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# Soft Semi # Generalized α-Connectedness in Soft Topological Spaces

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**Abstract**: Soft set theory is a newly emerging tool to deal with uncertain problems and has been studied by researchers in theory and practice. The concept of soft topological space is a very recently developed area having many research scopes. Soft sets have been studied in proximity spaces, multi-criteria decision-making problems, medical problems, mobile cloud computing networks, defense learning systems, and approximate reasoning, etc. The objective of this paper is to use $S_{ff}S_{emi}{}^{\#}g\alpha$-open sets and $S_{ff}S_{emi}{}^{\#}g\alpha$-closed sets to introduce the concepts of $S_{ff}S_{emi}{}^{\#}g\alpha$-separated sets, $S_{ff}S_{emi}{}^{\#}g\alpha$-connected space, $S_{ff}S_{emi}{}^{\#}g\alpha$-disconnected space, $S_{ff}S_{emi}{}^{\#}g\alpha$-component of a soft set, locally $S_{ff}S_{emi}{}^{\#}g\alpha$-connected space, and totally $S_{ff}S_{emi}{}^{\#}g\alpha$-disconnected space. We investigate and study the properties and characterizations of these spaces in soft topological spaces.

**Keywords:** $S_{ff}S_{emi}{}^{\#}g\alpha$-closed set, $S_{ff}S_{emi}{}^{\#}g\alpha$-separated set, $S_{ff}S_{emi}{}^{\#}g\alpha$-connected space

## I. Introduction

The real world is too complex for our immediate and direct understanding. We create ''models'' of reality that are simplifications of aspects of the real world. Unfortunately, these mathematical models are too complicated, and we cannot find the exact solutions. The uncertainty of data while modeling problems in engineering, physics, computer sciences, economics, social sciences, medical sciences, and many other diverse fields makes it unsuccessful to use the traditional classical methods. These may be due to the uncertainties of natural environmental phenomena, of human knowledge about the real world, or to the limitations of the means used to measure objects. For example, vagueness or uncertainty in the boundary between states or between urban and rural areas, or the exact growth rate of population in a country's rural area, or making decisions in a machine-based environment using database information. Thus, classical set theory, which is based on the crisp and exact case, may not be fully suitable for handling such problems of uncertainty. There are several theories, for example, the theory of fuzzy sets, theory of intuitionistic fuzzy sets, theory of vague sets, theory of interval mathematics, and the theory of rough sets. These can be considered as tools for dealing with uncertainties, but all these theories have their own difficulties. The reason for these difficulties is, possibly, the inadequacy of the parametrization tool of the theory as it was mentioned by Molodtsov. He initiated the concept of soft set theory as a new mathematical tool that is free from the problems mentioned above. He presented the fundamental results of the new theory and successfully applied it to several directions such as smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, theory of probability, etc. A soft set is a collection of approximate descriptions of an object. He also showed how soft set theory is free from the parametrization inadequacy syndrome of fuzzy set theory, rough set theory, probability theory, and game theory. Soft systems provide a very general framework with the involvement of parameters. Soft set theory has been applied in several directions. The researchers introduced the concept of soft sets to deal with uncertainty and to solve complicated problems in economics, engineering, sociology, and environment because of unsuccessful use of classical methods. The well-known theories that can be considered as mathematical tools for dealing with uncertainties and imperfect knowledge are theory of fuzzy sets, theory of vague sets, theory of interval



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mathematics, theory of intuitionists fuzzy sets, theory of rough sets and theory of probability. During recent years, soft set theory emerged as a best mathematical tool to deal with uncertainties, imprecision and vagueness. Many engineering, medical science, economics, environment problems have various uncertainties, and the soft set theory came up with the reasonable solutions to these problems. A soft set is a collection of approximate descriptions of objects. Some researchers have presented a systematic survey of the literature and the developments of Topological Spaces in soft set theory. They have also provided some applications of soft set theory in software engineering, innovation, medical diagnosis, data analysis, decision making etc. All these tools require the specification of some parameter to start with. The theory of soft sets gives a vital mathematical tool for handling uncertainties and vague concepts. Recently several researchers introduced the notion of soft topology and established that every soft topology induces a collection of topologies called the parametrized family of topologies induced by the soft topology. They discussed soft set-theoretical operations and gave an application of soft set theory to a decision-making problem. Several mathematicians published papers on applications of soft sets and soft topology. Soft sets and soft topology have applications in data mining, image processing, decision-making problems, spatial modeling, and neural patterns. Research works on soft set theory and its applications in various fields are progressing rapidly. Decision-making and topology have a long joint tradition since the modern statement of the classical Weierstrass extreme value theorem. It combines two topological concepts called continuity of a real-valued function and compactness of the domain (both with respect to a given topology). They represent a necessary and sufficient condition to guarantee the existence of the maximum and minimum values of the function. The success of the Mathematical Problems in Engineering technique was amplified by its adoption in fields like engineering sciences, computer sciences, and mathematical economics. This matter can be adopted in the version of soft setting by replacing the classical notions(compactness, function, and real numbers) by their soft counterparts (soft compactness, soft function, and soft real numbers). Some practical experiments in civil engineering require classification of the materials according to their characteristics (attribute set or parameter set E), which can be expressed using the concept of soft sets. We study the separation of them concerning the group of soft sets, which are constructed from the practical experiments. In this group of soft sets, we add the absolute and null soft sets to initiate a soft weak structure. The researchers in the communication engineering endeavor to select the best protocol to solve the noisy problems in wireless networks. They evaluate the performance of these protocols according to the proposed scenarios. The researchers in Soft Theory may plan with some engineers to propose some protocols using the appropriate soft structure to select the optimal protocol to solve the interference problems in wireless networks. The objective of this paper is to use $S_{ff}S_{emi}{}^{\#}g\alpha$-open sets and $S_{ff}S_{emi}{}^{\#}g\alpha$-closed sets to introduce the concepts of $S_{ff}S_{emi}{}^{\#}g\alpha$-separated sets, $S_{ff}S_{emi}{}^{\#}g\alpha$-connected space, $S_{ff}S_{emi}{}^{\#}g\alpha$-disconnected space, $S_{ff}S_{emi}{}^{\#}g\alpha$-component of a soft set, locally $S_{ff}S_{emi}{}^{\#}g\alpha$-connected space, and totally $S_{ff}S_{emi}{}^{\#}g\alpha$-disconnected space. We investigate and study the properties and characterizations of these in soft topological spaces.

## II. Preliminaries

**Definition 2.1.** Let $X$ be an initial universe set and E be a collection of all possible parameters with respect to $X$, where parameters are the characteristics or properties of objects in $X$. Let $P(X)$ denote the power set of $X$, and let $A$ be a non-empty subset of $E$. A pair $(F,A)$ is called a soft set over $X$, where $F$ is a mapping given by $F: A \to P(X)$. In other words, a soft set over $X$ is a parameterized family of subsets of the universe $X$. For $e \in A$, $F(e)$ may be considered as the set $e$-approximate elements of the soft set $(F,A)$. Clearly, a soft set is not a set. For two soft sets $(F,A)$ and $(G,B)$ over the common universe $X$, we say that $(F,A)$ is a soft subset of $(G,B)$ if $(i)$ $A \subseteq B$ and $(ii)$ for all $e \in A$, $F(e)$ and $G(e)$ are identical approximations. We write $(F,A) \subseteq (G,B)$. $(G,B)$ is said to be a soft superset of $(F,A)$, if $(F,A)$ is a soft subset of $(G,B)$. Two soft sets $(F,A)$ and $(G,B)$ over a common universe $X$ are said to be soft equal if $(F,A)$ is a soft subset of $(G,B)$ and $(G,B)$ is a soft subset of $(F,A)$.

**Definition 2.2.** The union of two soft sets of $(F,A)$ and $(G,B)$ over the common universe $X$ is soft set $(H,C)=(F,A)\bigcup(G,B)$, where $C=A\bigcup B$, and $H(e)=F(e)$ if $e\in A-B$, $H(e)=G(e)$ if $e\in B-A$ and $H(e)=F(e)\bigcup G(e)$ if $e\in A\bigcap B$.

**Definition 2.3.** The Intersection $(H,C)$ of two soft sets $(F,A)$ and $(G,B)$ over a common universe $X$ denoted $(F,A)\bigcap(G,B)$ is defined as $C=A\bigcap B$ and $H(e)=F(e)\bigcap G(e)$ for all $e\in C$.

**Definition 2.4.** For a soft set $(F,A)$ over the universe U, the relative complement of $(F,A)$ is denoted by $(F,A)^C$ and is defined by $(F,A)^C=(F^C,A)$, where $F^C:A\to P(X)$ is a mapping defined by $F^C(e)=X-F(e)$ for all $e\in A$.

**Definition 2.5**. A soft set $(F,E)$ over $X$ is said to be $(i)$ A null soft set, denoted by $\tilde{\phi}$, if $\forall$ $e\in E$, $F(e)=\phi$. $(ii)$ An absolute soft set, denoted by $\tilde{X}$, if $\forall$ $e\in E$, $F(e)=X$.

**Definition 2.6**. Let $\tilde{\tau}$ be the collection of soft sets over $X$. Then $\tilde{\tau}$ is said to be a soft Topology on $X$ if

$(i)$ $\tilde{\phi}$, $\tilde{X}$ belong to $\tilde{\tau}$

$(ii)$ The union of any number of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$

$(iii)$ The intersection of any two soft sets belongs to $\tilde{\tau}$.

The triplet $(\tilde{X},\tilde{\tau},E)$ is called a soft topological space over $X$. The complement of a soft open set is called soft closed set over $X$.

**Definition 2.7**. Let $(\tilde{X},\tilde{\tau},E)$ be a soft topological space over $X$ and $(F,E)$ be a soft set over $X$. Then

$(i)$ Soft interior of a soft set $(F,E)$ denoted by $S_{ft}Int(F,E)$ is defined as the union of all soft open sets over $X$ contained in $(F,E)$.

$(ii)$ Soft closure of a soft set $(F,E)$ denoted by $S_{ft}Cl(F,E)$ is defined as the intersection of all soft closed super sets over $X$ containing $(F,E)$.

**Definition 2.8**. A $S_{ft}$-subset $(F,A)$ of a $S_{ft}$-Top-Space $(\tilde{X},\tilde{\tau},E)$ is called a $S_{ft}\alpha$-closed set if $S_{ft}Cl\left[S_{ft}Int\left(S_{ft}Cl(F,A)\right)\right]\subseteq(F,A)$. The complement of a $S_{ft}\alpha$-closed set is called $S_{ft}\alpha$-open set.

**Definition 2.9**. A $S_{ft}$-subset $(F,A)$ of a $S_{ft}$-Top-Space $(\tilde{X},\tilde{\tau},E)$ is called a $S_{ft}g$-closed set if $S_{ft}Cl(A,E)\subseteq(U,E)$ whenever $(F,A)\subseteq(U,E)$ and $(U,E)$ is soft open in $(\tilde{X},\tilde{\tau},E)$.

**Definition 2.10.** A $S_{ft}$-subset $(F,A)$ of a $S_{ft}$-Top-Space $(\tilde{X},\tilde{\tau},E)$ is called a $S_{ft}S_{emi}$-closed set if $S_{ft}Int\left[S_{ft}Cl(F,A)\right]\subseteq(F,A)$. The complement of a $S_{ft}S_{emi}$-closed set is called a $S_{ft}S_{emi}$-open set.

**Definition 2.11.** A $S_{ft}$-subset $(F,A)$ of a $S_{ft}$-Top-Space $(\tilde{X},\tilde{\tau},E)$ is called a soft $S_{ft}g^{\#}\alpha$-closed set if $S_{ft}\alpha Cl(F,A)\subseteq(U,E)$, whenever $(F,A)\subseteq(U,E)$ and $(U,E)$ is $S_{ft}g$-open in $(\tilde{X},\tilde{\tau},E)$. The complement of a $S_{ft}g^{\#}\alpha$-closed set is called a $S_{ft}g^{\#}\alpha$-open set.

**Definition 2.12.** A $S_{ft}$ −subset $(F,A)$ of a $S_{ft}$ −Top −Space $(\tilde{X},\tilde{\tau},E)$ is called a soft $S_{ft}{}^{\#}g\alpha$ −closed set if $S_{ft}\alpha Cl(F,A)\subseteq(U,E)$, whenever $(F,A)\subseteq(U,E)$ and $(U,E)$ is $S_{ft}g^{\#}\alpha$ −open in $(X,\tau,E)$. The complement of a $S_{ft}{}^{\#}g\alpha$ −closed set is called a $S_{ft}{}^{\#}g\alpha$ −open set.

**Definition 2.13.** A $S_{ft}$ −subset $(F,A)$ of a $S_{ft}$ −Top −Space $(\tilde{X},\tilde{\tau},E)$ is called a soft $S_{ft}S_{emi}{}^{\#}g\alpha$ −closed set if $S_{ft}S_{emi}Cl(F,A)\subseteq(U,E)$, whenever $(F,A)\subseteq(U,E)$ and $(U,E)$ is $S_{ft}{}^{\#}g\alpha$ −open in $(X,\tau,E)$. The complement of a $S_{ft}S_{emi}{}^{\#}g\alpha$ −closed set is called a $S_{ft}S_{emi}{}^{\#}g\alpha$ −open set in $(X,\tau,E)$.

**Definition 2.14.** Let $(\tilde{X},\tilde{\tau},E)$ be a soft topological space over $X$ and $(F,A)$ be a soft set over $X$. Then

(i) $S_{ft}S_{emi}{}^{\#}g\alpha$ −interior of a soft set $(F,A)$ denoted by $S_{ft}S_{emi}{}^{\#}g\alpha-Int(F,A)$ is defined as the union of all soft $S_{ft}S_{emi}{}^{\#}g\alpha$ −open sets over $X$ contained in $(F,A)$.

(ii) $S_{ft}S_{emi}{}^{\#}g\alpha$ −closure of a soft set $(F,A)$ denoted by $S_{ft}S_{emi}{}^{\#}g\alpha-Cl(F,A)$ is defined as the intersection of all soft $S_{ft}S_{emi}{}^{\#}g\alpha$ −closed sets over $X$ containing $(F,A)$.

**Theorem 2.15.** (i) Every $S_{ft}$ −closed set in a soft topological space is $S_{ft}S_{emi}{}^{\#}g\alpha$ −closed set.
(ii) Every $S_{ft}$ −open set in a $S_{ft}$ −Top −Space is $S_{ft}S_{emi}{}^{\#}g\alpha$ −open set.

**Theorem 2.16.** Let $(F,A)$ and $(G,B)$ be two $S_{ft}$ −subsets of a $S_{ft}$ −Top −Space $(\tilde{X},\tilde{\tau},E)$. Then the following statements are true.
(i) $(F,A)$ is $S_{ft}S_{emi}{}^{\#}g\alpha$ −open if and only if $S_{ft}S_{emi}{}^{\#}g\alpha-Int(F,A)=(F,A)$.
(ii) $S_{ft}S_{emi}{}^{\#}g\alpha-Int(F,A)$ is $S_{ft}S_{emi}{}^{\#}g\alpha$ −open.
(iii) $(F,A)$ is $S_{ft}S_{emi}{}^{\#}g\alpha$ −closed if and only if $S_{ft}S_{emi}{}^{\#}g\alpha-Cl(F,A)=(F,A)$.
(iv) $S_{ft}S_{emi}{}^{\#}g\alpha-Cl(F,A)$ is $S_{ft}S_{emi}{}^{\#}g\alpha$ −closed..
(v) $S_{ft}S_{emi}{}^{\#}g\alpha-Cl[(X,E)-(F,A)]-(X,E)-S_{ft}S_{emi}{}^{\#}g\alpha-Int(F,A)$.
(vi) $S_{ft}S_{emi}{}^{\#}g\alpha-Int[(X,E)-(F,A)]=(X,E)-S_{ft}S_{emi}{}^{\#}g\alpha-Cl(F,A)$.
(vii) If $(F,A)$ is $S_{ft}S_{emi}{}^{\#}g\alpha$ −open in $(\tilde{X},\tilde{\tau},E)$ and $(G,B)$ is $S_{ft}$ −open in $(\tilde{X},\tilde{\tau},E)$, then $(F,A)\cap(G,B)$ is $S_{ft}S_{emi}{}^{\#}g\alpha$ −open in $(\tilde{X},\tilde{\tau},E)$.
(viii) A point $x_\alpha\in S_{ft}S_{emi}{}^{\#}g\alpha-Cl(F,A)$ if and only if every $S_{ft}S_{emi}{}^{\#}g\alpha$ −open set in $(\tilde{X},\tilde{\tau},E)$ containing $x$ intersects $(F,A)$.

(ix) Arbitrary intersection of $S_{ft}S_{emi}{}^{\#}g\alpha$ −closed sets in $(\tilde{X},\tilde{\tau},E)$ is $S_{ft}-S_{emi}-{}^{\#}g\alpha$ −closed in $(\tilde{X},\tilde{\tau},E)$.
(x) An arbitrary union of $S_{ft}S_{emi}{}^{\#}g\alpha$ −open sets in $(\tilde{X},\tilde{\tau},E)$ is $S_{ft}S_{emi}{}^{\#}g\alpha$ −open in $(X,\tilde{\tau},E)$.

**Definition 2.17**. A $S_{ft}$ −*Top* −*Space* $(\tilde{X},\tilde{T},E)$ is said to be soft connected if there does not exist a pair $F_A$ and $G_B$ of nonempty, disjoint soft open subsets of $(\tilde{X},\tilde{T},E)$ such that $\tilde{X}=F_A\tilde{\cup}G_B$.

## III. Soft $S_{emi}{}^{\#}g\alpha$ – Separated Sets

In this section, we will introduce and investigate the basic properties of $S_{ft}S_{emi}{}^{\#}g\alpha$ *–separated* sets.

**Definition 3.1.** Two non-null soft disjoint $S_{ft}$ *–sets* $F_E$ and $G_E$ in a $S_{ft}$ *–Top –Space* $(\tilde{X},\tilde{T},E)$ are called Soft Semi$^{\#}g\alpha$ *–separated* $(S_{ft}S_{emi}{}^{\#}g\alpha$ *–separated*$)$ sets if $G_E \tilde{\cap}[S_{ft}S_{emi}{}^{\#}g\alpha\text{-}Cl(F_E)] = [S_{ft}S_{emi}{}^{\#}g\alpha\text{-}Cl(G_E)]\tilde{\cap} F_E = \tilde{\phi}$

**Definition 3.2.** A Soft Semi$^{\#}g\alpha$ *–separation* $(S_{ft}S_{emi}{}^{\#}g\alpha$ *–separation*$)$ of a $S_{ft}$ *–Top –Space* $(\tilde{X},\tilde{T},E)$ is a pair of $S_{ft}S_{emi}{}^{\#}g\alpha$ *–separated* sets such that $F_E \tilde{\cup} G_E = \tilde{X}$.

**Definition 3.3.** Let $(\tilde{X},\tilde{T},E)$ be a $S_{ft}$ *–Top –Space.* A Soft Semi$^{\#}g\alpha$ *–connected* soft connected$(S_{ft}S_{emi}{}^{\#}g\alpha$ *–connected*$)$ set over X is a $S_{ft}$ *–set* $F_E \tilde{\in} S_{ft}S(X)_E$ which does not have a $S_{ft}S_{emi}{}^{\#}g\alpha$ *–separation* in the soft relative topology induced on the soft subset $F_E$.

**Proposition 3.4.** Every $S_{ft}S_{emi}{}^{\#}g\alpha$ *–connected* set in a $S_{ft}$ *–Top –Space* $(\tilde{X},\tilde{T},E)$ is a $S_{ft}$ *–connected* set.

**Proof.** Let $F_E$ be a $S_{ft}S_{emi}{}^{\#}g\alpha$ *–connected* set in a $S_{ft}$ *–Top –Space* $(\tilde{X},\tilde{T},E)$. Then there does not exist a $S_{ft}S_{emi}{}^{\#}g\alpha$ *–separation* of $F_E$. Since every $S_{ft}$ *–open* set is a $S_{ft}S_{emi}{}^{\#}g\alpha$ *–open* set, there does not exist a $S_{ft}$ *–separation* of $F_E$. Hence, $F_E$ is a $S_{ft}$ *–connected* set in the $S_{ft}$ *–Top –Space* $(\tilde{X},\tilde{T},E)$.

**Theorem 3.5.** Let $(\tilde{X},\tilde{T},E)$ be a $S_{ft}$ *–Top –Space* and $F_E$ be a $S_{ft}S_{emi}{}^{\#}g\alpha$ *–connected* set. Let $G_A$ and $H_B$ be $S_{ft}S_{emi}{}^{\#}g\alpha$ *–separated* sets. If $F_E \tilde{\subseteq} G_A \tilde{\cup} H_B$. Then either $F_E \tilde{\subseteq} G_A$ or $F_E \tilde{\subseteq} H_B$.

**Proof.** Let $(\tilde{X},\tilde{T},E)$ be a $S_{ft}$ *–Top –Space* and $F_E \tilde{\in} S_{ft}S(X)_E$ be a $S_{ft}S_{emi}{}^{\#}g\alpha$ *–connected* set. Let $G_A$ and $H_B$ be $S_{ft}S_{emi}{}^{\#}g\alpha$ *–separated* sets such that $F_E \tilde{\subseteq} G_A \tilde{\cup} H_B$. We have to prove either $F_E \tilde{\subseteq} G_A$ or $F_E \tilde{\subseteq} H_B$. Suppose not. Then $F_E \tilde{\not\subset} G_A$ and $F_E \tilde{\not\subset} H_B$. Then, $K_E = G_A \tilde{\cap} F_E \neq \phi$ and $L_E = H_B \tilde{\cap} F_E \neq \phi$ and $F_E = K_E \tilde{\cup} L_E$. Since $K_E \tilde{\subseteq} G_A$ implies that $S_{ft}S_{emi}{}^{\#}g\alpha\text{-}Cl(K_E) \tilde{\subseteq}$ $S_{ft}S_{emi}{}^{\#}g\alpha\text{-}Cl(G_A)$. Since $G_A$ and $H_B$ are $S_{ft}S_{emi}{}^{\#}g\alpha$ *–separated* sets, we have $[S_{ft}S_{emi}{}^{\#}g\alpha\text{-}Cl(G_A)]\tilde{\cap} H_B = \tilde{\phi}$. Therefore, $[S_{ft}S_{emi}{}^{\#}g\alpha\text{-}Cl(G_A)]\tilde{\cap} H_B = [S_{ft}S_{emi}{}^{\#}g\alpha\text{-}Cl(K_E)]\tilde{\cap} H_B = [S_{ft}S_{emi}{}^{\#}g\alpha\text{-}Cl(K_E)]\tilde{\cap} L_E = \tilde{\phi}$. Again $L_E \tilde{\subseteq} H_B$ implies $S_{ft}S_{emi}{}^{\#}g\alpha\text{-}Cl(L_E) \tilde{\subseteq} S_{ft}S_{emi}{}^{\#}g\alpha\text{-}Cl(H_B)$. Since $G_A$ and $H_B$ are $S_{ft}S_{emi}{}^{\#}g\alpha$ *–separted* sets, we have $G_A \tilde{\cap}[S_{ft}S_{emi}{}^{\#}g\alpha\text{-}S_{ft}(H_B)] = \tilde{\phi}$. Therefore, $G_A \tilde{\cap}[S_{ft}S_{emi}{}^{\#}g\alpha\text{-}Cl(H_B)] = G_A \tilde{\cap}[S_{ft}S_{emi}{}^{\#}g\alpha\text{-}Cl(L_E)] = K_E \tilde{\cap}[S_{ft}S_{emi}{}^{\#}g\alpha\text{-}Cl(L_E)] = \tilde{\phi}$. But $F_E = K_E \tilde{\cup} H_E$. Therefore, there exists a $S_{ft}S_{emi}{}^{\#}g\alpha$ *–separation* of $F_E$. Hence, $F_E$ is not a $S_{ft}S_{emi}{}^{\#}g\alpha$ *–connected* set. This is a contradiction. Therefore, either $F_E \tilde{\subseteq} G_A$ or $F_E \tilde{\subseteq} H_B$.

**Theorem 3.6.** If $F_E$ is $S_{ft}S_{emi}{}^{\#}g\alpha$ *–connected* set, then $S_{ft}S_{emi}{}^{\#}g\alpha\text{-}Cl(F_E)$ is $S_{ft}S_{emi}{}^{\#}g\alpha$ *–connected* set.

**Proof.** Let $F_E$ be $S_{ft}S_{emi}{}^{\#}g\alpha-connected$ set in a $S_{ft}-Top-Space$ $(\tilde{X},\tilde{T},E)$. We have to prove that $S_{ft}S_{emi}{}^{\#}g\alpha-Cl(F_E)$ is a $S_{ft}S_{emi}{}^{\#}g\alpha-connected$ set. Suppose not. Then there exists a $S_{ft}S_{emi}{}^{\#}g\alpha-separation$ of $S_{ft}S_{emi}{}^{\#}g\alpha-Cl(F_E)$. Therefore there exists a pair of $S_{ft}S_{emi}{}^{\#}g\alpha-separated$ sets $G_A$ and $H_B$ such that $\left[S_{ft}S_{emi}{}^{\#}g\alpha-Cl(F_E)\right]=G_A\tilde{\bigcup}H_B$. But $F_E\subseteq S_{ft}S_{emi}{}^{\#}g\alpha-Cl(F_E)=G_A\tilde{\bigcup}H_B$. Since $F_E$ is $S_{ft}S_{emi}{}^{\#}g\alpha-connected$ set, then by Theorem 3.5 either $F_E\tilde{\subseteq}G_A$ or $F_E\tilde{\subseteq}H_B$. If $F_E\tilde{\subseteq}G_A$ then $S_{ft}S_{emi}{}^{\#}g\alpha-Cl(F_E)\tilde{\subseteq}S_{ft}S_{emi}{}^{\#}g\alpha-Cl(G_A)$. Since $H_B\tilde{\subseteq}$ $S_{ft}S_{emi}{}^{\#}g\alpha-Cl(F_E)$, then $H_B=\tilde{\phi}$. This is a contradiction. Similarly, if $F_E\tilde{\subseteq}H_B$, we can prove $G_A=\tilde{\phi}$ which is a contradiction. Thus, there is not any $S_{ft}S_{emi}{}^{\#}g\alpha-separation$ of $S_{ft}S_{emi}{}^{\#}g\alpha-Cl(F_E)$. Hence $S_{ft}S_{emi}{}^{\#}g\alpha-Cl(F_E)$ is $S_{ft}S_{emi}{}^{\#}g\alpha-connected$ set.

**Theorem 3.7.** If $F_E$ is $S_{ft}S_{emi}{}^{\#}g\alpha-connected$ set and $F_E\tilde{\subseteq}G_E\tilde{\subseteq}S_{ft}S_{emi}{}^{\#}g\alpha-Cl(F_E)$, then $G_E$ is $S_{ft}S_{emi}{}^{\#}g\alpha-connected$ set.

**Proof.** Let $F_E\tilde{\in}S_{ft}S(X)_E$ be a $S_{ft}S_{emi}{}^{\#}g\alpha-connected$ set such that $F_E\tilde{\subseteq}G_E\tilde{\subseteq}S_{ft}S_{emi}{}^{\#}g\alpha-Cl(F_E)$. We have to prove that $G_E$ is $S_{ft}S_{emi}{}^{\#}g\alpha-connected$ set. Suppose $G_E$ is not a $S_{ft}S_{emi}{}^{\#}g\alpha-connected$ set. Then there exists a pair of $S_{ft}S_{emi}{}^{\#}g\alpha-separated$ sets $K_A$ and $L_B$ such that $G_E=K_A\tilde{\bigcup}L_B$. Since $F_E\tilde{\subseteq}G_E$ and $F_E\tilde{\subseteq}K_A\tilde{\bigcup}L_B$. We claim that either $F_E\tilde{\subseteq}K_A$ or $F_E\tilde{\subseteq}L_B$. For, $F_E\tilde{\bigcap}K_A\neq\tilde{\phi}$ and $F_E\tilde{\bigcap}L_B\neq\tilde{\phi}$. Then $F_E=\left(F_E\tilde{\cap}K_A\right)\tilde{\bigcup}\left(F_E\tilde{\cap}L_B\right)$. But $F_E\tilde{\bigcap}K_A$ and $F_E\tilde{\bigcap}L_B$ are $S_{ft}S_{emi}{}^{\#}g\alpha-separated$ sets. This is a contradiction to that $S_{ft}S_{emi}{}^{\#}g\alpha-connectivity$ of $F_E$. Hence, our claim is proved that $G_E$ is $S_{ft}S_{emi}{}^{\#}g\alpha-separated$ set. Suppose $F_E\tilde{\subseteq}K_A$, then $S_{ft}S_{emi}{}^{\#}g\alpha-Cl(F_E)\tilde{\subseteq}$ $S_{ft}S_{emi}{}^{\#}g\alpha-Cl(K_A)$. Since $K_A$ and $L_B$ are $S_{ft}S_{emi}{}^{\#}g\alpha-separated$ sets. $\left[S_{ft}S_{emi}{}^{\#}g\alpha-Cl(K_A)\right]\tilde{\bigcap}L_B=\tilde{\phi}$. Therefore, $\left[S_{ft}S_{emi}{}^{\#}g\alpha-Cl(F_E)\right]\tilde{\bigcap}L_B=\tilde{\phi}$. But $L_B\tilde{\subseteq}G_E$. Then by hypothesis $L_B\tilde{\subseteq}G_E\tilde{\subseteq}S_{ft}S_{emi}{}^{\#}g\alpha-Cl(F_E)$. Therefore, $\left[S_{ft}S_{emi}{}^{\#}g\alpha-Cl(F_E)\right]\tilde{\bigcap}L_B=L_B$. Thus, we have $\left[S_{ft}S_{emi}{}^{\#}g\alpha-Cl(F_E)\right]\tilde{\bigcap}L_B=\tilde{\phi}$ and $\left[S_{ft}S_{emi}{}^{\#}g\alpha-Cl(F_E)\right]\tilde{\bigcap}L_B=L_B$. Hence $L_B=\tilde{\phi}$, which is a contradiction. Similarly, if $F_E\tilde{\subseteq}L_B$, then we can prove $K_A=\tilde{\phi}$. This is a contradiction. Therefore, there does not exist a $S_{ft}S_{emi}{}^{\#}g\alpha-separation$ of $G_E$. Hence, $G_E$ is a $S_{ft}S_{emi}{}^{\#}g\alpha-connected$ set.

**Theorem 3.8.** The soft union $F_E$ of any family $\left\{F_{E_j}:j\in J\right\}$ of $S_{ft}S_{emi}{}^{\#}g\alpha-connected$ sets having a nonempty soft intersection is $S_{ft}S_{emi}{}^{\#}g\alpha-connected$ set.

**Proof.** Suppose that $F_E=G_A\tilde{\bigcup}H_B$ where $G_A$ and $H_B$ form a $S_{ft}S_{emi}{}^{\#}g\alpha-separation$ of $F_E$. By hypothesis, we may choose a soft point $x_e\tilde{\in}\tilde{\bigcap}_{j\in J}F_{E_j}$. Then $x_e\tilde{\in}F_{E_j}$ for all $j\in J$. If $x_e\tilde{\in}F_E$, then either $x_e\tilde{\in}G_A$ or $x_e\tilde{\in}H_B$ but not both. Since $G_A$ and $H_B$ are disjoint soft sets, we must have $F_{E_j}\tilde{\subseteq}G_A$, since $F_{E_j}$ is $S_{ft}S_{emi}{}^{\#}g\alpha-connected$ and it is true for all $j\in J$, and so $F_E\tilde{\subseteq}G_A$. From this, we obtain that $H_B=\tilde{\phi}$, which is a contradiction. Thus, there does not exist a $S_{ft}S_{emi}{}^{\#}g\alpha-separation$ of $F_E$. Therefore, $F_E$ is $S_{ft}S_{emi}{}^{\#}g\alpha-connected$.

## IV. Soft $S_{emi}{}^{\#}g\alpha$-Connected Space

In this section, we introduce the $S_{\pounds}S_{emi}{}^{\#}g\alpha$-*connected* space and discuss its properties in $S_{\pounds}$-*Top*-*Space*.

**Definition 4.1.** Let $(\tilde{X},\tilde{T},E)$ be a $S_{\pounds}$-*Top*-*Space*. If there does not exist a $S_{\pounds}S_{emi}{}^{\#}g\alpha$-*separation* of $\tilde{X}$, then it is said to be $S_{\pounds}S_{emi}{}^{\#}g\alpha$-*connected* space.

**Proposition 4.2.** Every $S_{\pounds}S_{emi}{}^{\#}g\alpha$-*connected* space $(\tilde{X},\tilde{T},E)$ is $S_{\pounds}S_{emi}{}^{\#}g\alpha$-*connected* space.

**Proof.** Let $(\tilde{X},\tilde{T},E)$ be a $S_{\pounds}S_{emi}{}^{\#}g\alpha$-*connected* space. Then there does not exist a $S_{\pounds}S_{emi}{}^{\#}g\alpha$-*separation* of $\tilde{X}$. Since every $S_{\pounds}$-*open* set is a $S_{\pounds}{}^{\#}g\alpha$-*open* set, there does not exist a $S_{\pounds}$-*separation* of $\tilde{X}$. Therefore, $(\tilde{X},\tilde{T},E)$ is a $S_{\pounds}$-*connected* space.

**Theorem 4.3.** Let $(\tilde{X},\tilde{T},E)$ be a $S_{\pounds}$-*Top*-*Space*. Then the following statements are equivalent:

(i) $\tilde{\phi}$ and $\tilde{X}$ are the only $S_{\pounds}S_{emi}{}^{\#}g\alpha$-*clopen* sets in $(\tilde{X},\tilde{T},E)$.

(ii) $\tilde{X}$ is not the soft union of two soft disjoint non-empty $S_{\pounds}S_{emi}{}^{\#}g\alpha$-*open* sets.

(iii) $\tilde{X}$ is not the soft union of two soft disjoint non-empty $S_{\pounds}S_{emi}{}^{\#}g\alpha$-*closed* sets.

(iv) $\tilde{X}$ is not the soft union of two $S_{\pounds}S_{emi}{}^{\#}g\alpha$-*separated* sets.

**Proof.** (i) $\Rightarrow$ (ii): Assume that $\tilde{\phi}$ and $\tilde{X}$ are the only $S_{\pounds}S_{emi}{}^{\#}g\alpha$-*clopen* sets in $(\tilde{X},\tilde{T},E)$. Suppose (ii) is false. Then $\tilde{X}=G_A\tilde{\bigcup}H_B$, where $G_A$ and $H_B$ are two soft disjoint, non-empty $S_{\pounds}S_{emi}{}^{\#}g\alpha$-*open* sets. Clearly $H_B=G_A^C$ is $S_{\pounds}S_{emi}{}^{\#}g\alpha$-*closed* and non-empty. Thus $H_B$ is a non-empty proper $S_{\pounds}S_{emi}{}^{\#}g\alpha$-*clopen* set in $\tilde{X}$, which shows that (i) is false.

(ii) $\Rightarrow$ (iii): Assume that, $\tilde{X}$ is not the soft union of two soft disjoint non-empty $S_{\pounds}S_{emi}{}^{\#}g\alpha$-*open* sets. Suppose (iii) is false. Then $\tilde{X}=G_A\tilde{\bigcup}H_B$ where $G_A$ and $H_B$ are two soft disjoint non-empty $S_{\pounds}S_{emi}{}^{\#}g\alpha$-*closed* sets. Now $G_A$ and $H_B$ being respectively the soft complement of $H_B$ and $G_A$. Therefore $G_A$ and $H_B$ are $S_{\pounds}S_{emi}{}^{\#}g\alpha$-*open* sets. This contradicts (ii).

(iii) $\Rightarrow$ (iv): Assume that $\tilde{X}$ is not the soft union of two soft disjoint non-empty $S_{\pounds}S_{emi}{}^{\#}g\alpha$-*closed* sets. If (iv) is false, then $\tilde{X}=G_A\tilde{\bigcup}H_B$, where $G_A$ and $H_B$ are $S_{\pounds}S_{emi}{}^{\#}g\alpha$-*separated* sets. Then $G_A\tilde{\bigcap}\left[S_{\pounds}S_{emi}{}^{\#}g\alpha\text{-}Cl(H_B)\right]=\tilde{\phi}$ and this implies that $S_{\pounds}S_{emi}{}^{\#}g\alpha\text{-}Cl(H_B)\tilde{\subseteq}H_B$. But $H_B\tilde{\subseteq}S_{\pounds}S_{emi}{}^{\#}g\alpha\text{-}Cl(H_B)$. Therefore, $S_{\pounds}S_{emi}{}^{\#}g\alpha\text{-}Cl(H_B)=H_B$ and hence $H_B$ is a $S_{\pounds}S_{emi}{}^{\#}g\alpha$-*closed* set. Similarly, $G_A$ is a $S_{\pounds}S_{emi}{}^{\#}g\alpha$-*closed* set, which is a contradiction to (iii). Therefor (iv) is true.

(iv) $\Rightarrow$ (i): Assume that $\tilde{X}$ is not the soft union of two $S_{\pounds}S_{emi}{}^{\#}g\alpha$-*separated* sets. Suppose (i) is false. Then there exists a non-empty proper $S_{\pounds}S_{emi}{}^{\#}g\alpha$-*clopen* subset $H_A$ of $\tilde{X}$. Then $H_B=H_A^C$ is a non-empty, $S_{\pounds}S_{emi}{}^{\#}g\alpha$-*clopen* set and $\tilde{X}=G_A\tilde{\bigcup}H_B$, where $G_A$ and $H_B$ are non-empty disjoint

$S_{\pm}S_{emi}{}^{\#}g\alpha-clopen$ sets. Since $G_A$ and $H_B$ are $S_{\pm}S_{emi}{}^{\#}g\alpha-clopen$ sets, $S_{\pm}S_{emi}{}^{\#}g\alpha-Cl(G_A)=G_A$ and $S_{\pm}S_{emi}{}^{\#}g\alpha-Cl(H_B)=H_B$. Therefore, $S_{\pm}S_{emi}{}^{\#}g\alpha-Cl(G_A)\tilde{\cap}H_B=G_A\tilde{\cap}H_B=\tilde{\phi}=G_A\tilde{\cap}\left[S_{\pm}S_{emi}{}^{\#}g\alpha-Cl(H_B)\right]$. Thus $\tilde{X}$ is a soft union of $S_{\pm}S_{emi}{}^{\#}g\alpha-separated$ sets. This is a contradiction to $(\mathrm{iv})$. Therefore $(\mathrm{i})$ must be true.

**Theorem 4.4.** Let $(\tilde{X},\tilde{T},E)$ be a $S_{\pm}-Top-Space$ such that any two soft points $x_\alpha$ and $y_\beta$ of $\tilde{X}$ are contained in some $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ subspace $\tilde{Y}$ of $\tilde{X}$. Then $\tilde{X}$ is $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ space.

**Proof.** Let $(\tilde{X},\tilde{T},E)$ be a $S_{\pm}-Top-Space$ and $(\tilde{Y},\tilde{T}_Y,E)$ be a $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ subspace of $\tilde{X}$. Let $x_\alpha$, $y_\beta$ be two disjoint soft points in $\tilde{X}$. Suppose that $x_\alpha$ and $y_\beta$ in $\tilde{X}$ are contained in the $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ subspace $\tilde{Y}$ of $\tilde{X}$, we have to prove, $(\tilde{X},\tilde{T},E)$ is a $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ space. Suppose $(\tilde{X},\tilde{T},E)$ is not a $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ space. Then there exists a $S_{\pm}S_{emi}{}^{\#}g\alpha-separation$ in $\tilde{X}$. Therefore, there exists a pair of non-empty, soft disjoint $S_{\pm}S_{emi}{}^{\#}g\alpha-separated$ sets $G_A$ and $H_B$. Since $G_A$ and $H_B$ are non-empty soft sets, there exist $x_\alpha$ and $y_\beta$ such that $x_\alpha\tilde{\in}G_A$ and $y_\beta\tilde{\in}H_\beta$. Since $(\tilde{Y},\tilde{T}_Y,E)$ is a $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ soft subspace of $(\tilde{X},\tilde{T},E)$, which contains $x_\alpha$ and $y_\beta$, therefore by Theorem 3.5, either $\tilde{Y}\tilde{\subseteq}G_A$ or $\tilde{Y}\tilde{\subseteq}H_B$. But $G_A\tilde{\cap}H_B=\tilde{\phi}$, which is a contradiction. Therefore $(\tilde{X},\tilde{T},E)$ is $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ space.

**Corollary 4.5.** If a $S_{\pm}-Top-Space\,(\tilde{X},\tilde{T},E)$ contains a $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ subspace $(\tilde{Y},\tilde{T}_Y,E)$ such that $S_{\pm}S_{emi}{}^{\#}g\alpha-Cl(\tilde{Y})=\tilde{X}$, then $\tilde{X}$ is $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ space.

**Proof.** Let $(\tilde{Y},\tilde{T}_Y,E)$ be a $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ subspace of $(\tilde{X},\tilde{T},E)$ such that $S_{\pm}S_{emi}{}^{\#}g\alpha-Cl(\tilde{Y})=\tilde{X}$. Since $(\tilde{Y},\tilde{T}_Y,E)$ is a $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ subspace, $\tilde{Y}$ is a $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ set. Then by Theorem 3.6, $S_{\pm}S_{emi}{}^{\#}g\alpha-Cl(\tilde{Y})$ is $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ set. Therefore, $\tilde{X}=S_{\pm}S_{emi}{}^{\#}g\alpha-Cl(\tilde{Y})$ is a $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ set. Hence $(\tilde{X},\tilde{T},E)$ is a $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ space.

**Theorem 4.6.** If $(\tilde{Y},\tilde{T}_Y,E)$ and $(\tilde{Z},\tilde{T}_Z,E)$ are $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ subspaces of a $S_{\pm}-Top-Space$ $(\tilde{X},\tilde{T},E)$ are such that $\tilde{Y}\tilde{\cap}\tilde{Z}\neq\tilde{\phi}$, then $\tilde{Y}\tilde{\cup}\tilde{Z}$ is $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ subspace.

**Proof.** Suppose $\tilde{Y}\tilde{\cup}\tilde{Z}$ is not $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ subspace, then there exists a $S_{\pm}S_{emi}{}^{\#}g\alpha-separation$ of $\tilde{Y}\tilde{\cup}\tilde{Z}$. Thus, there exists a pair of $S_{\pm}S_{emi}{}^{\#}g\alpha-separated$ sets $G_A$ and $H_B$ such that $G_A\tilde{\cup}H_B=\tilde{Y}\tilde{\cup}\tilde{Z}$. Since $\tilde{Y}\tilde{\subseteq}\tilde{Y}\tilde{\cup}\tilde{Z}=G_A\tilde{\cup}H_B$ and $\tilde{Y}$ is $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$, then by Theorem 3.5, either $\tilde{Y}\tilde{\subseteq}G_A$ or $\tilde{Y}\tilde{\subseteq}H_B$. Since $\tilde{Z}\tilde{\subseteq}\tilde{Y}\tilde{\cup}\tilde{Z}=G_A\tilde{\cup}H_B$ and $\tilde{Z}$ is $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$, then either $\tilde{Z}\tilde{\subseteq}G_A$ or $\tilde{Z}\tilde{\subseteq}H_B$.

$(\mathrm{i})$ If $\tilde{Y}\tilde{\subseteq}G_A$ and $\tilde{Z}\tilde{\subseteq}G_A$, then $\tilde{Y}\tilde{\cup}\tilde{Z}\tilde{\subseteq}G_A$. Hence $H_B=\tilde{\phi}$. This is a contradiction.

(ii) If $\tilde{Y} \tilde{\subseteq} G_A$ and $\tilde{Z} \tilde{\subseteq} H_B$, then $\tilde{Y} \tilde{\bigcap} \tilde{Z} \tilde{\subseteq} G_A \tilde{\bigcap} H_B = \tilde{\phi}$, which is a contradiction to our assumption. In the same way, we can get a contradiction if $\tilde{Y} \tilde{\subseteq} H_B$ and $\tilde{Z} \tilde{\subseteq} G_A$ or if $\tilde{Y} \tilde{\subseteq} H_B$ and $\tilde{Z} \tilde{\subseteq} H_B$. Therefore $\tilde{Y} \tilde{\bigcup} \tilde{Z}$ is $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ subspace of $\left(\tilde{X}, \tilde{T}, E\right)$.

**Corollary 4.7.** Let $\left(\tilde{X}, \tilde{T}_1, E\right)$ and $\left(\tilde{X}, \tilde{T}_2, E\right)$ be $S_{\pm}-Top-Spaces$ such that $\tilde{T}_2 \tilde{\subseteq} \tilde{T}_1$ and $\left(\tilde{X}, \tilde{T}_1, E\right)$ is $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ space. Then $\left(\tilde{X}, \tilde{T}_2, E\right)$ is a $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ space.

**Proof.** Suppose $\left(\tilde{X}, \tilde{T}_2, E\right)$ is not $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ space. Then there exists a $SS_{\pm}S_{emi}{}^{\#}g\alpha-separation$ $G_A$ and $H_B$ of $S_{\pm}S_{emi}{}^{\#}g\alpha-separated$ sets in $\tilde{X}$ with soft topology $\tilde{T}_2$. Since $\tilde{T}_2 \tilde{\subseteq} \tilde{T}_1$, $G_A$ and $H_B$ are $S_{\pm}S_{emi}{}^{\#}g\alpha-separated$ set in $\tilde{X}$ with soft topology $\tilde{T}_1$. This is a contradiction. Hence, the result follows.

**Proposition 4.8.** Let $\left(\tilde{X}, \tilde{T}, E\right)$ be a $S_{\pm}-Top-Space$ over $X$. Then the following statements are equivalent:

(1) $\left(\tilde{X}, \tilde{T}, E\right)$ is $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$.

(2) There exists no $F_E, G_E \tilde{\in} S_{\pm}S_{emi}{}^{\#}g\alpha-C\left(\tilde{X}, \tilde{T}, E\right)-\left\{\tilde{\phi}\right\}$ such that $F_E \tilde{\bigcap} G_E = \tilde{\phi}$ and $F_E \tilde{\bigcup} G_E = \tilde{X}$, where $S_{\pm}S_{emi}{}^{\#}g\alpha-C\left(\tilde{X}, \tilde{T}, E\right) = \left\{F_E^C \tilde{\in} S_{\pm}S_{emi}{}^{\#}g\alpha-O\left(\tilde{X}, \tilde{T}, E\right)\right\}$.

(3) There exists no $F_E, G_E \tilde{\in} S_{\pm}S\left(X\right)_E - \left\{\tilde{\phi}\right\}$ such that $\left[F_E \tilde{\bigcap} S_{\pm}S_{emi}{}^{\#}g\alpha-Cl\left(G_E\right)\right] \tilde{\bigcup} \left[S_{\pm}S_{emi}{}^{\#}g\alpha-Cl\left(F_E\right) \tilde{\bigcap} G_E\right] = \tilde{\phi}$ and $F_E \tilde{\bigcup} G_E = \tilde{X}$.

(4) There exists no $F_E \tilde{\in} S_{\pm}S\left(X\right)_E - \left\{\tilde{\phi}, \tilde{X}\right\}$ such that $F_E \tilde{\in} \left[S_{\pm}S_{emi}{}^{\#}g\alpha-O\left(\tilde{X}, \tilde{T}, E\right)\right] \tilde{\bigcap} \left[S_{\pm}S_{emi}{}^{\#}g\alpha-C\left(\tilde{X}, \tilde{T}, E\right)\right]$.

**Proof.** $(1) \Rightarrow (2)$. Assume there exist $F_E, G_E \tilde{\in} S_{\pm}S_{emi}{}^{\#}g\alpha-C\left(\tilde{X}, \tilde{T}, E\right)-\left\{\tilde{\phi}\right\}$ such that $F_E \tilde{\bigcap} G_E = \tilde{\phi}$ and $F_E \tilde{\bigcup} G_E = \tilde{X}$. Then $\forall\, e \in E$, $F_E(e) \tilde{\bigcap} G_E(e) = \tilde{\phi}$ and $F_E(e) \tilde{\bigcup} G_E(e) = \tilde{X}$. Thus $\forall\, e \in E$, $F^C(e) = X - F(e) = G(e)$ and $G^C(e) = X - G(e) = F(e)$, which implies that $F_E^C = G_E \tilde{\in} S_{\pm}S_{emi}{}^{\#}g\alpha-C\left(\tilde{X}, \tilde{T}, E\right)-\left\{\tilde{\phi}\right\}$ and $G_E^C = F_E \tilde{\in} S_{\pm}S_{emi}{}^{\#}g\alpha-C\left(\tilde{X}, \tilde{T}, E\right)-\left\{\tilde{\phi}\right\}$. Then there exist $F_E, G_E \tilde{\in} S_{\pm}S_{emi}{}^{\#}g\alpha-O\left(\tilde{X}, \tilde{T}, E\right)-\left\{\tilde{\phi}\right\}$ such that $F_E \tilde{\bigcap} G_E = \tilde{\phi}$ and $F_E \tilde{\bigcup} G_E = \tilde{X}$. However, $\left(\tilde{X}, \tilde{T}, E\right)$ is $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$. This is a contradiction.

$(2) \Rightarrow (3)$. Assume there exist $F_E, G_E \tilde{\in} S_{\pm}S\left(X\right)_E - \left\{\tilde{\phi}\right\}$ such that $\left[F_E \tilde{\bigcap} S_{\pm}S_{emi}{}^{\#}g\alpha-Cl\left(G_E\right)\right] \tilde{\bigcup} \left[S_{\pm}S_{emi}{}^{\#}g\alpha-Cl\left(F_E\right) \tilde{\bigcap} G_E\right] = \tilde{\phi}$ and $F_E \tilde{\bigcup} G_E = \tilde{X}$. Obviously, $F_E \tilde{\bigcap} G_E = \tilde{\phi}$. Now $S_{\pm}S_{emi}{}^{\#}g\alpha-Cl\left(G_E\right) = S_{\pm}S_{emi}{}^{\#}g\alpha-Cl\left(G_E\right) \tilde{\bigcap} \tilde{X} = S_{\pm}S_{emi}{}^{\#}g\alpha-Cl\left(G_E\right) \tilde{\bigcap} \left(F_E \tilde{\bigcup} G_E\right) = \left[S_{\pm}S_{emi}{}^{\#}g\alpha-Cl\left(G_E\right) \tilde{\bigcap} F_E\right] \tilde{\bigcup} \left[S_{\pm}S_{emi}{}^{\#}g\alpha-Cl\left(G_E\right) \tilde{\bigcap} G_E\right] = G_E$, which implies that $G_E$ is a $S_{\pm}S_{emi}{}^{\#}g\alpha-closed$ set. By using the same methods, we can show that $F_E$ is also a $S_{\pm}S_{emi}{}^{\#}g\alpha-closed$ set. Hence there exist $F_E, G_E \tilde{\in} S_{\pm}S_{emi}{}^{\#}g\alpha-C\left(\tilde{X}, \tilde{T}, E\right)-\left\{\tilde{\phi}\right\}$ such that $F_E \tilde{\bigcap} G_E = \tilde{\phi}$ and $F_E \tilde{\bigcup} G_E = \tilde{X}$. This is a contradiction. So (3) holds.

$(3)\Rightarrow(4)$. Assume there exists $F_E \tilde{\in} \left[S_{ft}S_{emi}{}^{\#}g\alpha-O(\tilde{X},\tilde{T},E)\right] \tilde{\cap} \left[S_{ft}S_{emi}{}^{\#}g\alpha-C(\tilde{X},\tilde{T},E)\right]-\{\tilde{\phi},\tilde{X}\}$. If we take $G_E=F_E^C$ then $F_E,\, G_E \tilde{\in} \left[S_{ft}S_{emi}{}^{\#}g\alpha-O(\tilde{X},\tilde{T},E)\right] \tilde{\cap} \left[S_{ft}S_{emi}{}^{\#}g\alpha-C(\tilde{X},\tilde{T},E)\right]-\{\tilde{\phi}\} \left(\tilde{\subseteq} S_{ft}S(X)_E-\{\tilde{\phi}\}\right)$. Besides, we have $\left[F_E \tilde{\cap} S_{ft}S_{emi}{}^{\#}g\alpha-Cl(G_E)\right] \tilde{\cup} \left[G_E \tilde{\cap} S_{ft}S_{emi}{}^{\#}g\alpha-Cl(F_E)\right]=F_E \tilde{\cap} G_E=\tilde{\phi}$ and $F_E \tilde{\cup} G_E=\tilde{X}$. This is a contradiction, so (4) holds.

$(4)\Rightarrow(1)$. Assume $(\tilde{X},\tilde{T}_2,E)$ is not $S_{ft}S_{emi}{}^{\#}g\alpha-connected$. Then there exist $F_E,\, G_E \tilde{\in} S_{ft}S_{emi}{}^{\#}g\alpha-O(\tilde{X},\tilde{T},E)-\{\tilde{\phi}\}$ such that $F_E \tilde{\cap} G_E=\tilde{\phi}$ and $F_E \tilde{\cup} G_E=\tilde{X}$. It is easy to see that $F_E^C=G_E$ and $G_E^C=F_E$. Thus $F_E, G_E \tilde{\in} \left[S_{ft}S_{emi}{}^{\#}g\alpha-O(\tilde{X},\tilde{T},E)\right] \tilde{\cap} \left[S_{ft}S_{emi}{}^{\#}g\alpha-C(\tilde{X},\tilde{T},E)\right]-\{\tilde{\phi},\tilde{X}\}$. This is a contradiction.

**Theorem 4.9.** A non-null proper subset $F_E$ of a $S_{ft}S_{emi}{}^{\#}g\alpha-connected$ space $(\tilde{X},\tilde{T},E)$ has a non-null $S_{ft}S_{emi}{}^{\#}g\alpha-boundary$ points.

**Proof.** Suppose, by contrary, that $S_{ft}S_{emi}{}^{\#}g\alpha-Bd(F_E)=\tilde{\phi}$. Then $S_{ft}S_{emi}{}^{\#}g\alpha-Cl(F_E)=S_{ft}S_{emi}{}^{\#}g\alpha-Int(F_E)$, which means that $F_E$ is both $S_{ft}S_{emi}{}^{\#}g\alpha-open$ and $S_{ft}S_{emi}{}^{\#}g\alpha-closed$. But this contradicts the $S_{ft}S_{emi}{}^{\#}g\alpha-connectedness$ of $(\tilde{X},\tilde{T},E)$. Hence, $S_{ft}S_{emi}{}^{\#}g\alpha-Bd(F_E)\neq\tilde{\phi}$, as required.

**Definition 4.10.** A soft mapping $f:(\tilde{X},\tilde{\tau},E)\to(\tilde{Y},\tilde{\sigma},K)$ is said to be $S_{ft}S_{emi}{}^{\#}g\alpha-continuous$ $\left(\text{resp.}, S_{ft}S_{emi}{}^{\#}g\alpha-irresolute\right)$ if $f^{-1}(H_K)$ is a $S_{ft}S_{emi}{}^{\#}g\alpha-open$ set where $H_K$ is a soft open $\left(S_{ft}S_{emi}{}^{\#}g\alpha-open\right)$ set.

**Proposition 4.11.** Let $f$ be a $S_{ft}S_{emi}{}^{\#}g\alpha-continuous$ mapping of a $S_{ft}S_{emi}{}^{\#}g\alpha-connected$ space $(\tilde{X},\tilde{T},E)$ onto a $S_{ft}-Top-Space$ $(\tilde{Y},\tilde{\sigma},K)$. Then $f(\tilde{X})$ is $connected$.

**Proof.** Suppose that $f(\tilde{X})=\tilde{Y}$ is $disconnected$. Then we get $\tilde{\sigma}$ contains two non-null disjoint soft open sets $F_K$ and $G_K$. By hypothesis, $f^{-1}(F_K)$ and $f^{-1}(G_K)$ are disjoint $S_{ft}S_{emi}{}^{\#}g\alpha-open$ sets in $(\tilde{X},\tilde{T},E)$. Since $f$ is surjective, these $S_{ft}S_{emi}{}^{\#}g\alpha-open$ sets are non-null. Thus $(\tilde{X},\tilde{T},E)$ is $S_{ft}S_{emi}{}^{\#}g\alpha-disconnected$. But this is a contradiction. Hence, $(\tilde{Y},\tilde{\sigma},K)$ is $S_{ft}S_{emi}{}^{\#}g\alpha-connected$.

**Theorem 4.12.** Let $f$ be a $S_{ft}S_{emi}{}^{\#}g\alpha-$ function from a $S_{ft}-Top-Space$ $(\tilde{X},\tilde{T},E)$ onto a soft space $(\tilde{Y},\tilde{\sigma},K)$. Suppose $(\tilde{X},\tilde{T},E)$ is $S_{ft}S_{emi}{}^{\#}g\alpha-connected$. Then $(\tilde{Y},\tilde{\sigma},K)$ is $S_{ft}S_{emi}{}^{\#}g\alpha-connected$.

**Proof.** Our assumption is that $(\tilde{Y},\tilde{\sigma},K)$ is not $S_{ft}S_{emi}{}^{\#}g\alpha-connected$. Therefore, there exists a non-empty proper subset $H_K$ of $(\tilde{Y},\tilde{\sigma},K)$ which is both $S_{ft}S_{emi}{}^{\#}g\alpha-open$ and $S_{ft}S_{emi}{}^{\#}g\alpha-closed$. Then the inverse image of $H_K$ under $f$ is both $S_{ff}S_{emi}{}^{\#}g\alpha-open$ and $S_{ff}S_{emi}{}^{\#}g\alpha-closed$ in $\left(\tilde{X},\tilde{T},E\right)$, which contradicts our hypothesis.

**Corollary 4.13.** A $S_{ff}S_{emi}{}^{\#}g\alpha-irresolute$ function maps $S_{ff}S_{emi}{}^{\#}g\alpha-connected$ set onto $S_{ff}S_{emi}{}^{\#}g\alpha-connected$ set.

**Theorem 4.14.** Consider a $S_{ff}S_{emi}{}^{\#}g\alpha-continuous$ function $f$ from a $S_{ff}-Top-Space$ $\left(\tilde{X},\tilde{T},E\right)$ onto a $S_{ff}-Top-Space$ $\left(\tilde{Y},\tilde{\sigma},K\right)$. If $\left(\tilde{X},\tilde{T},E\right)$ is $S_{ff}S_{emi}{}^{\#}g\alpha-connected$, then $\left(\tilde{Y},\tilde{\sigma},K\right)$ is soft connected.

**Proof.** Consider $\left(\tilde{Y},\tilde{\sigma},K\right)$ is not soft connected. Then there is a nonempty proper subset of $\left(\tilde{Y},\tilde{\sigma},K\right)$ which is both soft open and soft closed. Then the inverse image of $H_K$ under $f$ is both $S_{ff}S_{emi}{}^{\#}g\alpha-open$ and $S_{ff}S_{emi}{}^{\#}g\alpha-closed$ in $\left(\tilde{X},\tilde{T},E\right)$, a contradiction. Though the concept of soft continuity, $S_{ff}g{\star\star}-continuity$ and $S_{ff}S_{emi}{}^{\#}g\alpha-irresolute$ are independent of each other, but they behave similarly in the case of $S_{ff}S_{emi}{}^{\#}g\alpha-connectedness$, that is, these functions map a $S_{ff}S_{emi}{}^{\#}g\alpha-connected$ set onto a soft connected set.

**Definition 4.15.** A soft function $f:\left(\tilde{X},\tilde{\tau},E\right)\rightarrow\left(\tilde{Y},\tilde{\sigma},K\right)$ is called $S_{ff}S_{emi}{}^{\#}g\alpha-homeomorphism$ if $f$ is both $S_{ff}S_{emi}{}^{\#}g\alpha-continuous$ and $S_{ff}S_{emi}{}^{\#}g\alpha-open$.

**Remark 4.16.** A soft-homeomorphism preserves $S_{ff}S_{emi}{}^{\#}g\alpha-connectedness$.

**Theorem 4.17.** If $\left(\tilde{X},\tilde{T},E\right)$ is $S_{ff}S_{emi}{}^{\#}g\alpha-connected$ space, then $\left(\tilde{X},\tilde{T},E\right)\times\{a\}$ is also $S_{ff}S_{emi}{}^{\#}g\alpha-connected$.

**Proof.** Obviously, $\left(\tilde{X},\tilde{T},E\right)$ is soft homeomorphic to $\left(\tilde{X},\tilde{T},E\right)\times\{a\}$. Then by the previous remark, $\left(\tilde{X},\tilde{T},E\right)\times\{a\}$ is $S_{ff}S_{emi}{}^{\#}g\alpha-connected$.

**Theorem 4.18.** If $\left(\tilde{X},\tilde{T},E\right)$ and $\left(\tilde{Y},\tilde{\sigma},K\right)$ are two $S_{ff}S_{emi}{}^{\#}g\alpha-connected$ spaces, then $\left(\tilde{X},\tilde{T},E\right)\times\left(\tilde{Y},\tilde{\sigma},K\right)$ is also $S_{ff}S_{emi}{}^{\#}g\alpha-connected$.

**Proof.** For any soft point $\left(x_\alpha,y_\beta\right)$ in the product $\left(\tilde{X},\tilde{T},E\right)\times\left(\tilde{Y},\tilde{\sigma},K\right)$, each of the subspaces $\left(\tilde{X},\tilde{T},E\right)\times\{y_\beta\}\tilde{\bigcup}\left(\tilde{Y},\tilde{\sigma},K\right)\times\{x_\alpha\}$ is $S_{ff}S_{emi}{}^{\#}g\alpha-connected$ since it is the union of two $S_{ff}S_{emi}{}^{\#}g\alpha-connected$ subspaces with a point in common. Then by Theorem 4.6, $\left(\tilde{X},\tilde{T},E\right)\times\left(\tilde{Y},\tilde{\sigma},K\right)$ is also $S_{ff}S_{emi}{}^{\#}g\alpha-connected$.

## V. Soft $S_{emi}{}^{\#}g\alpha$-Disconnected Space

In this section, we present a new concept of $S_{ff}S_{emi}{}^{\#}g\alpha-disconnected$ space and discuss some of its properties.

**Definition 5.1.** A $\mathrm{S}_{ft}-Top-Space\left(\tilde{X},\tilde{T},E\right)$ is said to be a $\mathrm{SoftSemi}^{\#}\mathrm{g}\alpha-disconnected$ $\left(\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-disconnected\right)$ space if there exists a $\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-separation$ of $\tilde{X}$.

**Theorem 5.2.** A $\mathrm{S}_{ft}-Top-Space\left(\tilde{X},\tilde{T},E\right)$ is $\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-disconnected$ if and only if there exists a non-empty proper soft subset of $\tilde{X}$ which is both $\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-open$ and $\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-closed$.

**Proof.** Let $\left(\tilde{X},\tilde{T},E\right)$ be a $\mathrm{S}_{ft}-Top-Space$ and $F_E$ be a non-empty proper soft subset which is both $\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-open$ and $\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-closed$ set. Since $F_E$ is $\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-closed,$ $\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-Cl\left(F_E\right)=F_E...(*)$. Also, $F_E$ is $\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-open$ implies $F_E^C=G_E$ (say) is $\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-closed$. Therefore, $\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-Cl\left(G_E\right)=G_E...(**)$. Since $G_E$ is the soft complement of $F_E$ in $\tilde{X},$ therefore $F_E\tilde{\bigcup}G_E=\tilde{X}$ and $F_E\tilde{\bigcap}G_E=\tilde{\phi}.$ Now $F_E\tilde{\bigcap}G_E=\tilde{\phi}$ implies that $\left[\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-Cl\left(F_E\right)\right]\tilde{\bigcap}G_E=\tilde{\phi}\ by\ (*)$ and $\mathrm{F}_E\tilde{\bigcap}\left[\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-Cl\left(G_E\right)\right]=\tilde{\phi}\ by\ (**)$. Since $\tilde{X}=G_E\tilde{\bigcup}G_E$ and $\left[\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-Cl\left(F_E\right)\right]\tilde{\bigcap}G_E=\tilde{\phi},$ $F_E\tilde{\bigcap}\left[\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-Cl\left(G_E\right)\right]=\tilde{\phi},$ therefore $\tilde{X}$ is $\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-disconnected$. Therefore, we conclude that $\left(\tilde{X},\tilde{T},E\right)$ is $\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-disconnected$. Conversely, suppose that $\mathrm{S}_{ft}-Top-Space\left(\tilde{X},\tilde{T},E\right)$ is $\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-disconnected$ space. Therefore, there exists a pair of non-empty proper soft subsets of $\tilde{X}$ such that $\tilde{X}=G_A\tilde{\bigcup}H_B,$ $\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-Cl\left(G_A\right)\tilde{\bigcap}H_B=\tilde{\phi},$ $\mathrm{G}_A\tilde{\bigcap}\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-Cl\left(H_B\right)=\tilde{\phi}.$ Hence, we conclude that the $\mathrm{S}_{ft}-Top-Space\left(\tilde{X},\tilde{T},E\right)$ is $\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-disconnected$ space. Now $\mathrm{G}_A\tilde{\subseteq}\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-Cl\left(G_A\right)$. Therefore, $\left[\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-Cl\left(G_A\right)\right]\tilde{\bigcap}H_B=\tilde{\phi}$ implies that $G_A\tilde{\bigcap}H_B=\tilde{\phi}.$ Also $\tilde{X}=G_A\tilde{\bigcup}H_B.$ Therefore, $G_A=H_B^C.$ Thus, $G_A$ is a proper soft subset of $\tilde{X}$ which is non-empty. Again, therefore $G_A\tilde{\bigcup}H_B=\tilde{X}$ implies $\mathrm{G}_A\tilde{\bigcup}\left[\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-Cl\left(H_B\right)\right]=\tilde{X}$ and $\mathrm{H}_B\tilde{\subseteq}\left[\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-Cl\left(H_B\right)\right]$. We have $G_A\tilde{\bigcap}H_B=\tilde{\phi}$ implies $\mathrm{G}_A\tilde{\bigcap}\left[\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-Cl\left(H_B\right)\right]=\tilde{\phi}.$ We conclude that $\mathrm{G}_A=\tilde{X}-\left[\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-Cl\left(H_B\right)\right]$. But as $H_B$ is $\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-closed$ set. Therefore $\mathrm{G}_A=\tilde{X}-\left[\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-Cl\left(H_B\right)\right]$ is $\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-open$ $set...(***)$. Similarly, we can prove $H_B=\tilde{X}-\left[\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-Cl\left(G_A\right)\right]$ is $\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-open$ set. Now $G_A=H_B^C$, where $H_B$ is $\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-open$ set. Therefore, $G_A$ is $\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-closed$ and we have already shown in $(***)$ that $G_A$ is $\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-open.$ Hence, $G_A$ is a non-empty proper soft subset which is both $\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-open$ and $\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-closed$ set.

**Theorem 5.3.** A $\mathrm{S}_{ft}-Top-Space\left(\tilde{X},\tilde{T},E\right)$ is $\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-disconnected$ space if and only if any one of the following statements holds good.

(i) $\tilde{X}$ is the soft union of two non-empty disjoint $\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-open$ sets.

(ii) $\tilde{X}$ is the soft union of two non-empty disjoint $\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-closed$ sets.

**Proof.** Let $\left(\tilde{X},\tilde{T},E\right)$ be a $\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-disconnected$ space. We have to prove $\tilde{X}$ is the soft union of two non-empty disjoint $\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-open\left(\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-closed\right)$ subsets of $\tilde{X}.$ Since $\left(\tilde{X},\tilde{T},E\right)$ is a $\mathrm{S}_{ft}\mathrm{S}_{emi}{}^{\#}\mathrm{g}\alpha-disconnected$ space, then by Theorem 5.2, there exists a non-empty proper soft subset $F_E$ of

$\tilde{X}$ which is both $S_{\pm}S_{emi}{}^{\#}g\alpha-open$ and $S_{\pm}S_{emi}{}^{\#}g\alpha-closed$. Consequently, $F_E^C$ is non-empty, which is both $S_{\pm}S_{emi}{}^{\#}g\alpha-closed$ and $S_{\pm}S_{emi}{}^{\#}g\alpha-open$. Also, $F_E \tilde{\bigcup} F_E^C = \tilde{X}$ and $F_E \tilde{\bigcap} F_E^C = \tilde{\phi}$. From above, we conclude that $\tilde{X}$ is the soft union of two disjoint non-empty $S_{\pm}S_{emi}{}^{\#}g\alpha-open$ $\left(S_{\pm}S_{emi}{}^{\#}g\alpha-closed\right)$ sets. As $F_E$ and $F_E^C$ are both $S_{\pm}S_{emi}{}^{\#}g\alpha-open$ as well as $S_{\pm}S_{emi}{}^{\#}g\alpha-closed$. Conversely suppose, $\tilde{X}$ is the soft union of two non-empty disjoint $S_{\pm}S_{emi}{}^{\#}g\alpha-open$ sets. That is $\tilde{X} = G_A \tilde{\bigcup} H_B$ and $G_A \tilde{\bigcap} H_B = \tilde{\phi}$. We need to prove $\left(\tilde{X},\tilde{T},E\right)$ is a $S_{\pm}S_{emi}{}^{\#}g\alpha-disconnected$ space. Since $G_A = H_B^C$ is $S_{\pm}S_{emi}{}^{\#}g\alpha-open$ so that $G_A$ is $S_{\pm}S_{emi}{}^{\#}g\alpha-closed$ set. Also, $H_B$ is non-empty. Therefore $G_A = H_B^C$ is a proper soft subset of $\tilde{X}$ which is non-empty and is both $S_{\pm}S_{emi}{}^{\#}g\alpha-open$ and $S_{\pm}S_{emi}{}^{\#}g\alpha-closed$ set. Hence, by Theorem 5.2, the $S_{\pm}-Top-Space$ $\tilde{X}$ is $S_{\pm}S_{emi}{}^{\#}g\alpha-disconnected$ space. In the second case, if both $G_A$ and $H_B$ are both non-empty $S_{\pm}S_{emi}{}^{\#}g\alpha-closed$ sets, then $G_A = H_B^C$ and $H_B$ is $S_{\pm}S_{emi}{}^{\#}g\alpha-closed$ set. Therefore, $G_A$ is $S_{\pm}S_{emi}{}^{\#}g\alpha-open$ set. As above $G_A$ will be a proper non-empty soft subset that is both $S_{\pm}S_{emi}{}^{\#}g\alpha-open$ and $S_{\pm}S_{emi}{}^{\#}g\alpha-closed$ set. Hence, by Theorem 5.2, the $S_{\pm}-Top-Space\left(\tilde{X},\tilde{T},E\right)$ is $S_{\pm}S_{emi}{}^{\#}g\alpha-disconnected$ space.

## VI. Soft $S_{emi}{}^{\#}g\alpha-$Component of a Soft Set and Soft Topological Space

In this section, we introduce the concept of $S_{\pm}S_{emi}{}^{\#}g\alpha-component$ of a soft subset as well as $S_{\pm}S_{emi}{}^{\#}g\alpha-component$ of a $S_{\pm}-Top-Space$ and discuss their properties and characterizations.

**Definition 6.1.** Let $F_E$ be a soft subset of a $S_{\pm}-Top-Space\left(\tilde{X},\tilde{T},E\right)$ and $x_e$ be any soft point of $F_E$, then the soft union of all $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ subsets of $F_E$ containing $x_e$ is defined as the $S_{\pm}S_{emi}{}^{\#}g\alpha-component$ of $F_E$ with respect to $x_e$ and is denoted by $S_{\pm}S_{emi}{}^{\#}g\alpha-C\left(F_E,x_e\right)$.

$S_{\pm}S_{emi}{}^{\#}g\alpha-C\left(F_E,x_e\right)=\tilde{\bigcup}\left\{P_E \tilde{\subseteq} F_E : x_e \tilde{\in} P_E, P_E \ is\ S_{\pm}S_{emi}{}^{\#}g\alpha-connected\ set\right\}$.

**Proposition 6.2.** $S_{\pm}S_{emi}{}^{\#}g\alpha-C\left(F_E,x_e\right)$ is a $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ set.

**Proof.** The $S_{\pm}S_{emi}{}^{\#}g\alpha-component$ of $F_E$ with respect to the soft point $x_e$ is the soft union of all $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ subsets of $F_E$ which have a nonempty soft intersection (as each contains the soft point $x_e$). Then by Theorem 3.8, $S_{\pm}S_{emi}{}^{\#}g\alpha-C\left(F_E,x_e\right)$ is a $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ set.

**Remark 6.3.** $S_{\pm}S_{emi}{}^{\#}g\alpha-C\left(F_E,x_e\right)$ is the largest $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ subset of $F_E$ containing $x_e$.

**Theorem 6.4.** Let $\left(\tilde{X},\tilde{T},E\right)$ be a $S_{\pm}-Top-Space$. Then the following statements are true.

$(i)$ Each soft point in $\tilde{X}$ is contained in exactly one $S_{\pm}S_{emi}{}^{\#}g\alpha-component$ of $\tilde{X}$.

$(ii)$ The $S_{\pm}S_{emi}{}^{\#}g\alpha-components$ with respect to two different soft points of $\tilde{X}$ are either identical or soft disjoint.

**Proof.** $(i)$ Let $x_e$ be an arbitrary soft point of $\tilde{X}$ and $\{P_{E_\lambda} : \lambda \in \Lambda\}$ be the collection of all $S_{\pounds}S_{emi}{}^{\#}g\alpha-connected$ subsets of $\tilde{X}$ which contain $x_e$. Since the soft singleton $\{x_e\}$ is $S_{\pounds}S_{emi}{}^{\#}g\alpha-connected,$ the collection $\{P_{E_\lambda} : \lambda \in \Lambda\}$ of all soft $S_{\pounds}S_{emi}{}^{\#}g\alpha-connected$ subsets containing the soft point $x_e$ is non-empty. Then $\bigcap\{P_{E_\lambda} : \lambda \in \Lambda\} \neq \phi,$ as $x_e$ is contained in each $P_{E_\lambda}$. Now, $S_{\pounds}S_{emi}{}^{\#}g\alpha-C(\tilde{X},x_e)$ is the soft union of $\{P_{E_\lambda} : \lambda \in \Lambda\}$ having a non-empty soft intersection. Then by **Theorem 3.8,** $S_{\pounds}S_{emi}{}^{\#}g\alpha-C(\tilde{X},x_e)$ is also $S_{\pounds}S_{emi}{}^{\#}g\alpha-connected.$ Again $S_{\pounds}S_{emi}{}^{\#}g\alpha-C(\tilde{X},x_e)$ is the maximal $S_{\pounds}S_{emi}{}^{\#}g\alpha-connected$ subset of $\tilde{X}$ containing $x_e$. Therefore, $S_{\pounds}S_{emi}{}^{\#}g\alpha-C(\tilde{X},x_e)$ is $S_{\pounds}S_{emi}{}^{\#}g\alpha-component$ of $\tilde{X}$ with respect to $x_e$. Now we have to prove that there is no other $S_{\pounds}S_{emi}{}^{\#}g\alpha-component$ of $\tilde{X}$ containing $x_e$. Now, suppose $S_{\pounds}S_{emi}{}^{\#}g\alpha-C\star(\tilde{X},x_e)$ be another $S_{\pounds}S_{emi}{}^{\#}g\alpha-component$ of $\tilde{X}$ containing $x_e$. Then $S_{\pounds}S_{emi}{}^{\#}g\alpha-C\star(\tilde{X},x_e)$ is a $S_{\pounds}S_{emi}{}^{\#}g\alpha-connected$ subset of $\tilde{X}$ containing $x_e$. But $S_{\pounds}S_{emi}{}^{\#}g\alpha-C(\tilde{X},x_e)$ being a $S_{\pounds}S_{emi}{}^{\#}g\alpha-component$ is the maximal $S_{\pounds}S_{emi}{}^{\#}g\alpha-connected$ subset of $\tilde{X}$ containing $x_e$. Consequently $S_{\pounds}S_{emi}{}^{\#}g\alpha-C\star(\tilde{X},x_e) \tilde{\subseteq} S_{\pounds}S_{emi}{}^{\#}g\alpha-C(\tilde{X},x_e).$ Similarly $S_{\pounds}S_{emi}{}^{\#}g\alpha-C(\tilde{X},x_e) \tilde{\subseteq} S_{\pounds}S_{emi}{}^{\#}g\alpha-C\star(\tilde{X},x_e)$ and hence $S_{\pounds}S_{emi}{}^{\#}g\alpha-C(\tilde{X},x_e) = S_{\pounds}S_{emi}{}^{\#}g\alpha-C\star(\tilde{X},x_e).$ This implies $x_e$ is contained in exactly one $S_{\pounds}S_{emi}{}^{\#}g\alpha-component$ of $\tilde{X}.$

$(ii)$ $S_{\pounds}S_{emi}{}^{\#}g\alpha-C(\tilde{X},x_\alpha)$ and $S_{\pounds}S_{emi}{}^{\#}g\alpha-C(\tilde{X},y_\beta)$ be respectively the $S_{\pounds}S_{emi}{}^{\#}g\alpha-components$ of $\tilde{X}$ with respect to two different soft points $x_e$ and $y_\beta$ of $\tilde{X}$ with $x_e \neq y_\beta$. If $S_{\pounds}S_{emi}{}^{\#}g\alpha-C(\tilde{X},x_\alpha) \bigcap S_{\pounds}S_{emi}{}^{\#}g\alpha-C(\tilde{X},y_\beta) = \tilde{\phi},$ then the proof is over. Suppose $S_{\pounds}S_{emi}{}^{\#}g\alpha-C(\tilde{X},x_\alpha) \bigcap S_{\pounds}S_{emi}{}^{\#}g\alpha-C(\tilde{X},y_\beta) \neq \tilde{\phi}.$ We may choose a soft point $z_\gamma \tilde{\in} S_{\pounds}S_{emi}{}^{\#}g\alpha-C(\tilde{X},x_\alpha) \bigcap S_{\pounds}S_{emi}{}^{\#}g\alpha-C(\tilde{X},y_\beta).$ Then $z_\gamma \tilde{\in} S_{\pounds}S_{emi}{}^{\#}g\alpha-C(\tilde{X},x_\alpha)$ and $z_\gamma \tilde{\in} S_{\pounds}S_{emi}{}^{\#}g\alpha-C(\tilde{X},y_\beta).$ Since $S_{\pounds}S_{emi}{}^{\#}g\alpha-C(\tilde{X},z_\gamma)$ is the maximal $S_{\pounds}S_{emi}{}^{\#}g\alpha-component$ of $\tilde{X}$ with respect to $z_\gamma$. Therefore $S_{\pounds}S_{emi}{}^{\#}g\alpha-C(\tilde{X},x_\alpha) \subseteq S_{\pounds}S_{emi}{}^{\#}g\alpha-C(\tilde{X},z_\gamma)....(1).$ Since $x_\alpha \tilde{\in} S_{\pounds}S_{emi}{}^{\#}g\alpha-C(\tilde{X},x_\alpha) \subseteq S_{\pounds}S_{emi}{}^{\#}g\alpha-C(\tilde{X},z_\gamma),$ we have $x_\alpha \tilde{\in} S_{\pounds}S_{emi}{}^{\#}g\alpha-C(\tilde{X},z_\gamma).$ Also $S_{\pounds}S_{emi}{}^{\#}g\alpha-C(\tilde{X},z_\gamma)$ is a $S_{\pounds}S_{emi}{}^{\#}g\alpha-connected$ set containing $x_\alpha$. But, $S_{\pounds}S_{emi}{}^{\#}g\alpha-C(\tilde{X},x_\alpha)$ is the maximal $S_{\pounds}S_{emi}{}^{\#}g\alpha-component$ set containing $x_\alpha$. Therefore $S_{\pounds}S_{emi}{}^{\#}g\alpha-C(\tilde{X},z_\gamma) \subseteq S_{\pounds}S_{emi}{}^{\#}g\alpha-C(\tilde{X},x_\alpha)....(2).$ From (1) and (2), we have $S_{\pounds}S_{emi}{}^{\#}g\alpha-C(\tilde{X},x_\alpha) = S_{\pounds}S_{emi}{}^{\#}g\alpha-C(\tilde{X},z_\gamma).$ Similarly $z_\gamma \tilde{\in} S_{\pounds}g\star\star-C(\tilde{X},y_\beta)$ implies that $S_{\pounds}S_{emi}{}^{\#}g\alpha-C(\tilde{X},y_\beta) = S_{\pounds}S_{emi}{}^{\#}g\alpha-C(\tilde{X},z_\gamma).$ Thus we have $S_{\pounds}S_{emi}{}^{\#}g\alpha-C(\tilde{X},x_\alpha) = S_{\pounds}S_{emi}{}^{\#}g\alpha-C(\tilde{X},y_\beta).$

**Definition 6.5.** The $S_{\pounds}S_{emi}{}^{\#}g\alpha-component$ of a space $\tilde{X}$ of a $S_{\pounds}-Top-Space$ $(\tilde{X},\tilde{T},E)$ is the maximal $S_{\pounds}S_{emi}{}^{\#}g\alpha-connected$ subspace of $(\tilde{X},\tilde{T},E)$. The $S_{\pounds}S_{emi}{}^{\#}g\alpha-component$ of a $S_{\pounds}-Top-Space$ $(\tilde{X},\tilde{T},E)$ is denoted as $S_{\pounds}S_{emi}{}^{\#}g\alpha-C(\tilde{X}).$

**Theorem 6.6.** Each $S_{ft}S_{emi}{}^{\#}g\alpha-component$ of a $S_{ft}-Top-Space\left(\tilde{X},\tilde{T},E\right)$ is a $S_{ft}S_{emi}{}^{\#}g\alpha-closed$ set.

**Proof.** Let $S_{ft}S_{emi}{}^{\#}g\alpha-C\left(\tilde{X}\right)$ be a $S_{ft}S_{emi}{}^{\#}g\alpha-component$ of $\tilde{X}.$ Then, $S_{ft}S_{emi}{}^{\#}g\alpha-C\left(\tilde{X}\right)$ is the maximal $S_{ft}S_{emi}{}^{\#}g\alpha-connected$ subset of $\tilde{X}.$ By Theorem 3.6, $S_{ft}S_{emi}{}^{\#}g\alpha-Cl\left[S_{ft}S_{emi}{}^{\#}g\alpha-C\left(\tilde{X}\right)\right]$ is also $S_{ft}S_{emi}{}^{\#}g\alpha-connected$ and $S_{ft}S_{emi}{}^{\#}g\alpha-Cl\left[S_{ft}S_{emi}{}^{\#}g\alpha-C\left(\tilde{X}\right)\right]\subseteq S_{ft}S_{emi}{}^{\#}g\alpha-C\left(\tilde{X}\right).$ But $S_{ft}S_{emi}{}^{\#}g\alpha-C\left(\tilde{X}\right)\subseteq S_{ft}S_{emi}{}^{\#}g\alpha-Cl\left[S_{ft}S_{emi}{}^{\#}g\alpha-C\left(\tilde{X}\right)\right].$ Hence, $S_{ft}S_{emi}{}^{\#}g\alpha-C\left(\tilde{X}\right)= S_{ft}S_{emi}{}^{\#}g\alpha-Cl\left[S_{ft}S_{emi}{}^{\#}g\alpha-C\left(\tilde{X}\right)\right].$ Therefore, $S_{ft}S_{emi}{}^{\#}g\alpha-C\left(\tilde{X}\right)$ is a $S_{ft}S_{emi}{}^{\#}g\alpha-closed$ set.

**Theorem 6.7.** In a $S_{ft}-Top-Space\left(\tilde{X},\tilde{T},E\right)$, the set of all disjoint $S_{ft}S_{emi}{}^{\#}g\alpha-components$ of soft points of $\tilde{X}$ form a partition of $\tilde{X}.$

**Proof.** Let $S_{ft}S_{emi}{}^{\#}g\alpha-C\left(\tilde{X},x_{\alpha}\right)$ and $S_{ft}S_{emi}{}^{\#}g\alpha-C\left(\tilde{X},y_{\beta}\right)$ be two $S_{ft}S_{emi}{}^{\#}g\alpha-components$ of distinct soft points $x_{\alpha}$ and $y_{\beta}$ in $\tilde{X}.$ Now for any soft point $x_{\alpha}\tilde{\in}\tilde{X},$ $x_{\alpha}\tilde{\in} S_{ft}S_{emi}{}^{\#}g\alpha-C\left(\tilde{X},x_{\alpha}\right)$ and hence $\tilde{X}=\tilde{\bigcup}\left\{\left\{x_{\alpha}\right\}:x_{\alpha}\in\tilde{X}\right\}\tilde{\subseteq}\tilde{\bigcup}\left\{S_{ft}S_{emi}{}^{\#}g\alpha-C\left(\tilde{X},x_{\alpha}\right):x_{\alpha}\in\tilde{X}\right\},$ by considering only the disjoint $S_{ft}S_{emi}{}^{\#}g\alpha-components$ with respect to the soft point $x_{\alpha}\tilde{\in}\tilde{X}.$ This implies that $\tilde{X}\tilde{\subseteq}\tilde{\bigcup}\left\{S_{ft}S_{emi}{}^{\#}g\alpha-C\left(\tilde{X},x_{\alpha}\right):x_{\alpha}\in\tilde{X}\right\}\tilde{\subseteq}\tilde{X}.$ Therefore $\tilde{\bigcup}\left\{S_{ft}S_{emi}{}^{\#}g\alpha-C\left(\tilde{X},x_{\alpha}\right):x_{\alpha}\in\tilde{X}\right\}=\tilde{X}.$

## VII. Locally Soft $S_{emi}{}^{\#}g\alpha-$Connected Space

In this section, we introduce the concept locally $S_{ft}S_{emi}{}^{\#}g\alpha-connected$ space and discuss its properties. Also, we analyse locally $S_{ft}S_{emi}{}^{\#}g\alpha-connected$ space with other soft connectedness like $S_{ft}S_{emi}{}^{\#}g\alpha-connected$ space, soft connected space and locally soft connected space.

**Definition 7.1.** A $S_{ft}-Top-Space\left(\tilde{X},\tilde{T},E\right)$ is called locally $S_{ft}S_{emi}{}^{\#}g\alpha-connected$ at $x_{e}\in\tilde{X}$ if and only if for every $S_{ft}S_{emi}{}^{\#}g\alpha-open$ set $P_{E}$ containing $x_{e},$ there exist a $S_{ft}S_{emi}{}^{\#}g\alpha-connected$ soft open set $F_{E}$ such that $x_{e}\tilde{\in}F_{E}\tilde{\subseteq}P_{E}.$

**Definition 7.2.** A $S_{ft}-Top-Space\left(\tilde{X},\tilde{T},E\right)$ is said to be locally $S_{ft}S_{emi}{}^{\#}g\alpha-connected$ if and only if it is locally $S_{ft}S_{emi}{}^{\#}g\alpha-connected$ at each of its soft points.

**Theorem 7.3.** Every $S_{ft}S_{emi}{}^{\#}g\alpha-component$ of a locally $S_{ft}S_{emi}{}^{\#}g\alpha-connected$ space is $S_{ft}S_{emi}{}^{\#}g\alpha-open$ set.

**Proof.** Let $S_{ft}S_{emi}{}^{\#}g\alpha-C\left(\tilde{X}\right)$ be a $S_{ft}S_{emi}{}^{\#}g\alpha-component$ of a locally $S_{ft}S_{emi}{}^{\#}g\alpha-connected$ space $\left(\tilde{X},\tilde{T},E\right)$ and let $x_{e}\tilde{\in} S_{ft}S_{emi}{}^{\#}g\alpha-C\left(\tilde{X},x_{e}\right).$ Now $\tilde{X}$ is locally $S_{ft}S_{emi}{}^{\#}g\alpha-connected.$ Therefore, it is locally $S_{ft}S_{emi}{}^{\#}g\alpha-connected$ at each soft point $x_{e}\in\tilde{X}.$ This implies that each $S_{ft}S_{emi}{}^{\#}g\alpha-open$ set containing $x_e$ contains a $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ open set $P_E$ containing the soft point $x_e$. Since the $S_{\pm}S_{emi}{}^{\#}g\alpha-component$ $S_{\pm}S_{emi}{}^{\#}g\alpha-C\left(\tilde{X},x_e\right)$ is the maximal $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ set containing $x_e$, $x_e\,\tilde{\in}\,P_E\,\tilde{\subseteq}\,S_{\pm}S_{emi}{}^{\#}g\alpha-C\left(\tilde{X},x_e\right)$, where $P_E$ is $S_{\pm}S_{emi}{}^{\#}g\alpha-open$ set. Therefore, $S_{\pm}S_{emi}{}^{\#}g\alpha-C\left(\tilde{X},x_e\right)=\tilde{\bigcup}\left\{P_E : x_e\,\tilde{\in}\,P_E\subseteq S_{\pm}S_{emi}{}^{\#}g\alpha-C\left(\tilde{X},x_e\right)\right\}$. Thus, $S_{\pm}S_{emi}{}^{\#}g\alpha-C\left(\tilde{X},x_e\right)$ is the soft union of all $S_{\pm}S_{emi}{}^{\#}g\alpha-open$ sets containing $x_e$. Therefore, $S_{\pm}S_{emi}{}^{\#}g\alpha-C\left(\tilde{X},x_e\right)$ is $S_{\pm}S_{emi}{}^{\#}g\alpha-open$ set.

**Theorem 7.4.** A $S_{\pm}-Top-Space$ $\left(\tilde{X},\tilde{T},E\right)$ is locally $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ if and only if the $S_{\pm}S_{emi}{}^{\#}g\alpha-components$ of $S_{\pm}S_{emi}{}^{\#}g\alpha-open$ sets are soft open.

**Proof.** Suppose that $\left(\tilde{X},\tilde{T},E\right)$ is locally $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$. Let $P_E$ be a $S_{\pm}S_{emi}{}^{\#}g\alpha-open$ set and $F_E$ be a $S_{\pm}S_{emi}{}^{\#}g\alpha-component$ of $P_E$. If $x_e\,\tilde{\in}\,F_E$, then $x_e\,\tilde{\in}\,P_E$. Since $\left(\tilde{X},\tilde{T},E\right)$ is locally $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ space, there is a $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ open set $O_E$ such that $x_e\,\tilde{\in}\,O_E\,\tilde{\subseteq}\,P_E$. Since $F_E$ is the $S_{\pm}S_{emi}{}^{\#}g\alpha-component$ of $x_e$ and $O_E$ is $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$, we have $x_e\,\tilde{\in}\,O_E\,\tilde{\subseteq}\,F_E$. This shows that $F_E$ is soft open.

Conversely, let $x_e\,\tilde{\in}\,\tilde{X}$ be arbitrary and let $P_E$ be a $S_{\pm}S_{emi}{}^{\#}g\alpha-open$ set containing $x_e$. Let $F_E$ be the $S_{\pm}S_{emi}{}^{\#}g\alpha-component$ of $P_E$ such that $x_e\,\tilde{\in}\,F_E$. Now $F_E$ is a $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ open set such that $x_e\,\tilde{\in}\,F_E\,\tilde{\subseteq}\,P_E$.

**Theorem 7.5.** A $S_{\pm}-Top-Space$ $\left(\tilde{X},\tilde{T},E\right)$ is locally $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ if and only if, given any soft point $x_e\,\tilde{\in}\,\tilde{X}$ and a $S_{\pm}S_{emi}{}^{\#}g\alpha-open$ set $P_E$ containing $x_e$, there is a soft open set $O_E$ containing $x_e$ such that $O_E$ is contained in a single $S_{\pm}S_{emi}{}^{\#}g\alpha-component$ of $P_E$.

**Proof.** Let $\tilde{X}$ be locally $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$, $x_e\,\tilde{\in}\,\tilde{X}$ and $P_E$ be a $S_{\pm}S_{emi}{}^{\#}g\alpha-open$ set containing $x_e$. Let $F_E$ be a $S_{\pm}S_{emi}{}^{\#}g\alpha-component$ of $P_E$ that contains $x_e$. Since $\tilde{X}$ is locally $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ and $P_E$ is $S_{\pm}S_{emi}{}^{\#}g\alpha-open$, there is a $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ open set $O_E$ such that $x_e\,\tilde{\in}\,O_E\,\tilde{\subseteq}\,P_E$. By Remark 6.3, $F_E$ is the maximal $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ set containing $x_e$ and so $x_e\,\tilde{\in}\,O_E\,\tilde{\subseteq}\,F_E\,\tilde{\subseteq}\,P_E$. Since $S_{\pm}S_{emi}{}^{\#}g\alpha-components$ are disjoint soft sets, it follows that $O_E$ is not contained in any other soft $S_{\pm}S_{emi}{}^{\#}g\alpha-component$ of $P_E$. Conversely , we suppose that given any soft point $x_e\,\tilde{\in}\,\tilde{X}$ and any $S_{\pm}S_{emi}{}^{\#}g\alpha-open$ set $P_E$ containing $x_e$, there is a soft open set $O_E$ containing $x_e$ which is contained in a soft singleton $S_{\pm}S_{emi}{}^{\#}g\alpha-component$ $F_E$ of $P_E$. Then, $x_e\,\tilde{\in}\,O_E\,\tilde{\subseteq}\,F_E\,\tilde{\subseteq}\,P_E$. Let $y_\beta\,\tilde{\in}\,F_E$, then $y_\beta\,\tilde{\in}\,P_E$. Thus, there is a soft open set $O_A$ such that $y_\beta\,\tilde{\in}\,O_A$ and $O_A$ is contained in a single $S_{\pm}S_{emi}{}^{\#}g\alpha-component$ of $P_E$. Since the $S_{\pm}S_{emi}{}^{\#}g\alpha-components$ are disjoint soft sets and $y_\beta\,\tilde{\in}\,F_E$, $y_\beta\,\tilde{\in}\,G_E\,\tilde{\subseteq}\,F_E$. Thus $F_E$ is soft open. Thus, for every $x_e\,\tilde{\in}\,\tilde{X}$ and for every $S_{\pm}S_{emi}{}^{\#}g\alpha-open$ set $P_E$ containing $x_e$, there is a $S_{\pm}S_{emi}{}^{\#}g\alpha-connected$ open set $F_E$ such that $x_e\,\tilde{\in}\,F_E\,\tilde{\subseteq}\,P_E$. Thus $\left(\tilde{X},\tilde{T},E\right)$

is locally $S_{ft}S_{emi}{}^{\#}g\alpha-connected$ at $x_e$. Since $x_e \tilde{\in} \tilde{X}$ is arbitrary, $(\tilde{X},\tilde{T},E)$ is locally $S_{ft}S_{emi}{}^{\#}g\alpha-connected$ space.

## VIII. Totally Soft $S_{emi}{}^{\#}g\alpha$-Disconnected Space

In this section, we introduce the concept of totally $S_{ft}S_{emi}{}^{\#}g\alpha-disconnected$ space in a $S_{ft}-Top-Space$ and discuss its properties and characterizations.

**Definition 8.1.** $S_{ft}S_{emi}{}^{\#}g\alpha-disconnected$ means $\tilde{X}$ is the soft union of two nonempty disjoint $S_{ft}S_{emi}{}^{\#}g\alpha-open$ sets or $S_{ft}S_{emi}{}^{\#}g\alpha-closed$ sets in $\tilde{X}$.

**Definition 8.2.** A $S_{ft}-Top-Space$ $(\tilde{X},\tilde{T},E)$ is said to be totally $S_{ft}S_{emi}{}^{\#}g\alpha-disconnected$ space if and only if for each pair of distinct soft points $x_\alpha, y_\beta \tilde{\in} \tilde{X}$, there exists a $S_{ft}S_{emi}{}^{\#}g\alpha-disconnection$ $F_E \bigcup G_E$ of $\tilde{X}$ such that $x_\alpha \in F_E$ and $y_\beta \in G_E$, where $F_E$, $G_E$ are both either $S_{ft}S_{emi}{}^{\#}g\alpha-connected$ open sets or $S_{ft}S_{emi}{}^{\#}g\alpha-connected$ closed sets.

**Example 8.3.** Every discrete $S_{ft}-Top-Space$ $\tilde{X}$ is totally $S_{ft}S_{emi}{}^{\#}g\alpha-disconnected$ space. Indeed, let $x_\alpha, y_\beta \tilde{\in} \tilde{X}$ and $x_\alpha \neq y_\beta$. Then $\{x_\alpha\}\tilde{\bigcup}(\tilde{X}-\{x_\alpha\})$ is $S_{ft}S_{emi}{}^{\#}g\alpha-disconnection$ of $\tilde{X}$.

**Proposition 8.4.** A soft subspace of a totally $S_{ft}S_{emi}{}^{\#}g\alpha-disconnected$ space is totally $S_{ft}S_{emi}{}^{\#}g\alpha-disconnected$ space. Each totally soft disconnected space is a totally $S_{ft}S_{emi}{}^{\#}g\alpha-disconnected$ space. The $S_{ft}S_{emi}{}^{\#}g\alpha-components$ of a totally $S_{ft}S_{emi}{}^{\#}g\alpha-disconnected$ space $(\tilde{X},\tilde{T},E)$ are soft singleton subsets of $\tilde{X}$.

**Proof.** Let $(\tilde{X},\tilde{T},E)$ be a totally $S_{ft}S_{emi}{}^{\#}g\alpha-disconnected$ space and $\tilde{Y}$ be a soft subset of $\tilde{X}$ consisting of two or more than two soft points. Let $x_\alpha$ and $y_\beta$ be two distinct soft points of $\tilde{Y}$. Then $x_\alpha$ and $y_\beta$ are also distinct soft points of $\tilde{X}$. Now $(\tilde{X},\tilde{T},E)$ is totally $S_{ft}S_{emi}{}^{\#}g\alpha-disconnected$. Therefore, there exists a $S_{ft}S_{emi}{}^{\#}g\alpha-disconnection$ $F_A \tilde{\bigcup} G_B$ of $\tilde{X}$ such that $x_\alpha \tilde{\in} F_A$ and $y_\beta \tilde{\in} G_B$. Let $P_C = F_A \tilde{\bigcap} \tilde{Y}$ and $Q_D = G_B \tilde{\bigcap} \tilde{Y}$. We claim that $P_C \tilde{\bigcup} Q_D$ is a $S_{ft}S_{emi}{}^{\#}g\alpha-disconnection$ of $\tilde{Y}$. For, since $x_\alpha \tilde{\in} \tilde{Y}$, $x_\alpha \tilde{\in} F_A$, $y_\beta \tilde{\in} \tilde{Y}$, $y_\beta \tilde{\in} G_B$. Both $P_C$ and $Q_D$ are non-empty. Also, $P_C \tilde{\bigcap} Q_D = (F_A \tilde{\bigcap} \tilde{Y})\tilde{\bigcap}(G_B \tilde{\bigcap} \tilde{Y}) = (F_A \tilde{\bigcap} G_B)\tilde{\bigcap} \tilde{Y} = \tilde{\phi} \tilde{\bigcap} \tilde{Y} = \tilde{\phi}$. That is $P_C$ and $Q_D$ are soft disjoint. Thus, $\tilde{Y}$ is $S_{ft}S_{emi}{}^{\#}g\alpha-disconnected$ and hence $\tilde{Y}$ a soft subset containing two or more than two soft points cannot be a $S_{ft}S_{emi}{}^{\#}g\alpha-component$ of $\hat{X}$. Therefore, the $S_{ft}S_{emi}{}^{\#}g\alpha-components$ of $\tilde{X}$ are soft singleton subsets of $\tilde{X}$ which are the only $S_{ft}S_{emi}{}^{\#}g\alpha-connected$ subsets of $\tilde{X}$.

## Conclusions

Connectedness is an important and major area of topology, and it can give many relationships between other scientific areas and mathematical models. The notion of connectedness captures the idea of the hanging-togetherness of image elements in an object by giving a firmness of connectedness to every feasible path between every possible pair of image elements. It is an essential tool for designing algorithms for image segmentation. We have used the notions of $S_{ff}S_{emi}{}^{\#}g\alpha-\text{open}$ sets and $S_{ff}S_{emi}{}^{\#}g\alpha-\text{closed}$ sets to introduce $S_{ff}S_{emi}{}^{\#}g\alpha-\text{separated}$ sets, $S_{ff}S_{emi}{}^{\#}g\alpha-\text{connected}$ space, $S_{ff}S_{emi}{}^{\#}g\alpha-\text{disconnected}$ space, $S_{ff}S_{emi}{}^{\#}g\alpha-\text{component}$ of a soft set and soft topological space, locally $S_{ff}S_{emi}{}^{\#}g\alpha-\text{connected}$ space and totally $S_{ff}S_{emi}{}^{\#}g\alpha-\text{disconnected}$ space in soft topological spaces. We have investigated properties and characterizations of these new spaces in soft topological spaces. We hope that the concepts initiated herein will be beneficial for researchers and scholars to promote and progress the study of soft topology and decision-making problems with applications in many fields soon.

## Recommendations

In the future, it is recommended to utilize soft semi #generalized α-open sets to initiate new kinds of soft semi #generalized α-compactness, namely soft semi #generalized α-Lindelofness, almost (approximately, mildly) soft semi #generalized α-compactness, and almost (approximately, mildly) soft semi #generalized α-Lindelofness. The equivalent conditions of each one of them can be investigated. Also, the behavior of these spaces under soft semi #generalized α-irresolute maps can be investigated. Furthermore, the enough conditions for the equivalence among the four sorts of soft semi #generalized α- compact spaces and for the equivalence among the four sorts of soft semi #generalized α-Lindelof spaces can be explored. The relationships between enriched soft topological spaces and the initiated spaces can be investigated in different cases. Some properties which connect some of these spaces with some soft topological notions, such as soft semi #generalized α-$T_2$-spaces and soft subspaces, can be obtained.

## Scientific Ethics Declaration

The author declares that the scientific ethical, and legal responsibility of this article published in EPSTEM journal belongs to the author.

## Conflict of Interest

* The authors declare that they have no conflicts of interest

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