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Spectrum Estimation of Spatial Velocity Component Pulsations

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Abstract: One of the objectives in the study of turbulent flows is to determine the spectral features of flow parameters. Typically, the longitudinal and transverse components of the velocity vector are parameters for which spectral analysis is carried out. In the present study, the spectrum of pulsations of a turbulent velocity sampled in the spatial domain is considered. An application with the Reynolds number of 3900 is being examined for a turbulent flow past a 3D cylindrical surface. Numerical simulation is performed using ANSYS Fluent commercial code and based on the Navier-Stokes equations. The spatial signal is sampled on the centerline downstream of the flow. The energy of the transversal component of the velocity vector is analyzed using Fourier transforms of a signal sampled in the spatial domain. Two approaches are considered to study the spectral properties of the signal. The traditional Energy Signal Density (ESD) estimation is analyzed using Parseval's theorem. The alternative Energy Signal Spectra (ESS) estimation approach is based on the local signal energy definition. The energy spectra performances are compared to the "–5/3" law of A.N. Kolmogorov.

Keywords: Cylinder, Navier-Stokes equations, Turbulence, Spatial spectrum.

Introduction

Typically, the longitudinal and transverse components of the velocity vector of the turbulent flow are parameters for which spectral analysis is carried out. The spectral characteristics of the flow depend on the Reynolds number and the object geometry. For low Reynolds numbers (about 10^3), it is possible to use the Direct Numerical Simulation (DNS) based on solution of the Navier-Stokes equations (without involving turbulence models). A 3D cylindrical surface is a canonical object for numerical simulation. The potential to replicate different physical aspects of the flow, such as wake vortex generation, separation of the laminar boundary layer, and the creation of shear layers (including the Karman vortex street), explains the interest in this task. The results of the numerical simulation for Re =3900 can be found, for example, in Wissink and Rodi (2008). For the given value of the Reynolds number, some studies are also based on the Reynolds-averaged Navier-Stokes equations (Parnaudeau et al., 2008). To describe the Energy Spectral Density (ESD) of isotropic turbulent flow, one can use the "-5/3" law of Kolmogorov (1991). It should be noted that the "-5/3" law was obtained for the spatial spectrum of the longitudinal and transverse components of the velocity vector. In most studies, the "-5/3" law is also used to estimate the spectrum of a signal sampled in the temporal domain.

This paper presents an alternative approach for estimating the energy signal spectrum. As an illustration, estimates of the energy spectrum of the transverse velocity vector component are given for the flow near a 3D circular cylinder at the Reynolds number of 3900. Analysis of spectral characteristics is carried out for data obtained as a result of numerical modeling of the flow in the ANSYS Fluent package based on the Navier-Stokes equations without involving turbulence models. The spectral characteristics of the flow are estimated in the MATLAB code using the discrete Fourier transform of the signal sampled in the spatial domain.

- Selection and peer-review under responsibility of the Organizing Committee of the Conference

Methodology for Evaluating of Spectral Characteristics

For a continuous spatial real signal $u(x)$, the signal energy is defined by the expression

$$E_\infty = \int_{-\infty}^{\infty} U(x)\, dx, \tag{1}$$

where $U(x) = u^2(x)$. Spectral analysis for a spatial-limited signal (in the range $-X/2 \le x \le X/2$) is carried out using the Fourier transform (Lathi & Ding, 2010):

$$\hat{u}(k) = \int_{-X/2}^{X/2} u(x)\, e^{-2\pi i x k} dx,$$

Where k is the wave number.

Using Parseval's theorem (Prandoni & Vetterli, 2008). Allows for the determination of the signal ESD (Lathi & Ding, 2010):

$$E_D(k) = |\hat{u}(k)|^2. \tag{2}$$

In references are also used the signal PSD estimation (Lathi & Ding, 2010):

$$P_D(k) = \frac{1}{X}|\hat{u}(k)|^2. \tag{3}$$

From expressions (2) and (3) it is follows that the ESD and PSD characteristics of the signal have the similar spectrum distributions and differ only in the amplitude. From (1), it follows that the signal energy E is evaluated by the integral of $U(x)$. Along with expressions (2) and (3), the Energy Signal Spectra (ESS) estimation was introduced in Kusyumov et al. (2023). To evaluate the energy characteristics of the time-sampled signal:

$$E_S(f) = \left|\widehat{U}(f)\right|, \tag{4}$$

Where

$$\widehat{U}(f) = \int_{-T/2}^{T/2} u^2(t)\, e^{-2\pi i t f} dt. \tag{5}$$

From the analysis of expressions for ESD, PSD, ESS it follows that these characteristics are obtained using Fourier transforms of the function $u^m(x)$: $m = 1$ for ESD and PSD, whereas $m = 2$ for ESS. Note that to determine the spatial spectrum, one can introduce the Fourier transform for the spatial distribution of a spatial signal $u(x)$ (in the range $-X/2 \le x \le X/2$):

$$E_S(k) = \left|\widehat{U}(k)\right|. \tag{6}$$

Here $U(x) = u^2(x)$. The function $\widehat{U}(k)$ is defined by the spatial Fourier transform

$$\widehat{U}(k) = \int_{-X/2}^{X/2} U(x)\, e^{-2\pi i x k} dx. \tag{7}$$

Traditionally, the spectral characteristics of the ESD and PSD signals are aimed at analyzing the spectrum of velocity fluctuations. The obtained spectrum is usually compared with the spectral distribution determined by the K41 law of Kolmogorov. The K41 law was formulated by Kolmogorov (1991) within the framework of the statistical theory of turbulence. Based on this theory, Kolmogorov (1991) obtained the reference law "-5/3" for the spatial energy spectrum (in the inertial range of the spectrum):

$$EI_{ij}(k) \sim \alpha_{ij}\epsilon^{2/3}k^{-5/3}, \tag{8}$$

where ϵ is the turbulence energy dissipation rate. For isotropic flows $\alpha_{ij} = \alpha_i\delta_{ij}$, where δ_{ij} is the Kronecker symbol, $\alpha_1 = \alpha_2 = \alpha_3 = \alpha \approx 0.5$. Note that the dimension of the $EI_{ij}(k)$ coincides with the dimension of the function $E_S(k)$. Therefore, the spatial spectrum $E_S(k)$ can be compared with the K41 law.

From the K41 law, it follows that the reference spatial spectrum $EI_{ij}(k)$ of the signal energy is determined by the energy dissipation rate ϵ and the wave number k. Hence the resulting dimension of the reference estimation $EI_{ij}(k)$ is

$$[EI_{ij}(k)] = \frac{m^3}{s^2}.$$

One can determine also dimension of the $E_D(k)$ and $E_S(k)$ signal performances:

$$[E_D(k)] = \frac{m^4}{s^2}, \quad [E_S(k)] = \frac{m^3}{s^2}. \tag{9}$$

The $E_S(k)$ and $E_D(k)$ signal performances are used below to analyze turbulent flow properties obtained as a result of modeling the flow around a section of a circular cylinder.

Numerical Simulation Results

Numerical modeling was carried out for the Reynolds number $\mathrm{Re} = V_\infty d/\nu = 3900$, where V_∞ is the free stream velocity, d is the cylinder diameter, and ν is the kinematic viscosity coefficient. When carrying out calculations in the ANSYS Fluent code, the unsteady incompressible Navier-Stokes equations were used. Navier-Stokes equations were solved with second order discretization in space and first order in time. Longitudinal and transversal coordinates are denoted as x, y, and z. The distance from the inlet face to the cylinder axis was $5d$. The transversal dimension $l_z = \pi d$ was chosen in accordance with the recommendations of (Kravchenko & Moin, 2000; Maet al., 2000). At the inlet boundary, a uniform flow field was set with the velocity vector $(u, v, w)^T = (1{,}0{,}0)^T V_\infty$. On the sides of the computational domain (orthogonal to z coordinate), "symmetry" conditions were applied. The computational grid was built in the commercial ANSYS ICEM generator.

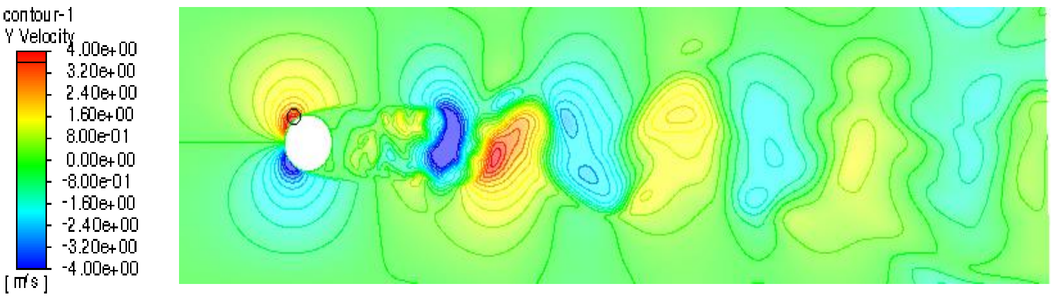

Figure 1. Instant field of the transversal velocity v in the symmetry plane.

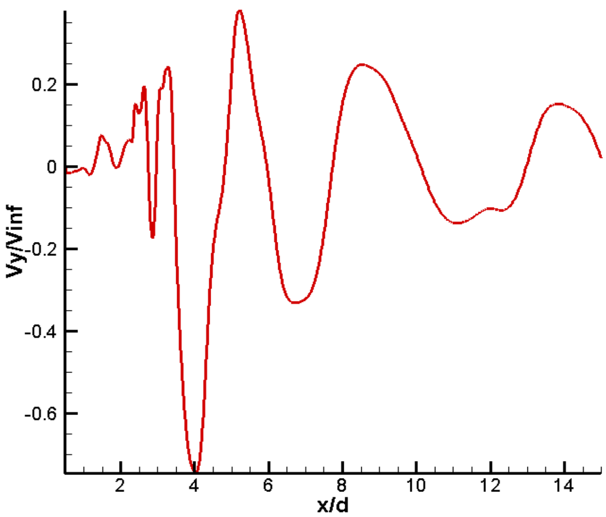

Figure 2. Instant distribution of the transversal velocity v in the symmetry plane.

The computational hexa-grid consisting of $16 \cdot 10^6$ elements was created. To resolve the boundary layer, the computational mesh was refined near the cylinder surface: the size of the first cell normal to the surface is about $10^{-3}d$. The time-step used in the simulation is $\Delta t = 0.0565\, d/V_\infty$. The period of integration corresponded to 100 cycles of vortex generation ($T \approx 500d/V_\infty$). Some other details of the simulation are presented in (Kusyumov et al., 2023). Figure 1 presents the distribution of the transverse component$v$of the velocity vector in the symmetry plane. From Figure1, it follows the inhomogeneities inside and outside of the recirculation zone. Figure 2 shows the spatial signal sampled at the center line (in the symmetry plane) downstream of the flow. The spatial distribution of the transversal velocity component in Figures 1 and 2 shows that small vortex structures are localized close to the cylinder surface ($x/d > 1$). Increasing the vortex structures dimension far from the cylinder surface is determined by the grow of the computational grid cells size and the dissipation of the turbulent energy process. Figure 3 shows the distribution of normalized functions $E_S(k)$ (ESS) and $E_D(k)$ (ESD) compared to the "-5/3" law.

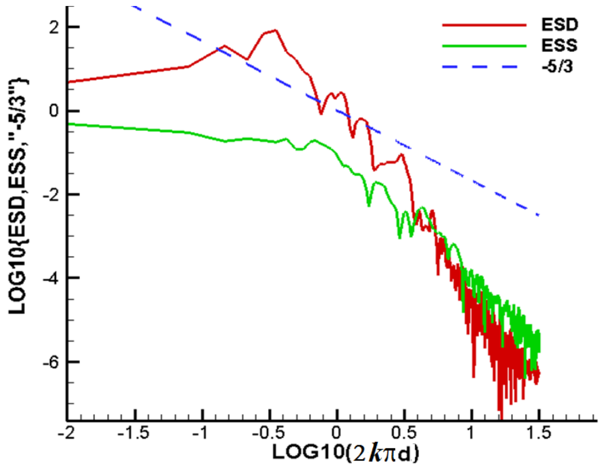

Figure 3. Spatial spectrum of the transversal velocity v.

In the inertial subrange, there are regions where the slope of both functions $E_S(k)$ (ESS) and $E_D(k)$ (ESD) curves corresponds to the "-5/3" law. Wherein, the gradient of the $E_S(k)$ curve in the inertial subrange is in better agreement with the "-5/3" law in comparison with the $E_D(k)$ gradient.

Conclusion

Three-dimensional flow around a section of a circular cylinder at Re = 3900 is studied numerically based on the unsteady incompressible Navier-Stokes equations using the Fluent commercial code. The main attention is paid to the spectrum of transverse flow velocity downstream of the cylinder. An alternate formulation (Energy Signal Spectrum) estimates the signal energy spectrum based on the square of the spatial signal function, in addition to the Energy Spectral Density (in the widely known formulation).The signal energy spectrum is compared with the "-5/3" law of Kolmogorov. Analysis of the numerical simulation results shows that the gradient of the Energy Signal Spectrum curve in the inertial subrange is in better agreement with the "-5/3" law in comparison with the Energy Spectral Density.

Scientific Ethics Declaration

The authors declare that the scientific ethical and legal responsibility of this article published in EPSTEM journal belongs to the authors.

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