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On the Finite Element Modeling of Metal/Ceramic Functionally Graded Beams Subjected to Non-Uniform Static Bending

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Abstract: Currently, several approaches focused on the optimization of materials design and solutions to respond to the ecological and economic concerns through the development of novel materials and structures. In this connection, functionally graded materials (FGM) known as a new generation of composites with optimized mechanical properties which vary according to a continuous function-law to avoid interfacial debonding and stresses concentration were largely investigated. Accordingly, the main purpose of this work is to advocate a new way to simulate the mechanical behavior of simply supported FGM beams submitted to non-uniform static bending through a finite element modeling (FEM). The power-law function which governs the stiffness distribution of the used metal and ceramic materials are explicitly presented. In addition, a mesh sensitivity was established to assess the optimal mesh size of the modeled FGM based-beams. The simulation outcomes presented in the form stress and strain cartographies were correlated with a good agreement to the analytical results from the literature in term of qualitative and quantitative validations.

Keywords: Composite based-beams, FGM, Non-uniform bending, FEM simulations, Elastic behavior.

Introduction

Specific applications and environmental conditions lead manufacturers to choose a new material that guarantees economy, performance and extended service life (Bouzeboudja & Salem, 2023; Si Salem et al., 2022). Researchers are mostly applied to optimize solutions of already exciting materials (Ait Taleb et al., 2022; Ali Ahmed et al., 2022). However, in some cases scientific were leaded to develop and design completely new materials (Bagheri et al., 2023; Medjmadj et al., 2022). Consequently, functional gradient materials (FGMs) were appeared as a novel generation of composite materials with optimized mechanical and physical properties which transit according to a gradually continuous function law through the material thickness (Medjmadj et al., 2023; Shinde & Sayyad, 2022).

Regarding the experimental and manufacturing difficulties, FGMs based-structures have been deeply investigated in the literature using analytical formulation (Do & Lee, 2018; Garg et al., 2021; Katili & Katili, 2020). Accordingly, Medjmadj et al. (2023) introduced a theoretical approach to predict the flexural response of functionally graded beams validated using microscopic images and experimental observations. In addition,

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Katili & Katili (2020) proposed a novel beam element based on two-node approach by using the first-order deformation model applied on FGM based-beams. By using analytical and numerical methods, Vo et al. (2014) focused the post-elastic behavior of a fully composited beam with metal facesheets under tangential loading. Based on the discrete shear projection approach Katili et al. (2023) studied the buckling of functionally graded material multilayer structures by means of quadrilateral elements.

However, these analytical resolution methods lead to an underestimation of the exact response of such structures under loading. To overcome these convergence problems, simulation procedures using finite elements approaches were investigated in the literature (Do & Lee, 2018; Jongpradist et al., 2024; Sharma and Singh 2021). In this regards, Sharma & Singh (2021) used the explicit differential quadrature technique to simulate the features functionally graded beams with various boundary conditions. The authors outcomes were validated on the basis of several mathematical approaches issued form the open literature. In addition, Sadowski et al. (2015) presented several numerical models of FGM structural parts using finite element analysis and the mechanical response were confronted with previously advocated simplified analytical model. Chaker et al. (2021) established a numerical model able to solve the convergence problems due to the finite elements meshes of solid and shell members.

According to the above literature review, the numerical investigations on the flexural behavior of FGM based-beam using 3D simulations were few. In this connection, the aim of this work is to highlight the mechanical behavior of simply supported FGM based-beams loaded under uniform and no uniform bending, using fully 3D simulations on the basis on finite elements analysis. The originality of this work is to propose a numerical approach to resolve the differential equation governing the bending response of FGM beams, while considering the impacts of shear and buckling. Consequently, 3D modeling based on the finite element method is carried out on ABAQUS and validated by using previously published theoretical and numerical works. In order to demonstrate the effectiveness of the model in terms of strength and deformability prediction, results in terms of deflections, strains and normal/shear stress are highlighted and deeply discussed.

Problem Formulation

In order to develop a reliable numerical procedure, the analytically studied and designed functionally graded materials based-beam by (Katili & Katili, 2020) was simulated in the present investigation. The beam was made of a metal phase in the flexural zone with is functionally graded to reach a ceramic phase in the beam's top compressive zone, as shown in Figure. 1. Rectangular Timoshenko beam with L length was modeled, with a beam non-dimensional width b = 1 and L/h=16 ratio to represent performance in thin problems. It is important to notice that the two materials phases are entirely continuous without any bonding interface and slip behavior.

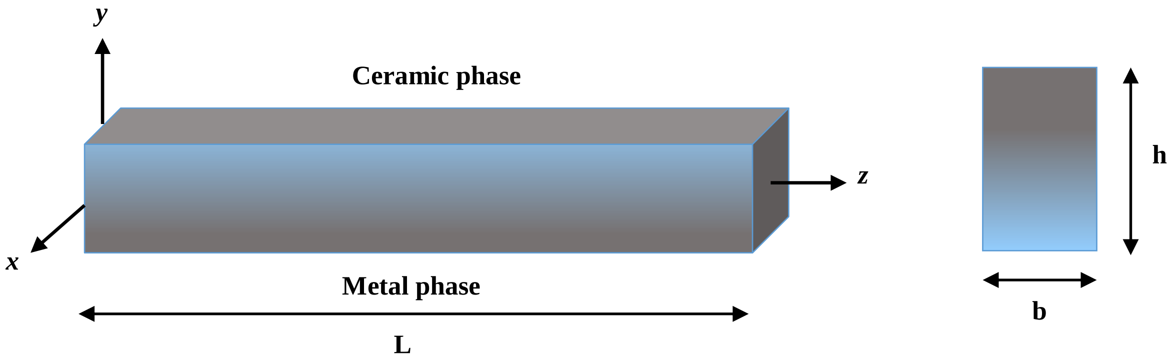


Figure 1. Geometric features of the simulated and modeled FGM based-beam.

The stiffens E(y) and density d(y) variation through the beam thickness was largely modeled in the literature using the power law functions in Equation. 1 and 2 respectively. Nevertheless, the Poisson's ratio is considered not varying according to the FGM beam height.

$$E(y) = E_c + (\frac{y}{h} + \frac{1}{2})^p (E_m - E_c) \quad (1)$$

$$d(y) = d_c + (\frac{y}{h} + \frac{1}{2})^p (d_m - d_c) \quad (2)$$

In which E_m and d_m are correspondingly the Elastic modulus and the relative-density of the metal phase. While E_c and d_c are separately the Elastic modulus and the density of the ceramic phase, the p-parameter governs the functionally transition from the metal to the ceramic phases. When p- tends to null value, the beam is full metal rich-composition. The full ceramic rich-composition was reached when high p-values were considered. The elastic behavior of the such FGM based-beams was generally evaluated using analytical approaches, such as high-order beam theories, which is based on many assumptions (Ait Taleb et al., 2017; Ait Taleb et al., 2020; Taleb et al., 2015). In this respect, a new three-dimensional numerical-way to model the power law function of the Equation 1 and 2 using finite element simulations in the below sections.

Simulation Procedure

Full 3D simulations and modeling were conducted on simply supported FGM based-beams using ABAQUS as used by Djenad et al (2022); Djenad et al. (2023); Salem et al.(2015); Taleb et al.(2015). The different steps followed in the modeling, starting from the geometric conditions creation to the results extraction are listed in the below section:

Geometry, Boundaries and Materials

The geometric model and the adoption of finite element models for the mesh are generated in three-dimensional (3D) space, while taking into account the actual behavior of the constituent material. Rectangular Timoshenko beam with non-dimensional span and unit width was simulated. While, L/h=16 was considered to represent performance in thin problems as studied by Katili & Katili (2020); Nguyen et al. (2013) and validated by Katili et al. (2020); Vo et al. (2014). It is significant to clarify that the two materials phases are modeled to be continuous without any bonding interface and tangential behavior during the transition.

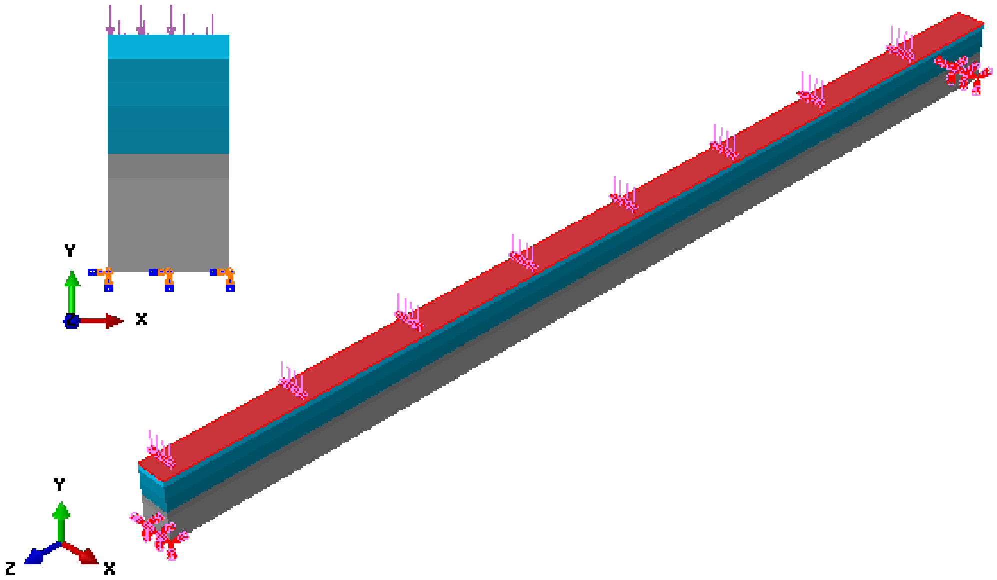


Figure 2. Geometric and boundaries conditions of the modeled FGM based-beam

The frontiers conditions and the loading configuration in the present simulations were correspondingly to the theoretical models of (Katili & Katili, 2020). In order to ensure the boundary conditions and the non-uniform bending load, reference points were assigned to analytical-rigid shells located on the at the position z = 0 and z = L. The interface among the reference points and the beam was ensured by the rigid body constraint of (ABAQUS, 2014) which may allocate regularly the support to all nodes of the modeled FGM based-beam depicted in Figure. 2. The rotations and axial displacement of the right reference point were allowed, while, the vertical translations in the direction of the Y axis was restrained. The general static step was assumed to prevent to convergence problems. The fully-static analysis method was implemented with tiny loading increments with a smooth-step way. The loading was applied on the ceramic phase surface as illustrated in Figure. 2

The numerical analysis is carried out on a ceramic-metal phases constituting a graded beam submitted to a non-uniformly scattered bending load. Considered materials are metal at the bottom surface of the beam with Young modulus Em = 70 GPa, density dm = 2702 kg/cm3 and poison's ratio υm = 0.3. The ceramic phase of the top beam surface was modeled with: Young modulus Ec = 200 GPa, density dc = 3960 kg/cm3 and poison's ratio υc = 0.3. Several configuration of the functional degradation were considered according to the p-parameter values, as depicted in Figure 3, which illustrates the variation of the bema stiffness through the thickness. In addition, full rich ceramic and metal beam were also modeled.

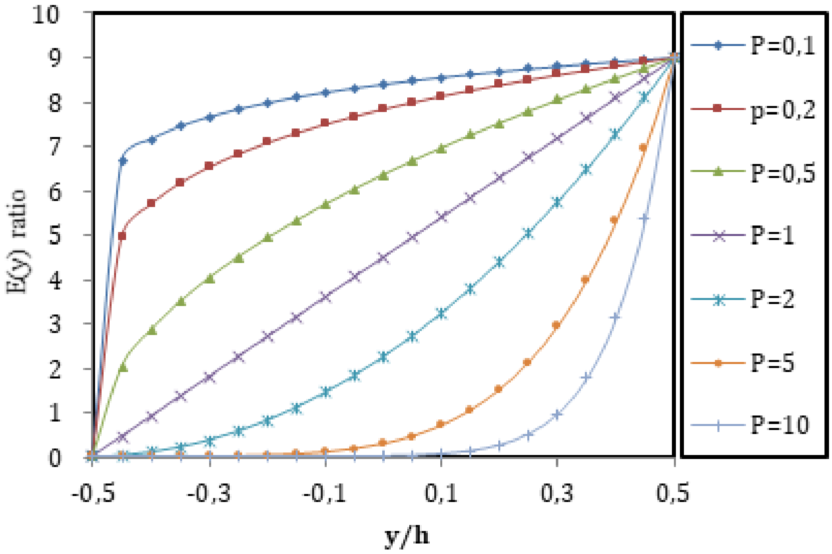


Figure 3. Evolution of the stiffness according to the FGM based-beam thickness.

Finite Elements and Stability

Hexahedral elements with first-order linear shape functions and 8 nodes were used to mesh the beam. These 3D HEX8, ABAQUS elements C3D8 are very efficient for linear geometry and offer the possibility of reduced integration (Djenad, et al., 2023; Si Salem et al., 2017). It gives also the ability to mesh the same part differently to highlight the most deformable surfaces. A mesh sensitivity study has been carried out for the beam, in order to define the optimum size of finite elements to best converge towards the exact solution. To this end, a comparative study was carried out with various dimensions, namely: 20 mm, 15 mm, 10 mm, 8 mm, 6 mm, 4mm, 2mm, 1mm and 0.5mm. Observation shows that the curves of the Figure 4 stabilized with a quasi-constant displacement at an approximately 10 mm value. As a result, the beam is meshed with 5mm finite elements in all three spatial directions, providing a highly refined mesh.

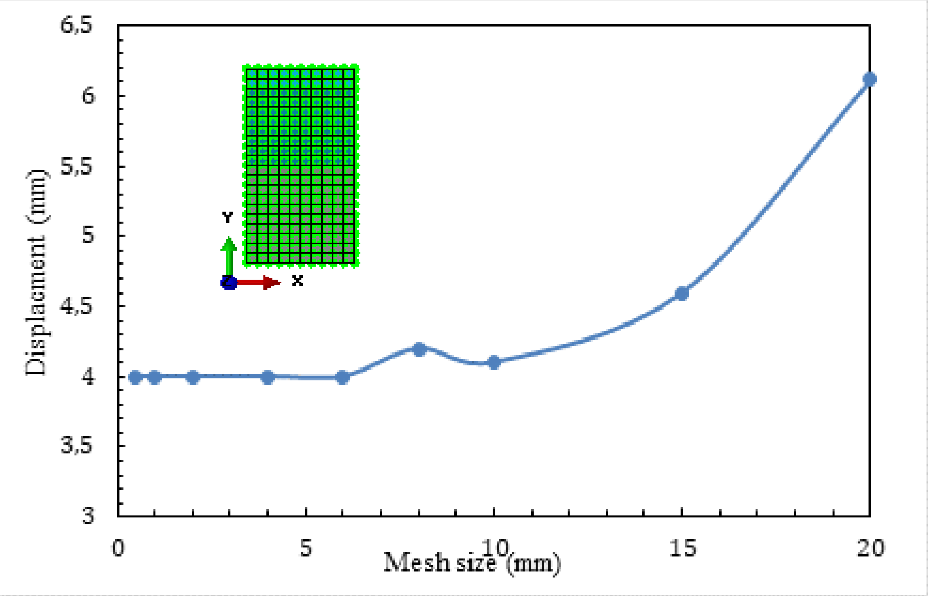


Figure 4. Mesh stability curve of the used C3D8 FE elements.

Numerical Outcomes

In this section, the obtained results from the numerical simulations carried out on the mechanical behavior of the FGM based-beams under bending were presented and interpreted. In this respect, all the findings and results in terms of displacements and stresses under mechanical loading are quantitatively and qualitatively discussed.

FEM Validation

The numerical procedure and the proposed finite element model are validated using analytical results issued from the literature review is carried out. Accordingly, a comparison between the present study results and the theoretical model is done in Figure 5.

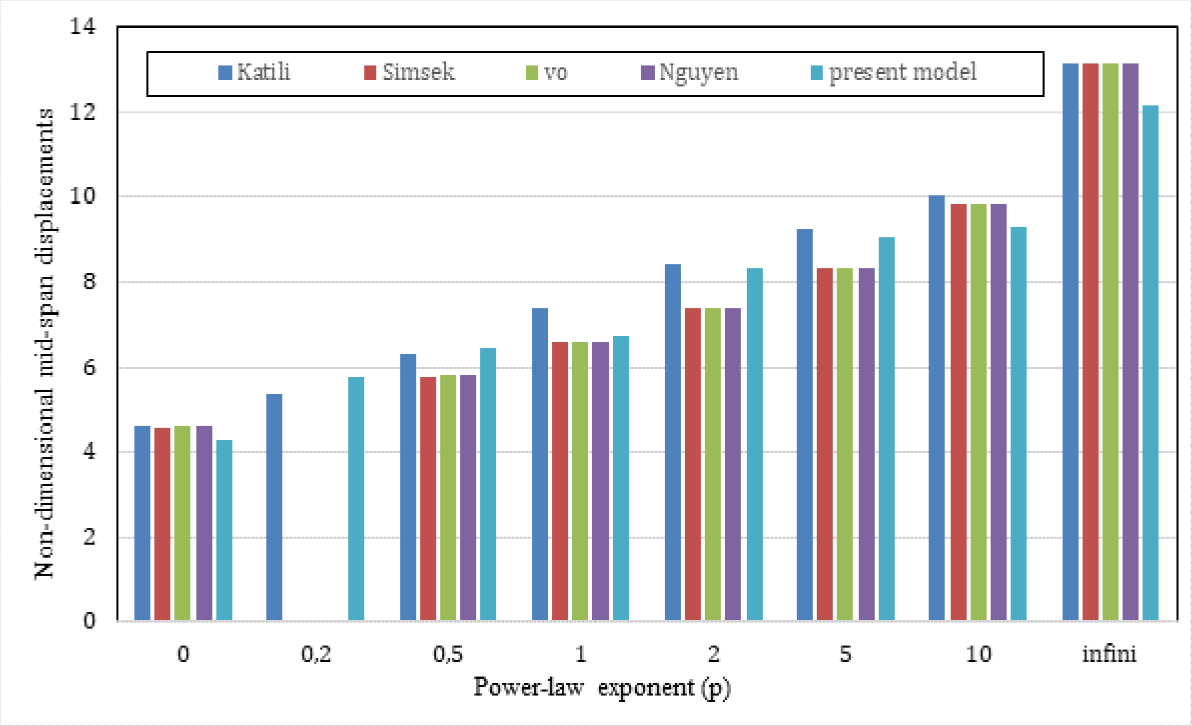


Figure 5. Histogram of non-dimensional mid span deflections comparison.

The obtained results in Table 1 and Figure 5 reveal a good agreement between the present numerical approach and analytical ones in terms of the general behavior of the FGM beams below flexural loading. Analysis of these results shows that displacement increases proportionally with the material p-parameter. The maximum error of the proposed model is less than 1% in all cases, validating the numerical models developed to predict the response of FGM beams under static bending. Physically this error is due to the perfect conditions of the analytical formulation in terms of geometric and loading values.

Mid-Span Deflections Evolution

The FGM beam non-dimensional displacements under mechanical loading for various p-parameters according to the z/L ratio are shown in Figure 6. From these curves, one can conclude that the displacement increases with the material p-parameter, and the ultimate value was reached at the beam mid-span. As the p-value increases, the transition from the ceramic to the metallic phase becomes higher, giving the FGM beam low stiffness which leads to high displacement values. In terms of qualitative observation, the cartographies of displacement evolution through the functionally graded material based-beam are shown in Figure 7.

From the mappings, it is observed that the maximum displacement for all cases of p-parameters appears at the beam mid-span, then propagates respectively towards the simply-supports, the displacement value was null at the supports. The displacement is greater for full-ceramic beams corresponding to p = infinite with 12.216 mm, compared to one of the full-metal beams (p=0) with 4.257 mm. This is due to the influence of the Young's modulus, which is higher for the ceramic phase than for the metal one. Based on the obtained outcomes in terms of displacement, it can be concluded that the suggested model is capable to predict the response of FGM beams.

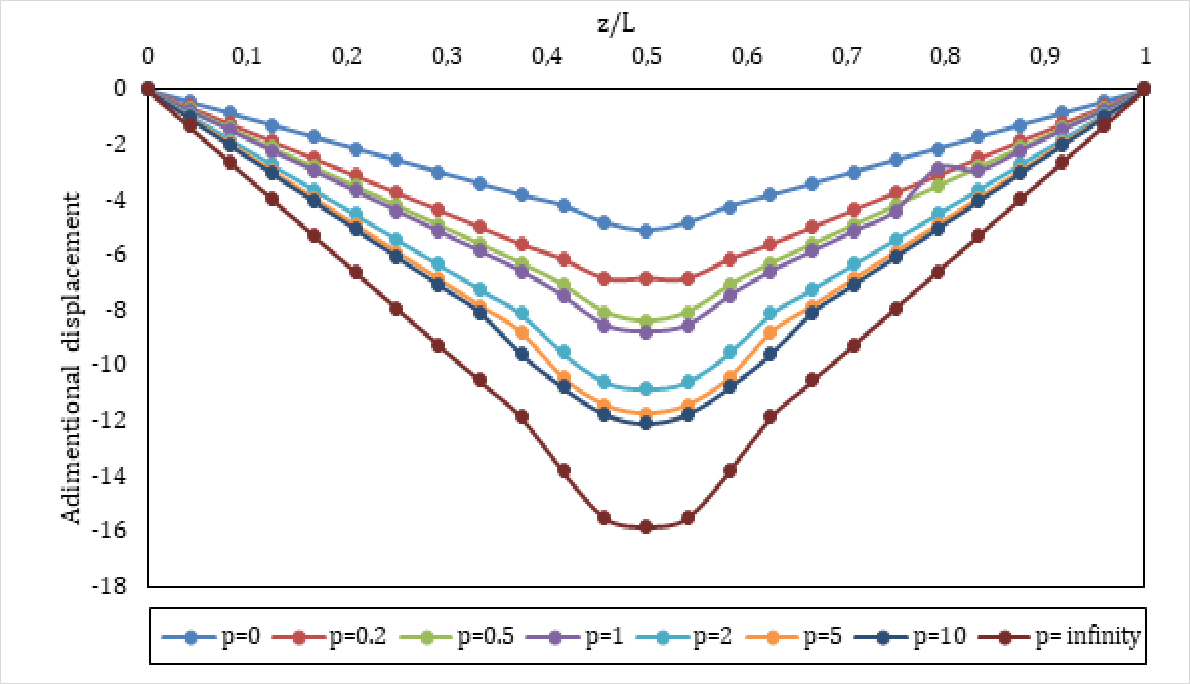
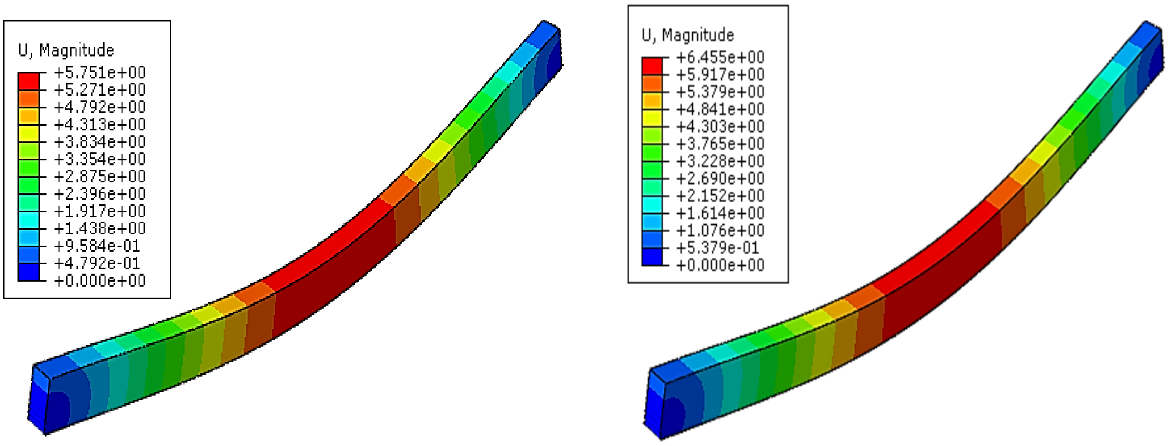


Figure 6. Non-dimensional deflections according to the beam span.


a)

b)

Figure 7. Cartography maps of non-dimensional deflections according to the beam span: a) p = 0.2; b) p = 0.5

Conclusion

In this work, a numerical approach is developed under FEM ABAQUS code to propose and validate a model susceptible to resolve the behavior governed by the power law function of FGM based-beams under non-uniform bending. Consequently, the various achieved outcomes enabled us to list the below conclusions:

- The established numerical model predicts with a satisfactory agreement the mid-span displacements of the FGM based-beam, through confrontation with analytical models available in the scientific literature;
- By increasing the p-parameter value of the power law function, great degradation from the ceramic matrix to the metal core is observed., which confers to the FGM beam low stiffness and accordingly large displacements;
- As the power-law exponent p-value decreases, the functionally degradation tends to a smooth transition from the ceramic phase to the metal matrix, conferring to the FGM beam low strengths and stresses;

Recommendations

The critical analysis of the obtained outcomes allowed us to identify future aspects for improving and overcoming the limitations of the present work. Indeed, the further perspectives should be formulated to take into account the effect of material porosity when evaluating the stiffness of the FGM based-beam.

Scientific Ethics Declaration

The authors declare that the scientific ethical and legal responsibility of this article published in EPSTEM Journal belongs to the authors.

Acknowledgements or Notes

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