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On the Solution of the Generalized Symmetric Woods-Saxon Potential in the Dirac Equation

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Abstract: In this work, we present a solution to the Dirac equation that is coupled with vector and scalar generalized symmetric Woods-Saxon potential energy in one plus one space-time. The chosen potential energy has a flexible structure to examine four different physical problems. The potential energy can be a barrier or well depend on being attractive or repulsive. Furthermore, the included surface effects can be attractive or repulsive. Therefore, in one class a potential barrier occurs with a pocket or an extra barrier nearby the effective radius. Similar effects occur in the potential well whether the surface effects are repulsive or attractive. Here we use the usual two-component approach. We obtain the solutions in hypergeometric function form.

Keywords: Dirac equation, Scalar and vector potential energies, Generalized symmetric, Woods-Saxon potential energy

Introduction

In 1928, only after two years of the introduction of the second order Klein-Gordon equation [Klein 1926], Dirac declared the first order relativistic wave equation [Dirac 1928]. Basically, Dirac equation describes the dynamics of spin half particles and possesses results in the high accuracy of the fine details of the hydrogen spectrum. The time passed but its fashion did not pass, and it is still in used to describe the natural laws in many areas in physics [Lee 2009, Hosseinpour 2017].

In 1954, Woods and Saxon introduced a potential energy alternative to the square barrier, later will be called with their names (WSP), to calculate the differential cross section of elastically scattered protons from medium or heavy nuclei [Woods & Saxon 1954]. The potential energy obtained its fame with the highly accurate results especially in nuclear [Brandan et al 1997] , atom and molecule physics [Costa et al. 1999] within relativistic[Bayrak et al. 2015, Rojas et al.2005] and non-relativistic [Zaichenko et. Al 1976] approaches. Besides those studies, Satchler declared a more generalized form of the WSP, namely GWSP, by introducing the surface interactions in addition to the core effects. [Satchler 1983].

In one dimension we investigate the symmetric form of the GWSP (GSWSP) energy is given form

$$V(x) = \Theta(-x)\left[\frac{-V_0}{1+e^{-\alpha(x+L)}} + \frac{W\, e^{-\alpha(x+L)}}{(1+e^{-\alpha(x+L)})^2}\right] + \Theta(x)\left[\frac{-V_0}{1+e^{\alpha(x-L)}} + \frac{W\, e^{\alpha(x-L)}}{(1+e^{\alpha(x-L)})^2}\right]$$

Here, $\Theta(x)$ denotes the step function. The GSWSP energy has three common parameters with WSP energy. V_0, α, L are the strength, the slope and the effective distance of the potential energy.

Recently, we studied the GWSP energy within non-relativistic [Lütfüoğlu, Akdeniz, Bayrak 2016] and relativistic approaches [Lütfüoğlu, Lipovsky, Kriz 2018, Lütfüoğlu 2018]. Furthermore, we analyzed the comparison of the WSP and GSWSP in terms of thermodynamic functions [Lütfüoğlu Commun.Theor. Phys. 2018]. Finally, we used the GSWSP energy to obtain energy spectra by solving Schrödinger and Klein-Gordon equations and discussed the results in the statistical mechanic point of view [Lütfüoğlu Can. J. Phys. 2018].

In this paper, we investigate the solution of the Dirac equation within the presence of scalar and vector GSWSP energy. In method section we derive the Dirac equation. We discuss the solution in the Results and Discussion section.

Method

We use analytical algebraic methods to solve the Dirac equation in the natural units.

Dirac Equation

We start with the non-interacting Dirac equation

$$\left(i\gamma^{\mu}\frac{\partial}{\partial x^{\mu}}-m\right)\Psi(x,t)=0\,.$$

Here, m is the mass of the particle. We couple an external potential which describes the interaction via the four momentum and mass.

$$\left[\gamma^{\mu}\left(i\frac{\partial}{\partial x^{\mu}}-eA_{\mu}\right)-(m+gV_s)\right]\Psi(x,t)=0\,.$$

Here, e and g are coupling constants. In 1+1 dimension, the four-vector A_{μ} has two components, namely time and spatial components. We propose a nonzero time component and call it as the vector potential. Furthermore, we choose the spatial component as zero. Moreover, we take Pauli matrices σ^3 and $i\sigma^1$ instead of the gamma matrices. Since the GSWSP energy does not depend on time, we can separate the wave function into time and space functions.

$$\Psi(x,t)=\psi(x)e^{-iEt}$$

Then, we substitute them into the Dirac equation and we find

$$\left[\sigma^3(E-eA_0)-\sigma^1\frac{d}{dx}-(m+gV_s)\right]\psi(x)=0\,.$$

We choose the external potential to be equal to GSWSP energy with equal magnitude. Then, we decompose the Dirac spinor, into upper $u_1(x)$ and lower $u_2(x)$, spinors as

$$\psi(x)=\begin{pmatrix}u_1(x)\\ u_2(x)\end{pmatrix}.$$

and we obtain coupled first order equations

$$\frac{du_2(x)}{dx}=\left[\big(E-V(x)\big)-\big(m+V(x)\big)\right]u_1(x),$$
$$\frac{du_1(x)}{dx}=\left[-\big(E-V(x)\big)+\big(m+V(x)\big)\right]u_2(x).$$

We follow the path that was introduced in the book of Flügge [Flügge 1974]

$$\phi(x)=u_1(x)+i\,u_2(x),$$
$$\chi(x)=u_1(x)\,-\,i\,u_2(x).$$

After some algebra we obtain

$$\frac{d^2\phi(x)}{dx^2}+\left[\big(E-V(x)\big)^2-\big(m+V(x)\big)^2+i\,\frac{d}{dx}\,V(x)\right]\phi(x)-K(x)\phi(x)=0,$$

$$\frac{d^2\chi(x)}{dx^2} + \left[\left(E - V(x)\right)^2 - \left(m + V(x)\right)^2 - i\,\frac{d}{dx}\,V(x)\right]\chi(x) - L(x)\chi(x) = 0.$$

Here

$$K(x)\phi(x) = \frac{\frac{d}{dx}V(x)}{m + V(x)}\left[\frac{d}{dx} - i\left(E - V(x)\right)\right]\phi(x),$$
$$L(x)\chi(x) = \frac{\frac{d}{dx}V(x)}{m + V(x)}\left[\frac{d}{dx} + i\left(E - V(x)\right)\right]\chi(x).$$

Chabab et al. showed that these terms can be ignored when the strength of interaction is very small to the mass term [Chabab et al. 2016].

Results and Discussion

Since GSWSP energy is symmetric, we study only negative region. Moreover, we examine the solutions only for one of the composite spinors, $\phi(x)$. All results can be extended. We apply the following transformation

$$z = \left[1 + e^{-\alpha(x+L)}\right]^{-1},$$

and we find that the GWSP energy and its derivative become

$$V(x) \xrightarrow{yields} -(V_0 - W_0)z - W_0 z^2,$$
$$\frac{d}{dx}V(x) \xrightarrow{yields} \alpha\, z\,(z-1)[(V_0 - W_0) + 2W_0\, z].$$

Then we obtain a dimensionless differential equation as follows.

$$\left[\frac{d^2}{dz^2} + \left(\frac{1}{z} + \frac{1}{z-1}\right)\frac{d}{dz} - \frac{\epsilon^2}{z^2(z-1)^2} + \frac{C^2}{(z-1)^2} + \frac{D^2}{z(z-1)} + \frac{F^2}{z-1}\right]\phi_L(z) = 0,$$

Here, we abbreviate the following terms:

$$\epsilon^2 = \frac{m^2 - E^2}{\alpha^2},$$
$$C^2 = \frac{2(E + m - i\alpha)W_0 + i\alpha V_0}{\alpha^2},$$
$$D^2 = \frac{(2E + 2m - i\alpha)V_0 + (E + m - i\alpha)W_0}{\alpha^2},$$
$$F^2 = \frac{2\mathrm{i}W_0}{\alpha},$$

One can obtain the general solution with the ansatz

$$\phi_L(z) = z^{\mu}(z-1)^{\nu} f(z),$$

It is a very well-known result that such differential equations yield to be a hypergeometric differential equation. Therefore, we obtain the function $f(z)$ in terms of Gauss hypergeoemetric functions 2F1. Here, we avoid discussing the results in details since it is going to be published in a separate paper.

Conclusion

In this work, we discuss the solution of the Dirac equation when it is coupled with vector and scalar GSWSP energy in 1+1 dimension. We obtain that the resulting wave function is expressed in terms of Gauss 2F1 hypergeometric functions.

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References

Bayrak, O. & Aciksoz, E.. (2015). Corrected analytical solution of the generalized Woods-Saxon potential for arbitrary ℓ states. *Phys. Scr*., 90, 015302.

Brandan, M. E. & Satchler, G. R.. (1997). The interaction between light heavy-ions and what it tells us, *Phys. Rep.,* 285, 143–243.

Chabab, M., El Batoul, A., Hassanabadi, H., Oulne, M., & Zare, S.. (2016). Scattering states of Dirac particle equation with position-dependent mass under the cusp potential. *Eur. Phys. J. Plus,* 131(11).

Costa, L.S., Prudente, F. V., Acioli, P. H., Soares Neto, J. J. & Vianna, J. D. M.. (1999). A study of confned quantum systems using the Woods-Saxon potential, *J. Phys. B: At., Mol. Opt. Phys.,* 32, 2461–2470.

Dirac, P. A. M.. (1928). The quantum theory of the electron, *Proc. Roy. Soc. A*117, 610-628.

Flügge, S.. (1974). *Pratical Quantum Mechanics*. Berlin: Spinger.

Hosseinpour, M., Andrade, F.M., Silva, E.O., & Hassanabadi, H. (2017). Scattering and bound states for the Hulthén potential in a cosmic string background. *Eur. Phys. J. C,* 77: 270.

Klein, O.. (1926). Quantentheorie und fünfdimensionale Relativitätstheorie, *Z. Phys.,* 37, 895-906.

Lee, D. -H.. (2009). Surface states of topological insulators: The dirac fermion in curved two-dimensional spaces. Phys. Rev. Lett., 103, 196804.

Lütfüoğlu, B. C., Akdeniz, F.& Bayrak, O.. (2016). Scattering, bound, and quasi-bound states of the generalized symmetric Woods-Saxon potential, *J. Math. Phys.,* 57, 032103.

Lütfüoğlu, B. C.. (2018). Comparative Effect of an Addition of a Surface Term to Woods-Saxon Potential on Thermodynamics of a Nucleon, *Commun. Theor. Phys*., 69, 23-27.

Lütfüoğlu, B. C.. (2018). Surface interaction effects to a Klein-Gordon particle embedded in a Woods-Saxon potential well in terms of thermodynamic functions, *Can. J. Phys*., 96, 843-850.

Lütfüoğlu, B. C, Lipovsky, J. & Kriz, J..(2018) Scattering of Klein-Gordon particles in the background of mixed scalar-vector generalized symmetric Woods-Saxon potential, *Eur. Phys. J. Plus*, 133, 17.

Lütfüoğlu, B. C.. (2018). An investigation of the bound-state solutions of the Klein-Gordon equation for the generalized Woods-Saxon potential in spin symmetry and pseudo-spin symmetry limits, *Eur. Phys. J. Plus,* 133, 309.

Rojas, C. & Villalba, V. M. (2005). Scattering of a Klein-Gordon particle by a Woods-Saxon potential, *Phys. Rev. A*, 71, 052101.

Satchler, G. R.. (1983). *Direct Nuclear Reaction*. Oxford: Oxford University Press.

Woods, R. D. & Saxon, D. S.. (1954). Diffuse Surface Optical Model for Nucleon-Nuclei Scattering, *Phys. Rev.,* 95, 577-578.

Zaichenko, A. K.& Ol'khovskii, V. S.. (1976) Analytic solutions of the problem of scattering by potentials of the Eckart class, *Theor. Math. Phys.,* 27, 475–477.

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