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Analysis of Delamination in Statically Indeterminate Steric Frames Subjected to Fixed Support Rotation

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Abstract: The paper is devoted to the problem of delamination in multilayered steric frames. The attention is focussed on statically indeterminate load-bearing frame structures that are under fixed support rotation. External mechanical loading is not applied on the frames under consideration. Only the stresses and strains induced by the fixed support rotation causes delamination. The layers of the frame are made of structural materials that continuously inhomogeneous in longitudinal direction. Thus, the elastic properties of the layers vary smoothly along the length of the frame portions. Due to steric geometry of the frame, the fixed support rotation generates torsion and bending in the frame portions. Therefore, in this paper we analyze delamination behaviour under combined action of bending and torsion moments. First, the mechanical behaviour of the frame portions is studied by using the equations of equilibrium. Then the strain energy release rate (SERR) is determined by analyzing the frame compliance.

Keywords: Steric frame, Multilayered material, Delamination, Fixed support rotation

Introduction

Steric frames are used as load-bearing engineering structures in various applications in construction, machine-building, shipbuilding, car industry, etc. Multilayered materials are potential candidates for application in various structures (Finot & Suresh, 1996; Rzhanitsyn, 1986; Sy-Ngoc Nguyen et al., 2020; Tokova et al., 2016). When steric frames are made of multilayered engineering materials, delamination fracture behaviour becomes one of the most important issues with great influence on the safety and reliability of structures. For example, in multilayered steric frame structures, the delamination is a frequent cause of a variety of uncertainties like reducing the load-bearing ability, worsening the stability performance, increasing deformations, reducing the life of structures, etc. The rapid growth of a delamination crack under certain conditions even may cause sudden collapse of the entire frame structure. The studies of delamination in pane load-bearing structures (mainly beams) are well documented in the scientific literature (Dolgov, 2005; Hutchinson & Suo, 1992; Rizov, 2018a, Rizov, 2018; Rizov, 2019). However, this is not the case with delamination in steric frames. There are many issues that need exploration when dealing with delamination problem in steric frames.

In this paper, we analyze delamination in a steric multilayered frame structure that is statically indeterminate. The frame layers are with continuous material inhomogeneity along the length. It should be noted here that continuous inhomogeneity is considered since the use of continuously inhomogeneous materials grows fast in many areas of engineering (Gandra et al., 2011; Udupa et al., 2014). The frame is under rotation of the fixed support. Since the frame is statically indeterminate, the fixed support rotation induces stressed and strained state in the frame. Besides, since the frame is steric, the fixed support rotation generates bending, and bending and torsion in the different members of the frame structure. The SERR in the frame is determined by the compliance method. Analysis of the SERR is performed with purpose to evaluate the influence of the fixed support rotation, frame geometry and layer inhomogeneity.

- Selection and peer-review under responsibility of the Organizing Committee of the Conference

Model of a Steric Frame with Delamination

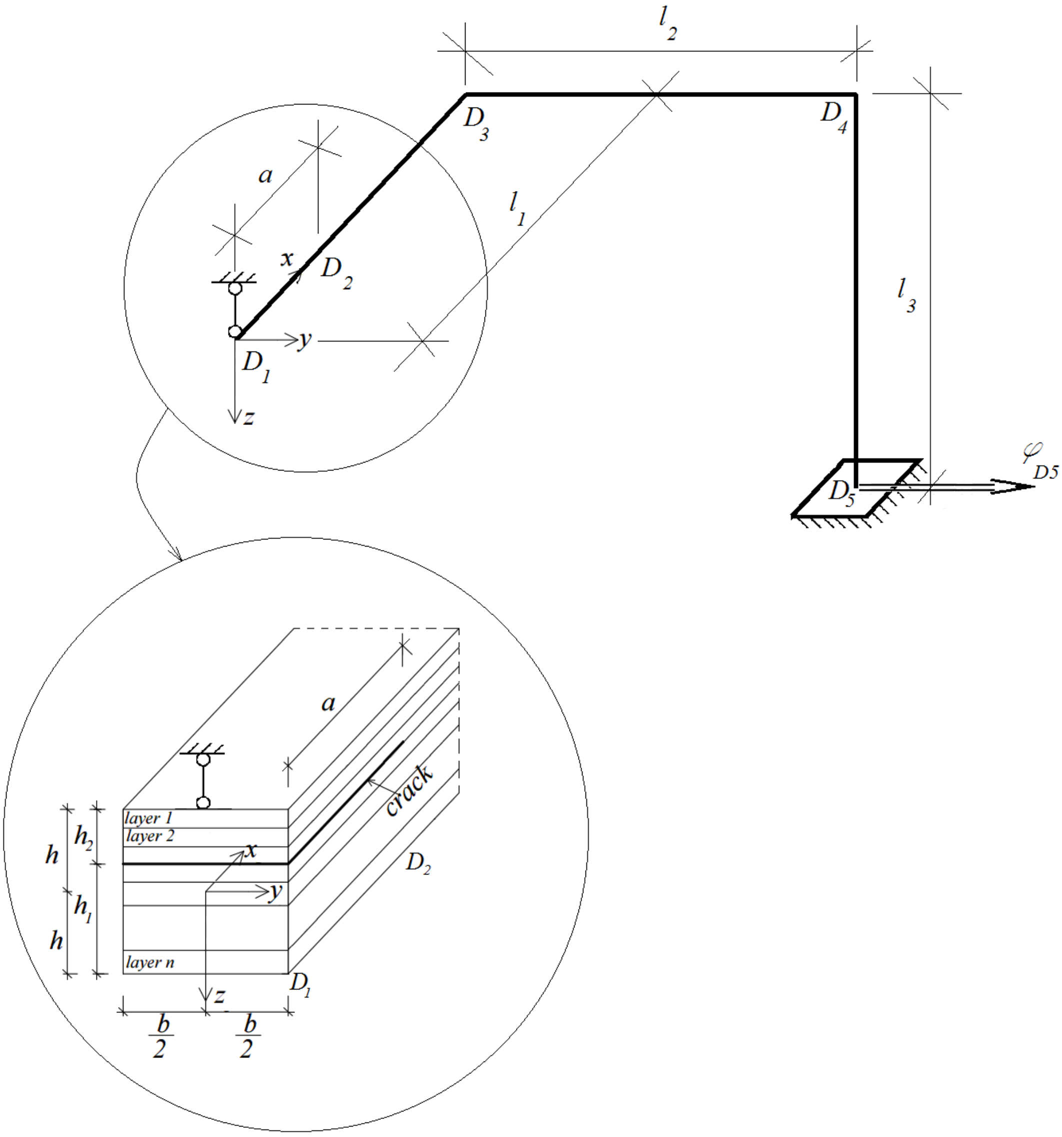

Figure 1. Steric frame under the fixed support rotation.

The steric frame structure in Fig. 1 is built-up by three members (two horizontal members, D_1D_3 and D_3D_4, and a vertical member, D_4D_5). The end, D_1 , of the member, D_1D_3, is supported by a vertical link, the lower end of the vertical member is fixed. Therefore, the frame represents a structure with one degree of static indeterminacy. The fixed support is under angle of rotation, φ_{D5}, as shown in Fig. 1. The delamination crack in portions, D_1D_2, of the frame has length of a. The SERR, G, for the delamination crack is obtained by formula (1) derived through the compliance method.

$$G = \frac{1}{2b}\left(M_{D5y4}^{2} \frac{dC_{MD5}}{da} \right) \tag{1}$$

where b is the beam width, M_{D5y4} is the bending moment in the fixed support along the rotation, C_{MD5} is the frame compliance. The latter is defined by formula (2).

$$C_{MD5} = \frac{\varphi_{D5}}{M_{D5y4}} \tag{2}$$

In order to apply formula (1), the static indeterminacy has to be considered first. This is done through the following equations. The angle of rotation, φ_{D5}, is written as

$$\varphi_{D5} = \int_0^a \kappa_{D1D2} \frac{1}{l_1} x dx + \int_0^{l_1} \kappa_{D2D3} \frac{1}{l_1} (a + x_2) dx_2 + \int_0^{l_2} \frac{T}{S} dx_3 + \int_0^{l_2} \kappa_{D3D4} \frac{1}{l_1} x_3 dx_3 + + \int_0^{l_3} \kappa_{D4D5} dx_4 + \int_0^{l_3} \kappa_{D4D5} \frac{1}{l_1} l_2 dx_4 \tag{3}$$

where κ_{D1D2}, κ_{D2D3}, κ_{D3D4} and κ_{D4D5} are the curvatures of the frame portions, l_1, l_2 and l_3 are the lengths (Fig. 1), T is the torsion moment in member, D_3D_4, of the frame, S is the stiffness in torsion (one can find the method for determination of S in Chobanian 1997). Formula (3) is obtained by the integrals of Maxwell-Mohr.

The stress, σ_j, in the layers of portion, D_1D_2, of the frame is

$$\sigma_j = E_j \varepsilon \tag{4}$$

where E_j is the modulus of elasticity, ε is the strain. The layers are inhomogeneous along the length. The distribution of the modulus of elasticity along the length is

$$E_j = E_{Lj} + \frac{E_{Qj} - E_{Lj}}{l_1^p} x^p \tag{5}$$

where E_{Qj} and E_{Lj} are the values of E_j in the points, D_1 and D_3, in portion, D_1D_3, of the frame, p is a parameter of the distribution, the axis, x, is shown in Fig. 1. The equilibrium of the elementary forces in the cross-sections of the frame portions is considered to formulate the following equations.

$$N_{D1D2} = \sum_{j=1}^{j=n_1} \iint_{(A_j)} \sigma_j dA, \tag{6}$$

$$M_{D1D2} = \sum_{j=1}^{j=n_1} \iint_{(A_j)} \sigma_j z dA, \tag{7}$$

$$N_{D2D3} = \sum_{j=1}^{j=n} \iint_{(A_j)} \sigma_{D2D3j} dA, \tag{8}$$

$$M_{D2D3} = \sum_{j=1}^{j=n} \iint_{(A_j)} \sigma_{D2D3j} z_2 dA, \tag{9}$$

$$N_{D3D4} = \sum_{j=1}^{j=n} \iint_{(A_j)} \sigma_{D3D4j} dA, \tag{10}$$

$$M_{D3D4} = \sum_{j=1}^{j=n} \iint_{(A_j)} \sigma_{D3D4j} z_3 dA, \tag{11}$$

$$N_{D4D5} = \sum_{j=1}^{j=n} \iint_{(A_j)} \sigma_{D4D5j} dA, \tag{12}$$

$$M_{D4D5y4} = \sum_{j=1}^{j=n} \iint_{(A_j)} \sigma_{D4D5j} z_4 dA, \tag{13}$$

$$M_{D4D5z4} = \sum_{j=1}^{j=n} \iint_{(A_j)} \sigma_{D4D5j} y_4 dA, \tag{14}$$

where n_1 and n are the number of layers in upper delamination arm and in the frame. The quantities, N_{D1D2}, N_{D2D3}, N_{D3D4} and N_{D4D5}, are the axial forces, M_{D1D2}, M_{D2D3}, M_{D3D4}, M_{D4D5y4} and M_{D4D5z4} are the bending moments in the frame portions. These quantities are

$$N_{D1D2} = 0 , \tag{15}$$

$$N_{D2D3} = 0 , \tag{16}$$

$$N_{D3D4} = 0 , \tag{17}$$

$$N_{D4D5} = R_{D1} , \tag{18}$$

$$M_{D1D2} = R_{D1} x , \tag{19}$$

$$M_{D2D3} = R_{D1}\left(a + x_2\right), \tag{20}$$

$$M_{D3D4} = R_{D1} x_3 , \tag{21}$$

$$M_{D4D5y4} = R_{D1} l_1 , \tag{22}$$

$$M_{D4D5z4} = R_{D1} l_2 , \tag{23}$$

where R_{D1} is the support reaction in the vertical link. Equations (3), (6) – (14) are used to determine the curvatures and the neutral axis coordinates in the frame portions, the strain in the centre of cross-section of portion, D_4D_5, of the frame, and the support reaction in the vertical link by the MatLab. Then the SERR is obtained by using formula (2).

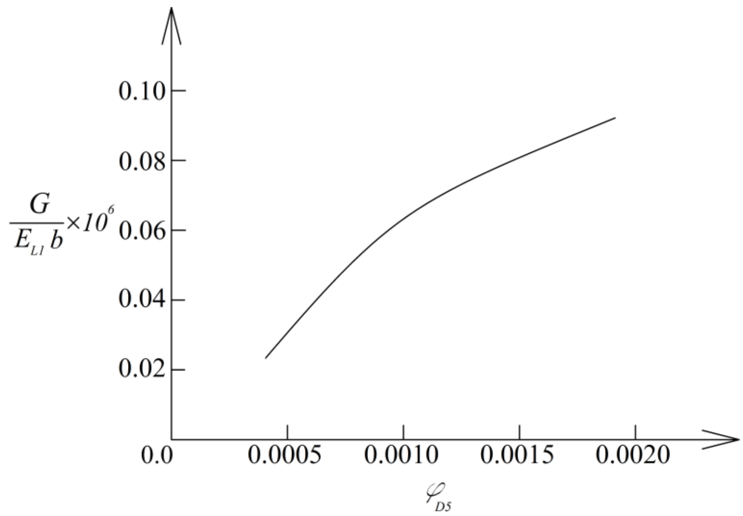

Figure 2. The non-dimensional SERR - φ_{D5} curve.

The SERR is checked-up by formula (24).

$$G = \frac{dU}{bda}, \tag{24}$$

where U is the strain energy in the frame.

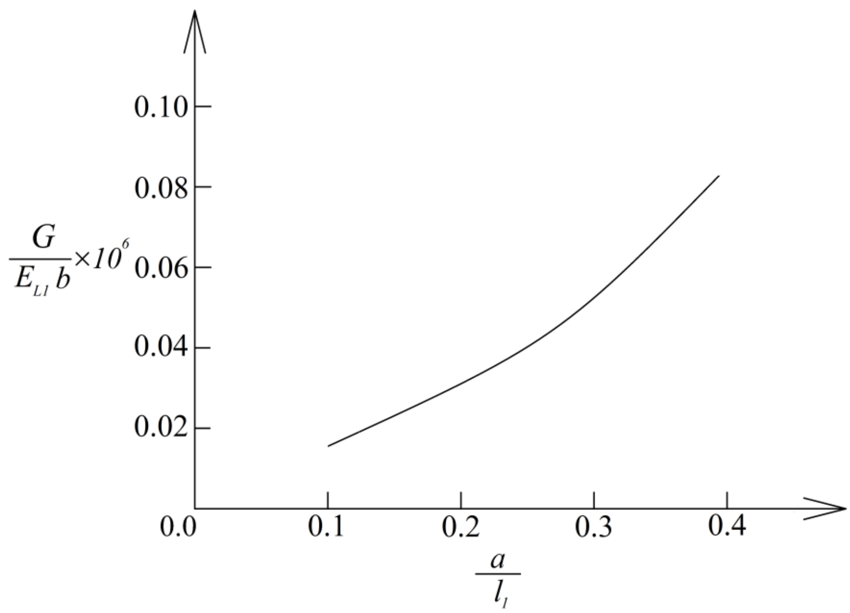

Figure 3. The non-dimensional SERR - a/l_1 curve.

Numerical Results

Numerical results are obtained and used to plot the curves shown in the next figures. These curves illustrate how the SERR changes when the basic parameters of the model vary. The data used for obtaining the SERR are $a = 0.120$ m, $b = 0.012$ m, $h = 0.008$ m, $l_1 = 0.300$ m, $l_2 = 0.400$ m, $l_3 = 0.500$ m, $n_2 = 2$, $n = 4$ and $p = 0.2$.

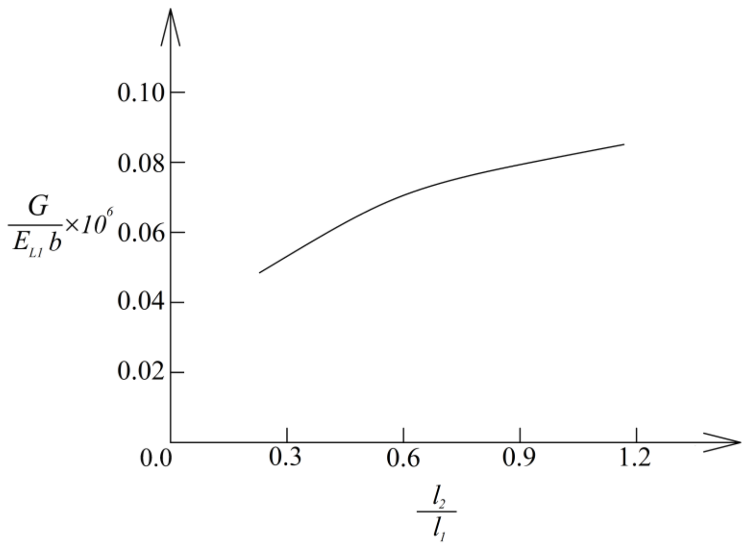

Figure 4. The non-dimensional SERR - l_2 / l_1 curve.

The influence of the value of the angle of rotation, φ_{D5}, on the SERR is shown in Fig. 2. The strong sensitivity of the SERR with respect to φ_{D5} indicates that the fixed support rotation has to be considered in delamination analyses of the statically indeterminate frames.

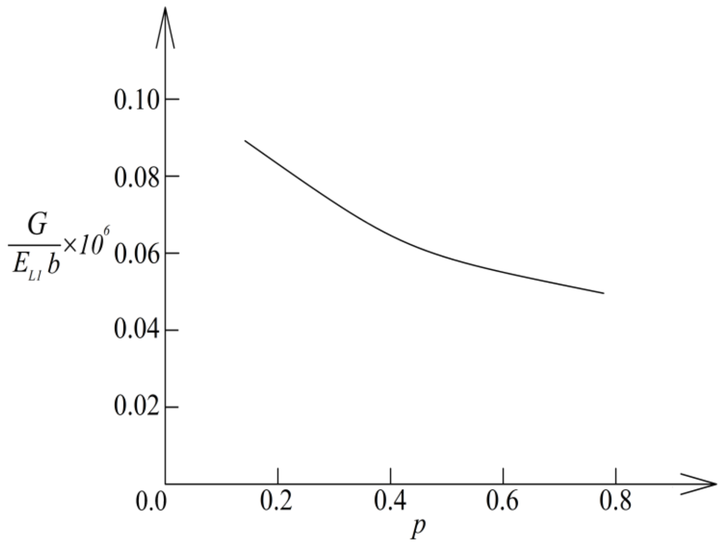

Figure 5. The non-dimensional SERR - p curve.

Other parameters whose influence on the SERR is studied are the non-dimensional delamination length, a / l_1, the ratio, l_2 / l_1, and the parameter of the distribution, p . The corresponding curves are shown in Fig. 3, Fig. 4 and Fig. 5, respectively. The sensitivity of the SERR with respect to these factors is also significant as can be seen in Fig. 3, Fig. 4 and Fig. 5.

Conclusion

Delamination in statically indeterminate steric multilayered frames under fixed support rotation is analyzed. In particular, a frame with three members (two horizontal members and a vertical member) is studied in detail. The lower end of the vertical member is fixed. Besides, the left end of the frame is supported by a vertical link. The SERR is derived. The influence of basic parameters of the model of the steric frame on the SERR is investigated. The significant sensitivity of the SERR with respect to parameters, φ_{D5}, a / l_1, l_2 / l_1 and p , detected by the investigation shows that the fixed support rotation has to be considered when analyzing delamination behaviour of statically indeterminate steric multilayered frames.

Recommendations

The analysis can be applied in design of streic statically indeterminate frames with considering the delamination behaviour under fixed support rotation.

Scientific Ethics Declaration

The author declares that the scientific ethical and legal responsibility of this article published in EPSTEM Journal belongs to the author.

Acknowledgements or Notes

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