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A Numerical Approach to Predict the Flexural Response of Simply Supported Nonhomogeneous and Non-Slender Graded Beams

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Abstract: In recent years, one promising avenue to optimize material design is the development of functionally graded materials (FGMs), which represent a new generation of composites with tailored mechanical properties. These properties vary continuously according to a function-law, allowing FGMs to mitigate issues like interfacial debonding and stress concentration. Accordingly, this study focuses on simulating the mechanical behavior of simply supported FGM non-slender beams under bending using finite element modeling (FEM). The numerical procedure and the loading setup as well as the implementation the power-law function which governs the stiffness distribution of the used metal and ceramic materials are explicitly presented. In addition, an optimal mesh size is determined for the modeled FGM beams. The numerical results show the effect of material exponent index and beam span ration on displacements and stresses distributions. The validated FEM-tool developed in this work provides a reliable means for estimating the elastic flexural response of non-slender graded beams under bending.

Keywords: Non-slender beams, Functionally graded materials, Three-point bending, Finite element approach, Mechanical properties.

Introduction

Functionally graded materials (FGMs) constitute a new category of nonhomogeneous composite materials made of different material components. Their microstructure and composition gradually and continuously vary to optimize the mechanical and thermal performance of the structures. Scientific research has focused on developing their mechanical and thermal properties (Ameryan et al., 2020). The concept of FGMs was invented in 1984 by Japanese scientists were looking for a material capable of withstanding high temperatures for space applications (Mahamood et al., 2017). FGMs possess a special structure, known for their lightweight, strength and durability, as well as their ability to combine contradictory thermomechanical properties in a single structure. By combining different materials, FGMs enhance both mechanical and thermal properties. Notably, FGMs exhibit two contrasting properties: conductivity and thermal insulation (Medjmadj et al., 2022; 2023).

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The scientific literature shows that the mechanical behavior of FGM beams has been extensively investigated and analytically solved the differential equation which governs the behavior of beams in bending (Ait Taleb, et al., 2017; 2020). In this connection, Ktili, Irwan and Katili (2020) presented a study which consists of determining the mechanical characteristics of an FGM beam. Vo et al. (2013) established a static analysis of an FGM beam using a refined shear deformation theory. In addition, Garg et al. (2020) made a comparative study of the bending of FGM sandwich beams composed of different materials variation laws using refined layer theory. The difficulty of implementing these materials, along with environmental and economic conditions, has prompted scientific researchers (Zhou et al., 2021; Li. et al.; 2021). To analytically investigate resolving mechanical problems associated with structures designed using FGM materials. Compared to analytical methods, numerical investigations are few. Therefore, simulation and numerical analysis were currently prioritized. These approaches play crucial role in studying complex mechanical behaviors (Djenad, et al,. 2022, 2023). Additionally, these simulations are more cost-effective than extensive laboratory testing with optimized number of trials.

To address these convergence issues, the objective of this study is to analyze the mechanical behavior of simply supported and non-slender FGM beams subjected to bending. We achieve this through a combination of analytical modeling and numerical simulation using the finite element method. The novelty of this approach lies in proposing a numerical solution that accounts for the differential equation governing the bending behavior of FGM beams while considering shear and warping effects. Specifically, 3D modeling using (Abaqus, 2014) were performed which is then validated against previous research. This validation demonstrates the model's effectiveness in predicting strength and deformability of the studied structures.

FEM Procedure

In this study, an analytically studied beam made of FGMs based on (Katili & Irwan Katili, 2020)'s research will be simulated. Indeed, a rectangular Timoshenko beam, with a width b = 1 (unit), length-to-height ratio, L/h = 4 is modeled to represent the performance in a simply supported ceramic-metal FGM beam subjected to a uniformly distributed load f_0. The composting materials consist of the metal at the bottom and ceramic on the upper surface of the beam. The lower surface of the beam has an elastic modulus E_m = 70 GPa, and a poisson's ratio υ_m = 0.3, while the upper ceramic surface has an elastic modulus E_c = 200 GPa, and a poisson's ratio υ_c = 0.3. The bouadary and loafing configutation of the simulated beam is given in Figure 1.

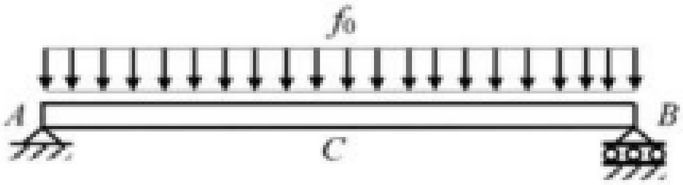

Figure 1. Boundary and loading conditions of the FGM beam.
A : Simple support ; **B** : Non-slender support

Model for FGM Beam Materials

A parametric study is carried out to study the effect of varying the number of layers using the power-law with different value of p and the exponent laws given by the following equation according to following equations. The parameters sought are the density and Young's modulus for each material.

$$E(z)_{p-FGM} = E_m e^{\beta(\frac{z+H}{2})} \tag{1}$$

$$\beta = \frac{1}{H}\ln(\frac{E_c}{E_m}) \tag{2}$$

Table 1. and Figure 2 show the variation of densities and the Young's modulus in the modeled beam according to the p values and the z/h position.

Table 1. Densities (D: kg/m3) and Young's modulus (E: Mpa) according z/h using the power-law function.

P/ z		**1**	**2**	**3**	**4**	**6**	**7**	**8**	**9**
0	**D**	3960	3960	3960	3960	3960	3960	3960	3960
	E	200	200	200	200	200	200	200	200
0.2	**D**	2702	3531.970	3655.385	3735.919	3847.134	3889.662	3926.848	3960
	E	70	155.768	168.521	176.844	188.337	192.731	196.574	200
0.5	**D**	2702	3146.77	3331	3472.364	3696.536	3791.456	3878.751	3960
	E	70	115.962	135	149.608	172.774	182.583	191.604	200
1	**E**	2702	2859.25	3016.5	3173.75	3488.25	3645.5	3802.75	3960
	D	70	86.25	102.5	118.75	151.25	167.5	183.75	200
2	**D**	2702	2721.656	2780.625	2878.906	3193.406	3409.625	3665.156	3960
	E	70	72.031	78.125	88.281	120.781	143.125	169.531	200
5	**D**	2702	2702.038	2703.228	2711.329	2821.972	3000.529	3347.239	3960
	E	70	70.004	70.127	70.964	82.398	100.850	136.678	200
10	**D**	2702	2702	2702.001	2702.069	2713.441	2772.842	3347.239	3960
	E	70	70	70	70.007	70.182	77.320	104.201	200
∞	**D**	2702	2702	2702	2702	2702	2702	2702	2702
	E	70	70	70	70	70	70	70	70

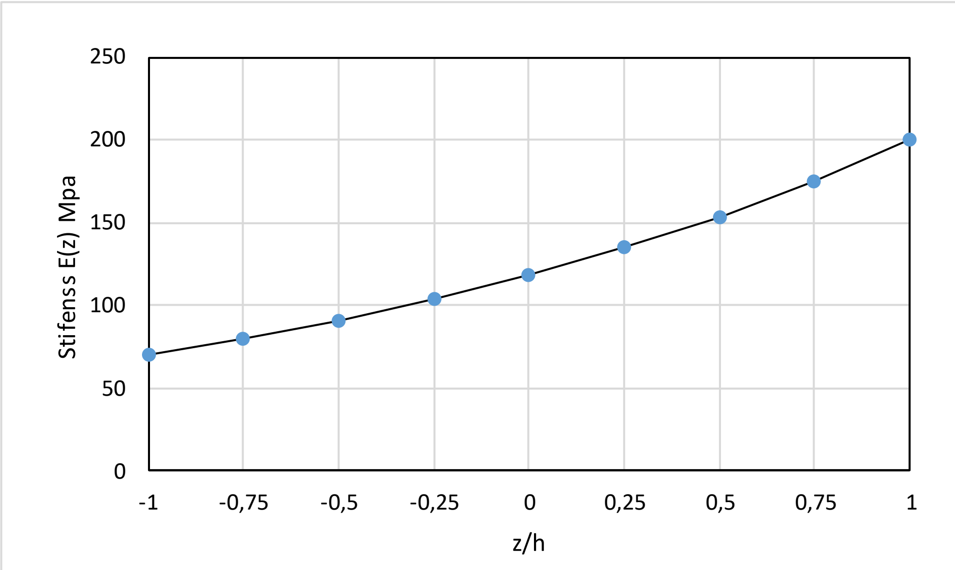

Figure 2. Stiffness variation in the FGM beam modeled using exponent-law function

Results and Discussion

In this section, a presentation and interpretation of the obtained results from numerical simulations are provided. The mechanical characteristics of the materials are mentioned in the numerical simulation section, the geometry variant considered is h *x* b *x* L corresponding to adimensional 2 *x* 1 *x* 8. Accordingly, all the obtained results in terms of displacements and stresses under mechanical loading are discussed and presented in the form of graphs and tables.

Calibration of the Numerical Model

The numerical procedure and the proposed finite element model are validated by a comparison with the experimental ones. A comparison between the test results and the theoretical model is deemed necessary to enhance the model. These results are presented in the Table 2. Based on the results, a good agreement emerges between the numerical and analytical outcomes regarding the overall behavior of beams subjected to bending loading. Indeed, the ultimate error rate between the adimensional mid-span displacement both approaches not exceed 0.23%. The displacement of the FGM beam increases linearly with the power-law exponent p. Additionally, the maximum error of the proposed model remains below 1% across all cases, thereby validating the accuracy of the numerical models used to predict the bending response of FGM beams.

Table 2. Comparison of numerical and theoretical adimensional mid-span displacement.

p-parameter value	**0**	**0,2**	**0,5**	**1**	**2**	**5**	**10**	∞
(Katili & Katili, 2020)	5,268	6,024	6,535	7,460	8,362	9,508	11,305	15,052
(Nguyen &Thai, 2013)	5,268	-	6.535	7,464	8,370	9,510	11,297	15,052
(Simsek 2009)	5,149	-	6.403	7,313	8,194	9,307	11,055	14,713
(Vo et al., 2014)	5,268	-	6,535	7,464	8,369	9,515	11,307	15,052
Present model	5,722	6,988	7,661	8,627	9,571	10,82	14,01	17,21
Error (%)	0.08	0.16	0.17	0.15	0.14	0.137	0.239	0.143

Displacements Distribution

Figure 4 illustrates the evolution of the displacements distribution of the FGM beam under mechanical bending load, with varying stiffness according to the law of rigidity variation and depicts the variation of deformations (displacements) as a function of x/L for P-FGM beams under different power-law exponent p. Indeed, when the value of p increases, the transition from the ceramic to metallic phase becomes rapid, resulting in low rigidity for the FGM beam and significant displacements. In order to validate the numerical results already obtained,

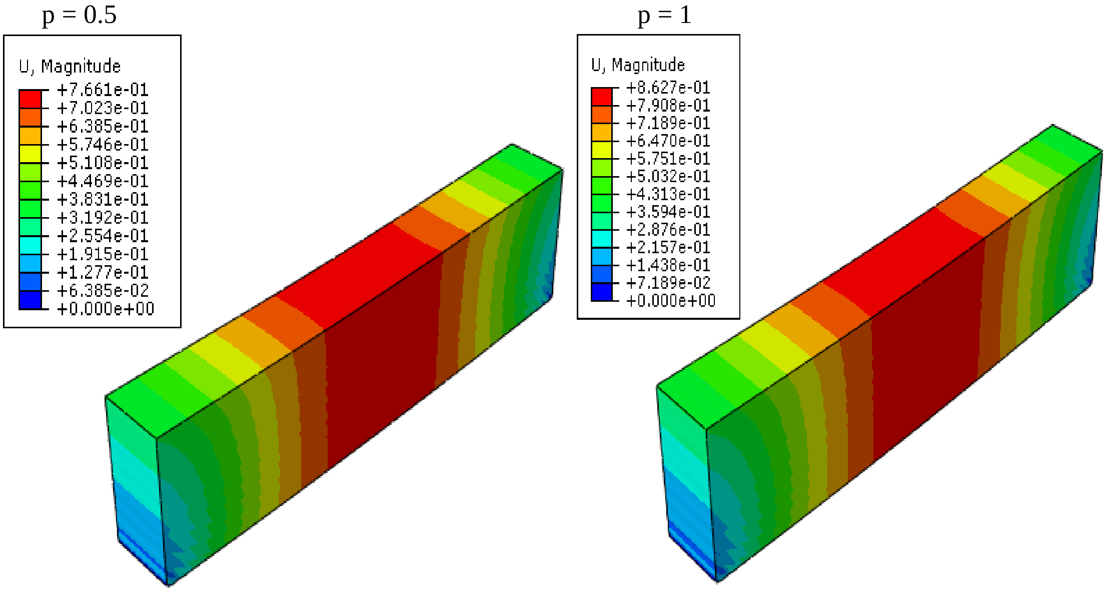

Figure 4. Cartography maps of displacements in longitudinal direction of non-slender FGM beam.

According to Figure 4, the maximum displacement for all variants appears in middle of the beam (at mid-span), then propagated respectively towards the supports, the value of the latter turns to zero at the supports. The displacement is greater for entirely ceramic beams which is equal to 17.21 mm, compared to the displacement of entirely metal beams which corresponds to 5.722 mm. This is due to the influence of the Young's modulus which is high for ceramic compared to that of metal. According to these results, which align with the analytical finding reported in scientific literature, this numerical model enables the prediction of the response of FGM beams subjected to bending.

Evolution of Normal Stresses

In this section, the variation of stresses according to the thickness of the FGM beam will be investigated. To achieve this, the same procedure as for displacement evolution will be employed, with the parameter p varying across the beam thickness. Figure 5, illustrates the evolution of the Von Mises stress distribution within the beam under mechanical loading for different parameter coefficients.

According to the evolution of normal stresses illustrated in Figure 5, the parameter p has a significant effect on the normal stresses. The stress ratio decreases as a function of p, reaching zero at the most tensile and compressive fibers. As p increases, the transition from the ceramic to the metallic phase becomes very rapid, resulting in reduced strength for the FGM beam and consequently lower stress levels.

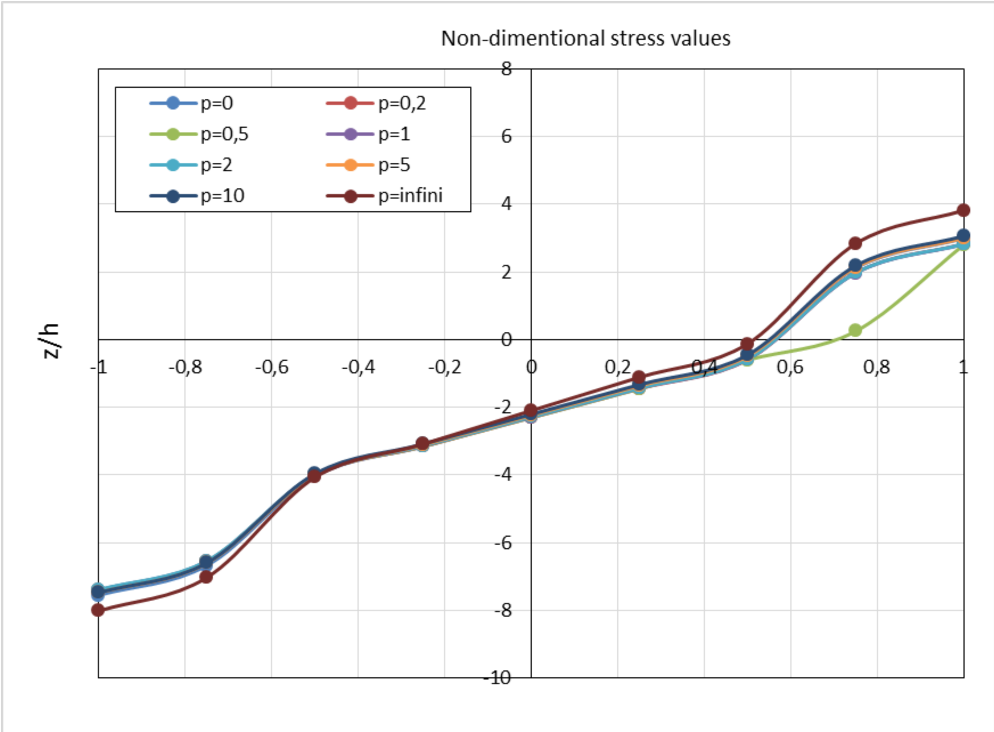

Figure 5. Evolution of normal stress according to the non-slender FGM beam thickness

The morphological distribution of the normal stresses in the FGM beams is represented using the cartography maps in the Figure 6. According to Figure 8, the maximum stress for all p variants appears at the supports of the beam and then propagated towards the beam's span; its value tends to diminish at the beam's mid-plane in the direction of its thickness. In the case of the FGM beams, the numerical displacements were marginally less than the analytical ones, this was caused by the geometric non-linearity conditions of loading application under the numerical procedure. as well as the convergence of the finite element model to the reliable solution.

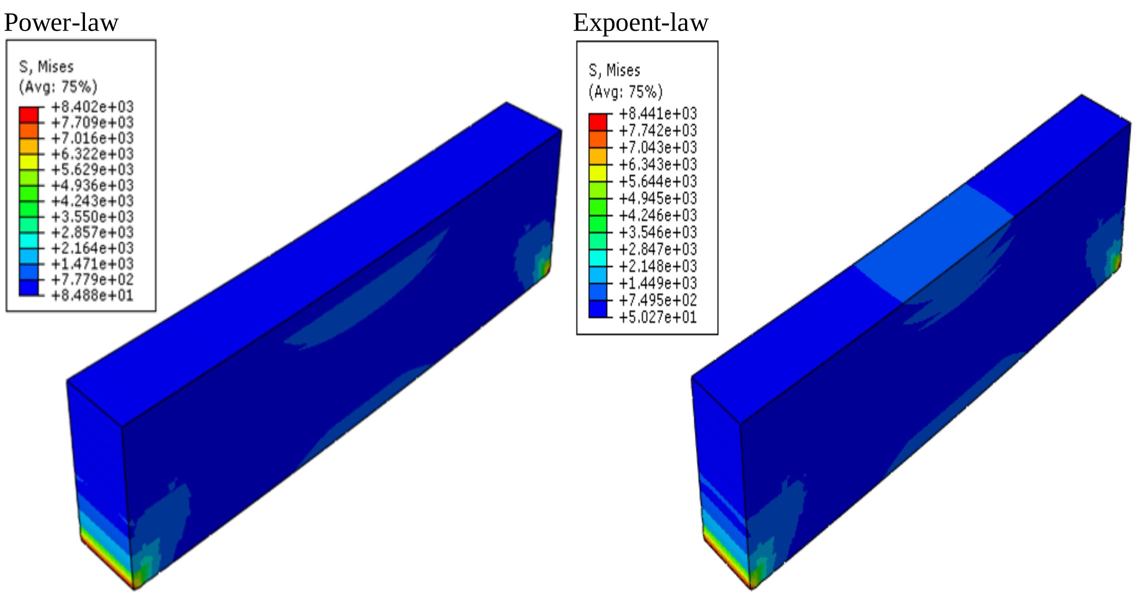

Figure 6. Cartography maps of normal stress according to the non-slender FGM beam thickness.

Conclusion

In this study, we conducted a numerical investigation using the ABAQUS code. Our focus was on the finite element method, aiming to propose and validate a model capable of analyzing the behavior of a functionally graded material (FGM) beam under bending. The key findings from our work are as follows:

- The established numerical model exhibited excellent agreement with analytical models from existing scientific literature when predicting mid-span displacements of the non-slender FGM beam.
- As the parameter p increases, the transition from the ceramic to metallic phase within the FGM beam occurs rapidly. Consequently, the beam exhibits low rigidity, resulting in significant displacements.
- Conversely, a good correlation between the results predicted by the proposed models when using power-law and exponential-law functions in terms of stresses values and arrangement.
- The normal stress values decrease proportionally to the p-parameter values. Nullity stress is observed on the center of gravity of the beam's transversal section. While, the stress achieves their ultimate values on the tensile and compressive zone;

Recommendations

In order to improve the findings of the present work, the authors advocate to introduce in the finite element simulations the real behavior of the constitutive materials namely: metal and ceramic to take into account the post-elastic phase. Consequently, predicting the failure and collapse mode of the studied non-slender FGM beams.

Scientific Ethics Declaration

The authors declare that the scientific ethical and legal responsibility of this article published in EPSTEM Journal belongs to the authors.

Acknowledgements or Notes

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