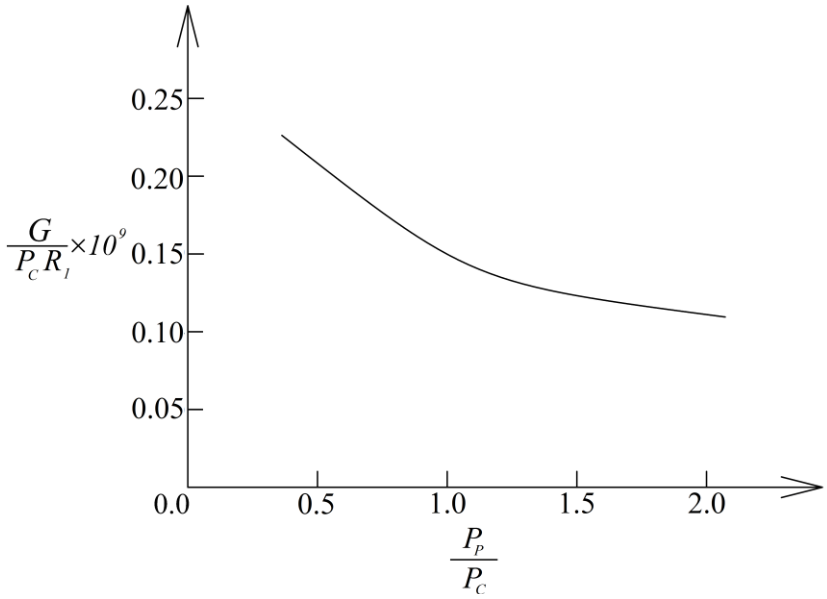

Figure 3. The non-dimensional SERR versus P_P / P_C ratio.

The numerical results determined via the solution of the SERR use to the following data: $a = 0.100$ m, $l_1 = 0.300$ m, $l_2 = 0.400$ m, $R_1 = 0.010$ m, $R_2 = 0.015$ m, $v_{B1} = 4 \times 10^{-8}$ m/s, $b = 0.4$, $f = 0.4$, $g = 0.6$, $h = 0.6$ and $\dot{\varepsilon}_0 = 2 \times 10^{-6}$ 1/s. These results reveal how the SERR for the rod in Fig. 1 varies under the influence of some basic parameters. This variation is shown in four figures. For example, the curve in Fig. 2 illustrates the behaviour of the SERR when the elongation speed, v_{B1}, increases. The quick rise of the SERR in Fig. 2 is due to growth of the rod elongation.

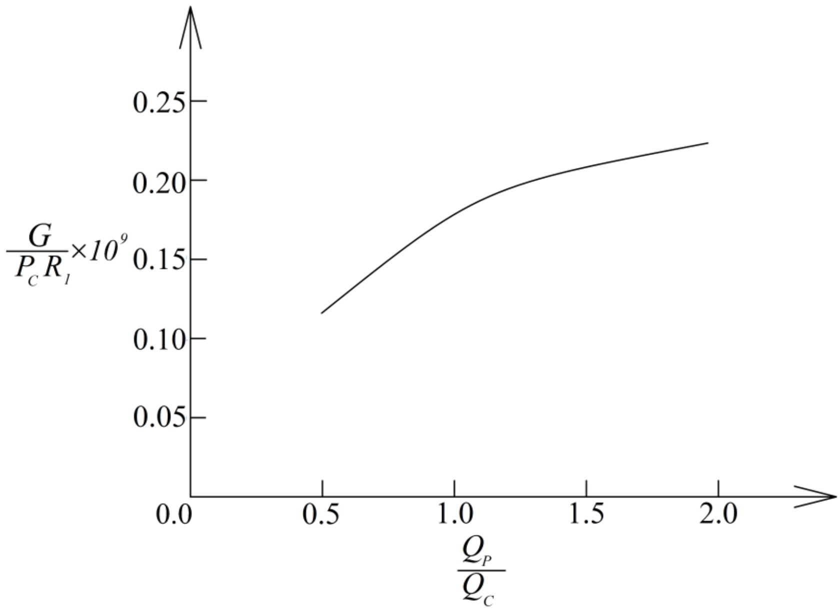

Figure 4. The non-dimensional SERR versus Q_P / Q_C ratio.

The variation of the SERR under the influence of parameters, P, Q and D, is studied too (these parameters are characterized by ratios, P_P / P_C, Q_P / Q_C and D_P / D_C, respectively, since the material of the rod is continuously inhomogeneous).

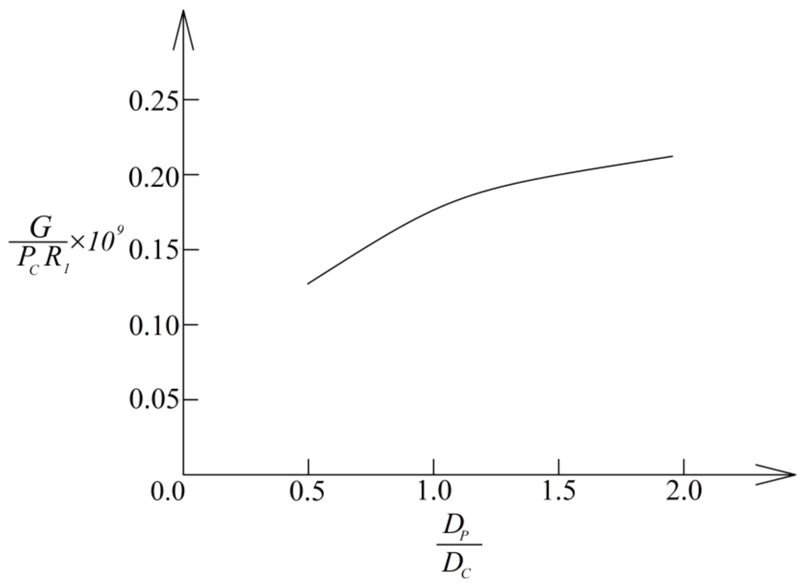

Figure 5. The non-dimensional SERR versus D_P / D_C ratio.

The variation of the SERR when the ratio, P_P / P_C, increases may be observed in Fig. 3. The behaviour of the SERR is characterized by a smooth reduction caused by the increase of P_P / P_C ratio (Fig. 3). One can see how the SERR changes when the ratio, Q_P / Q_C, grows in Fig. 4. The rise of the SERR in Fig. 4 is generated by reduction of the rod stiffness when Q_P / Q_C increases. The behaviour of the SERR when the ratio, D_P / D_C, rises is presented by the curve in Fig. 5. This behaviour is characterized by growth of the SERR.

Conclusion

The longitudinal fracture of an inhomogeneous rod under centric tension is studied theoretically with accounting the influence of the speed of rod elongation. The rod is non-linear elastic. The SERR is determined analytically and analyzed in full. The analysis performed gives important knowledge for the influence of some typical factors like elongation speed and material parameters involved in the non-linear constitutive law. Since the rod is continuously inhomogeneous, the material parameters are presented by the ratios of their values on the surface and the centre of the rod cross-section. The study shows that the SERR quickly increases when parameter, v_{B1}, grows. Increase of the SERR is found also when Q_P / Q_C and D_P / D_C ratios grow. Reduction of the SERR is observed when P_P / P_C ratio grows.

Recommendations

The elongation speed has to be incorporated in longitudinal fracture analyses of continuously inhomogeneous rods subjected to centric tension.

Scientific Ethics Declaration

The author declares that the scientific ethical and legal responsibility of this article published in EPSTEM journal belongs to the author.

Acknowledgements or Notes

This article was presented as an oral presentation at the International Conference on Research in Engineering, Technology and Science (www.icrets.net) held in Tashkent/Uzbekistan on August 22-25, 2024.

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To cite this article:

Rizov, V. (2024). Longitudinal fracture analysis with taking into account the elongation speed of ınhomogeneous rod subjected to centric tension. *The Eurasia Proceedings of Science, Technology, Engineering & Mathematics (EPSTEM), 29*, 69-75.