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Analysis of Supports Displacements Induced Delamination in Multilayered Indeterminate Beam Structures

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Abstract: This theoretical paper is focussed on the problem of delamination in a multilayered beam load-carrying structure that is statically indeterminate. The beam is supported by three supports. The middle support represents a vertical spring. There is no external mechanical loading on the beam. Instead of this, the beam is under vertical displacement of the left-hand support. The main goal of the paper is to investigate the delamination due to the displacement of the left-hand support. Another issue that is under consideration is the effect of the vertical spring. The beam has non-linear elastic mechanical behaviour. The parameters of the non-linear constitutive law that is applied for treating the beam mechanical behaviour vary continuously along the beam length since the beam layers are made of engineering materials that are continuously inhomogeneous in the length direction. Equations for determination of the curvatures and coordinates of the neutral axis in the portions of the beam are constituted. The strain energy release rate (SERR) induced by the support displacement is derived and checked-up by the integral *J*.

Keywords: Support displacement, Multilayered material, Beam, Delamination

Introduction

The problem of delamination of beam load-bearing engineering structures has been investigated mainly for statically determinate beams (Broek, 1986; Dowling, 2007; Rizov, 2018; Rizov, 2021). In the case of statically indeterminate beam structures, the delamination analysis becomes more complicated since the static indeterminacy problem has to be treated first. Also, if the statically indeterminate beams are non-linear elastic, this puts some additional complications in the delamination problem investigation (Rizov, 2019). One of the specific peculiarities that need a detailed examination when studying delamination in statically indeterminate beam structures is the effect of displacements of supports on the SERR.

In this theoretical paper, we give a solution of the SERR for a delamination crack in a multilayered statically indeterminate beam structure on three supports for the case when one of the supports undergoes displacement (we are dealing here with inhomogeneous multilayered beams since multilayered systems are widely used in engineering (Kaul, 2014; Kissiov, 1997; Lloyd & Molina-Aldareguia, 2003; Yu et al., 2003; Rizov, 2005; Rzhanitsyn, 1986; Sy-Ngoc Nguyen et al., 2020; Tokova et al., 2016)). Another issue that is examined in this paper is the influence of the elastic spring on the SERR (it is assumed that the middle support of the beam represents a vertical spring). The SERR solution derived here incorporates both the displacement of the left support and the spring constant of the middle support. Besides, the solution accounts for the non-linear elastic behaviour of the beam structure and the material inhomogeneity of the beam layers along the length. The SERR induced by the support displacement is checked-up by the integral *J*. The analysis leads to results clarifying how the SERR magnitude is influenced by the value of the support displacement and the spring constant.

Analysis

- Selection and peer-review under responsibility of the Organizing Committee of the Conference

The engineering structure shown in Fig. 1 represents a multilayered beam on three supports in points, H_1, H_3 and H_4.

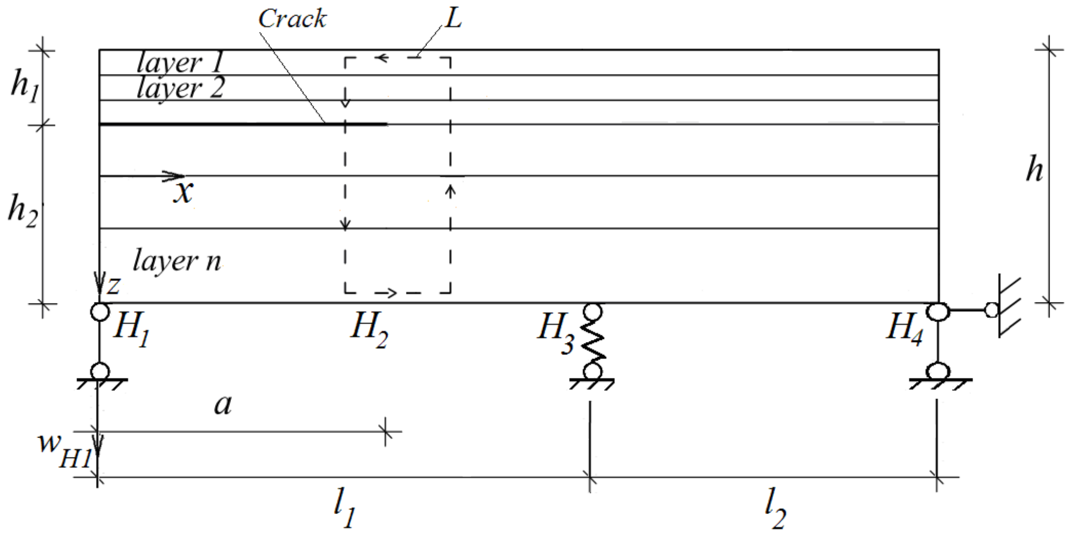

Figure 1. Multilayered beam with delamination.

The support in point, H_3, is a vertical spring with spring constant, C. The beam layers are non-linear elastic. Their behaviour is described by the non-linear constitutive law (1) (Lukash, 1997).

$$\sigma_i = \frac{\varepsilon}{S_i + B_i \varepsilon}, \tag{1}$$

$$i = 1, 2, ..., n, \tag{2}$$

where σ_i is the stress, ε is the strain, S_i and B_i are material properties, n is the number of layers in the beam.

The layers are inhomogeneous along the x axis. Formulas (3) and (4) present the change of material properties along x.

$$S_i = S_{i0} e^{f_i \frac{x}{l_1 + l_2}}, \tag{3}$$

$$B_i = B_{i0} e^{g_i \frac{x}{l_1 + l_2}}, \tag{4}$$

$$0 \leq x \leq l_1 + l_2, \tag{5}$$

where S_{i0} and B_{i0} are the values of S_i and B_i at the left end of the beam, f_i and g_i are parameters, l_1, l_2 and x are shown in Fig. 1. The support in point, H_1, undergoes vertical displacement, w_{H1}. Our purpose is the derive the SERR for the delamination crack (the latter is located in part, $H_1 H_2$, of the beam (Fig. 1)) with considering w_{H1}. The beam has one degree of static indeterminacy that is treated by the following equations. First, w_{H1} is expressed by integrals of Maxwell-Mohr, i.e.

$$w_{H1} = \int_0^a \kappa_{H1H2} x_1 dx_1 + \int_0^{l_1 - a} \kappa_{H2H3} (x_2 + a) dx_2 + \int_0^{l_2} \kappa_{H3H4} (l_1 + x_3) dx_3 + \frac{w_{H3}(l_1 + l_2)}{l_2}, \tag{6}$$

where κ_{H1H2}, κ_{H2H3} and κ_{H3H4} are the curvatures of the corresponding parts of the beam, x_1, x_2 and x_3 are the local centroidal axes of the beam parts, w_{H3} is the displacement of point, H_3.
The support reaction, R_{H3}, in the spring in point, H_3, is given by

$$R_{H3} = R_{H1}\frac{l_1 + l_2}{l_2}, \tag{7}$$

where R_{H1} is the support reaction in point, H_1.

Formula (8) calculates w_{H3}, i.e.

$$w_{H3} = \frac{R_{H3}}{C}. \tag{8}$$

The next six equations represent conditions for equilibrium of the elementary forces in the cross-sections of the beam parts.

$$N_{H1H2} = b\sum_{i=1}^{i=n_1}\iint_{(A_i)}\sigma_i dA, \tag{9}$$

$$M_{H1H2} = b\sum_{i=1}^{i=n_1}\iint_{(A_i)}\sigma_i z_1 dA, \tag{10}$$

$$N_{H2H3} = b\sum_{i=1}^{i=n}\iint_{(A_i)}\sigma_{H2H3i} dA, \tag{11}$$

$$M_{H2H3} = b\sum_{i=1}^{i=n}\iint_{(A_i)}\sigma_{H2H3i} z_2 dA, \tag{12}$$

$$N_{H3H4} = b\sum_{i=1}^{i=n}\iint_{(A_i)}\sigma_{H3H4i} dA, \tag{13}$$

$$M_{H3H4} = b\sum_{i=1}^{i=n}\iint_{(A_i)}\sigma_{H3H4i} z_3 dA, \tag{14}$$

Where

$$N_{H1H2} = 0, \tag{15}$$

$$M_{H1H2} = R_{H1}x_1, \tag{16}$$

$$N_{H2H3} = 0, \tag{17}$$

$$M_{H2H3} = R_{H1}(a + x_2), \tag{18}$$

$$N_{H3H4} = 0, \tag{19}$$

$$M_{H3H4} = R_{H1}(l_1 + x_3). \tag{20}$$

In equations (9) – (20), b is the beam width, n_1 is the number of layers in the lower delamination arm, N_{H1H2}, N_{H2H3} and N_{H3H4} are the axial forces in the beam parts, M_{H1H2}, M_{H2H3} and M_{H3H4} are the bending moments.

The curvatures and the coordinates of the neutral axes in beam parts, H_1H_2, H_2H_3 and H_3H_4, that are determined from Eqs. (6), (9), (10), (11), (12), (13) and (14) are used to calculate the complementary strain energy, U^*, in the beam via formula (21).

$$U^* = \sum_{i=1}^{i=n_1} \iiint_{(V_i)} u_{0i}^* dV + \sum_{i=1}^{i=n} \iiint_{(V_i)} u_{0H2H3i}^* dV + \sum_{i=1}^{i=n} \iiint_{(V_i)} u_{0H3H4i}^* dV , \tag{21}$$

where u_{0i}^*, u_{0H2H3i}^* and u_{0H2H3i}^* are the complementary strain energy densities.
The SERR, G, is given by

$$G = \frac{dU^*}{bda} . \tag{22}$$

The SERR found by (22) is confirmed by the method of the integral *J* (Broek, 1986). This integral is solved along the contour of integration, L, shown in Fig. 1.

Numerical Results

The numerical results for the SERR reported here are for $l_1 = 0.400$ m, $l_2 = 0.300$ m, $a = 0.200$ m, $h = 0.015$ m, $b = 0.008$ m, $n = 5$ and $n_1 = 3$. The results indicating the influence of support displacement, spring constant, and the distribution of the material properties on the SERR magnitude are shown in the next figures. Figure 2 indicates the influence of the support displacement value (the latter is presented by w_{H1}/h ratio) on the SERR magnitude.

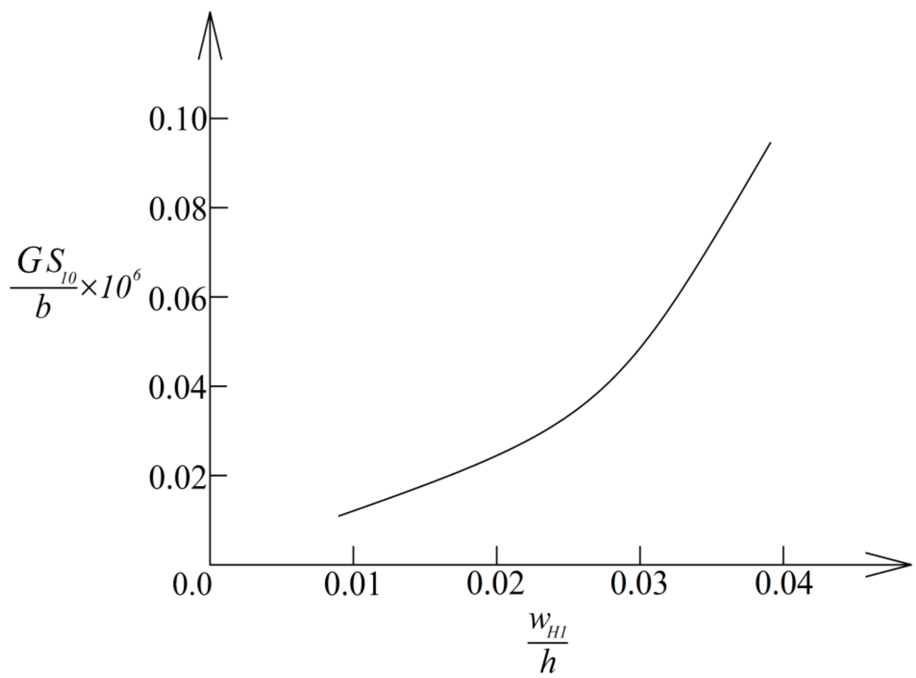

Figure 2. The non-dimensional SERR versus w_{H1}/h ratio.

The rapid growth of the SERR which can be observed in Fig. 2 shows that the support displacements have to be taken into account when analyzing delamination behaviour of statically indeterminate multilayered beam structures.

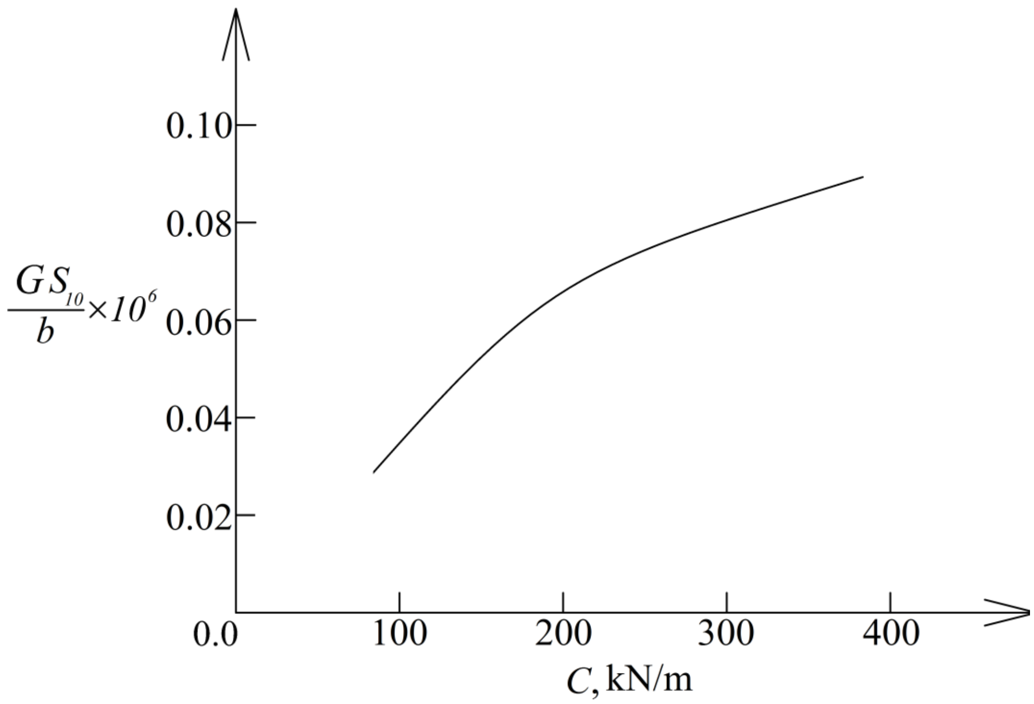

Figure 3. The non-dimensional SERR versus C .

Neglecting the effect of support displacements in delamination studies may have very negative influence on the safety and life of the structures and even may cause structural failure. The curve shown in Fig. 3 illustrates the influence of the value of the spring constant on the SERR magnitude. Since the SERR magnitude changes quickly with rise of the spring constant, it can be concluded that springs have to be considered carefully in the delamination analysis of statically indeterminate multilayered beam structures. The influence of continuous distribution of S_1 along the length of layer 1 of the beam on the SERR magnitude is shown in Fig. 4 (the distribution of S_1 is presented by the parameter, f_1).

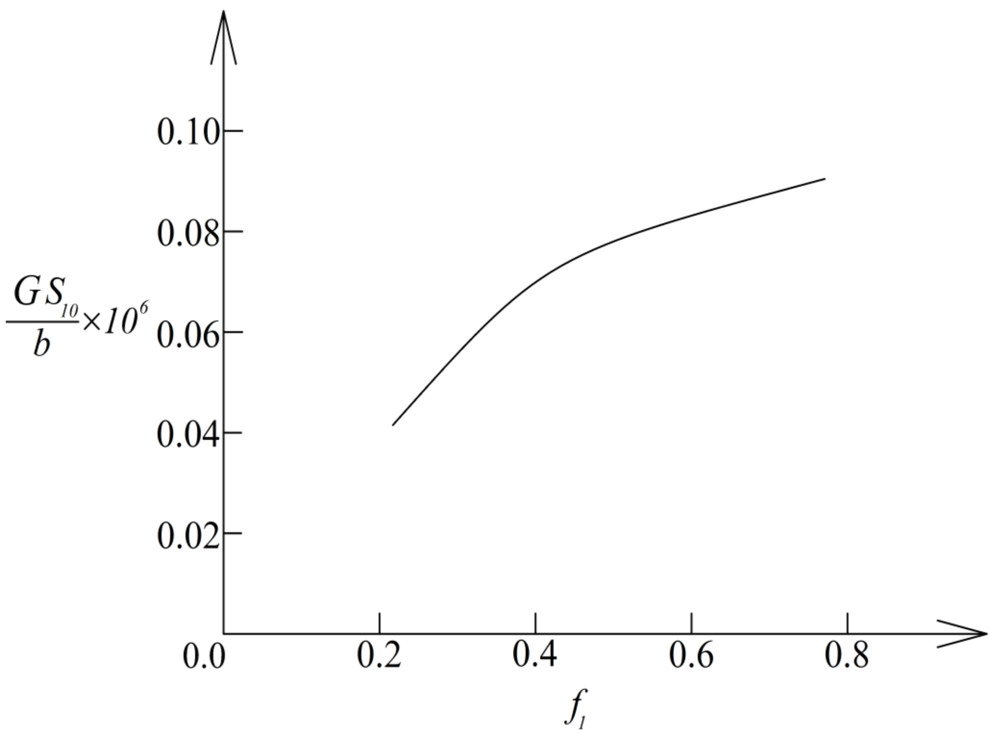

Figure 4. The non-dimensional SERR versus f_1 .

From the significant variation of the SERR in Fig. 4 it can be concluded that distribution of S_1 is an important factor for the delamination behaviour.

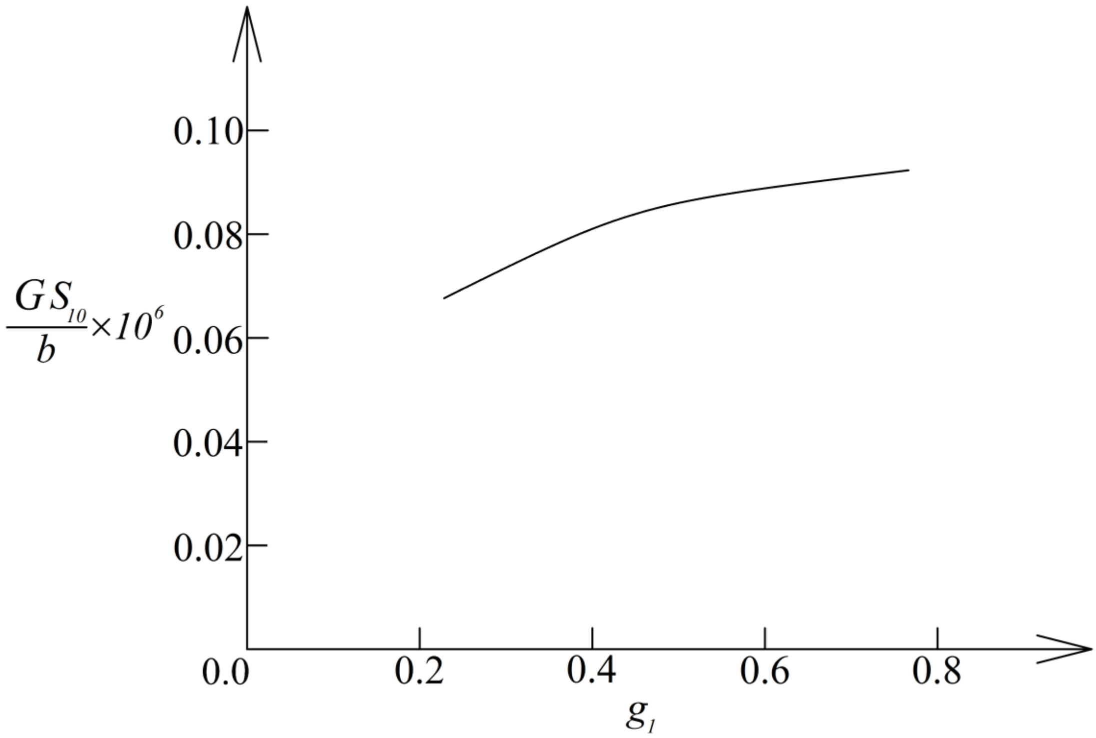

Figure 5. The non-dimensional SERR versus g_1.

Therefore, the S_1 distribution has to be considered in delamination analyses of statically indeterminate beams made of inhomogeneous layers. Another factor whose influence on the delamination is analyzed here is the continuous distribution of B_1 along the length of layer 1 of the beam structure (this distribution is presented by the parameter, g_1). The analysis yielded the curve shown in Fig. 5. It can be seen in Fig. 5 that the distribution of B_1 affects highly the SERR magnitude.

Conclusion

Theoretical study of delamination in a statically indeterminate multilayered beam structure on three supports is carried-out. The main point in the study is that the beam is under support displacement, i.e. delamination is due only to support displacement. Besides, the middle support of the beam is a spring. Thus, the influence of the spring on the delamination in a statically indeterminate beam is also analyzed. The effects of the support displacement and the spring on the delamination are taken into account when deriving the SERR. The study indicates that the SERR magnitude is strongly influenced by the value of the support displacement and the spring constant. A strong influence of the distribution of material properties along the layers length on the SERR magnitude is detected too.

Recommendations

The analytical approach presented in this paper can be recommended also for analyzing support displacements induced delamination in multilayered beams with more degrees of static indeterminacy.

Scientific Ethics Declaration

The author declares that the scientific ethical and legal responsibility of this article published in EPSTEM journal belongs to the author.

Acknowledgements or Notes

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