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Investigation Methods for a Family of Cubic Dynamic Systems

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Abstract: A broad family of differential dynamic systems is considered on a real plane of their phase variables x, y. The main common feature of systems under consideration is the follows: every particular system includes two equations with polynomial right parts of the third order in one equation and of the second order in another one. These polynomials are mutually reciprocal in the following understanding: their decomposition into forms of lower order does not contain common multipliers. The whole family of such dynamic systems has been split into subfamilies according to numbers of different multipliers in the abovementioned decomposition and depending on an order of sequence of different roots of polynomials. Every subfamily has been studied in a Poincare circle using especially developed investigation methods. As a result all possible for the dynamic systems belonging to this family phase portraits have been revealed and described. There appeared to exist more than 200 different topological types of phase portraits in a Poincare circle. The obtained results have a scientific interest as well as a methodical and educational one.

Keywords: Dynamic systems, Differential equations, Poincare circle, Phase portraits

Introduction

A differential dynamic system is interesting as a mathematical model of a phenomenon or a process, where statistical events (fluctuations) may be disregarded. Main characteristics of a dynamic system are: its initial state and a law of its transition to a different state. A totality of all possible (admissible) states for a given dynamic system is called a phase space of it.

The two fundamentally different categories of dynamic systems are: systems with continuous time and systems with discrete time. For cascades (dynamic systems with discrete time) their behavior is described as a sequence of states of a system. For flows (dynamic systems with continuous time) a state of a system is defined for each time point on a real (or an imaginary) axis. Flows and cascades are studied in the topological and the symbolic dynamics.

All types of dynamic systems (the both - with discrete and with continuous time) are commonly described using an autonomous system of differential equations. Such a system is defined in a certain domain. In that domain it satisfies the conditions of the Cauchy theorem of existence and uniqueness of solutions of differential equations. In this model singular points of differential equations will correspond to the positions of equilibrium of dynamic systems. Also periodical solutions of differential equations will correspond to dynamic systems' closed phase curves.

A global task of the dynamic systems theory is an investigation of curves which are defined by differential equations. It's necessary to split a phase space into trajectories and study a limit behavior of them. That means to find out equilibrium positions and make their classification; reveal attracting and repulsive manifolds (attractors and repellers, or the so-called sinks and sources).

Notions of the greatest importance in the theory of dynamic systems are: the notion of stability of equilibrium states, i.e. an ability of a system to remain near an equilibrium state under satisfactory small changes of initial data - or on a certain manifold - for an arbitrary long period of time; and the notion of a roughness of a system (preserving of a considered system's properties under some small changes of the model itself). A rough dynamic

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system preserves its qualitative character of motion despite of (satisfactory small) changes of its parameters. Special research methods described in this paper we consider effective; they may be useful for investigation of applied dynamic systems having polynomials in their right parts.

A bit of History

Studies of normal autonomous second-order differential systems with polynomial right parts were inspired by the great French mathematician-encyclopedist Jules Henri Poincare (April 29, 1854 – July 17, 1912). Jules H. Poincare (together with David Hilbert) is considered as one of the very last mathematicians, who were able to keep in mind all results in all areas of contemporary for them mathematics and mechanics. Poincare has created the topology, the qualitative theory of differential equations, he has developed new methods in celestial mechanics, also he has laid the mathematical foundations of the theory of relativity, etc.

Jules Poincare has revealed, that normal autonomous second-order differential system (having polynomial right parts) allows in principle its full qualitative investigation on an extended arithmetical plane $\overline{R}^2_{x,y}$ (A. Andronov and other authors, 1973).

Being inspired by Poincare's studies, next generations of mathematicians – followers, including contemporary scientists, have successfully studied some kinds of such systems. Among those, for example, are the quadratic dynamic systems (A. Andreev, I. Andreeva, 1997); systems, containing nonzero linear terms; cubic homogeneous systems; dynamic systems having nonlinear (of the odd degrees 3, 5, 7) homogeneous terms (A. Andreev, I. Andreeva, 2007), which have a center or a focus in a singular point O (0,0) (A. Andreev and other authors, 2017), and some additional systems' types.

The Article Subject Matter

We describe in the present paper results of the research of an extended family of dynamic systems on a real (arithmetical) plane *x, y*

$$\frac{dx}{dt} = X(x, y), \quad \frac{dy}{dt} = Y(x, y), \tag{1}$$

for which *X (x, y), Y (x, y)* are reciprocal forms of *x* and *y, X* be a cubic, and *Y* be a square form, such as *X (0,1) > 0, Y (0, 1) > 0.*

Our aim is to obtain and describe in a Poincare circle (A. Andreev, I. Andreeva, 2017) all types of phase portraits, possible – and different in the topological meaning - for the Eq. (1) systems, as well as to indicate special criteria of every portrait's appearance, close to coefficient criteria.

We follow the developed by Henri Poincare method of consecutive mappings: at first we perform a central mapping of a plane *x, y* (from a center (0, 0, 1) of a sphere ∑), augmented with a line at infinity ($\overline{R}^2_{x,y}$ plane) on a sphere ∑: $X^2 + Y^2 + Z^2 = 1$ with identified diametrically opposite points.
The first Poincare transformation helps us to do this:

$$x = \frac{1}{z}, \quad y = \frac{u}{z} \quad \left(u = \frac{y}{x}, \quad z = \frac{1}{x}\right)$$

The Eq.(1) system this mapping transforms into a system, which in the Poincare coordinates

u, z after a time change $dt = -z^2 d\tau$ looks like

$$\frac{du}{d\tau} = P(u)u - Q(u)z, \ \frac{dz}{d\tau} = P(u)z,$$

where $P(u) :\equiv X(1, u)$ and $Q(u) :\equiv Y(1, u)$ are reciprocal polynomials.

Secondly we use an orthogonal mapping of a lower enclosed semi sphere of a sphere ∑ to a circle $\overline{\Omega}: x^2 + y^2 \leq 1$ with identified diametrically opposite points of its boundary *Γ*. This is achieved with the help of the second Poincare transformation.

The second Poincare transformation

$$x = \frac{v}{z}, \quad y = \frac{1}{z} \quad \left(v = \frac{x}{y}, \quad z = \frac{1}{y}\right)$$

also anambiguosly maps a phase plane $R^2_{x,y}$ onto a Poincare sphere $\sum$ with the diametrically opposite points identified, considered without its equator, and every Eq.(1) system the second Poincare transformation converts into a system, which in the coordinates τ, v, z looks like:

$$\frac{dv}{d\tau} = -X(v,1) + Y(v,1)vz, \ \frac{dz}{d\tau} = Y(v,1)z^2.$$

The circle $\bar{\Omega}$ and the sphere $\sum$ are correspondingly called in this process the Poincare circle and the Poincare sphere (A. Andronov and other authors, 1973).

Main Notation Basic Definitions

$\varphi(t,p), \ p = (x,y)$ –a fixed point := a solution (a motion) of an Eq.(1) system with initial data $(0,p)$.
$L_p : \varphi = \varphi(t,p), t \in I_{max}$, - a trajectory of a motion $\varphi(t,p)$.
$L_p^{+(-)}$:= +(-)- a semi trajectory of a trajectory L_p.
O-curve of a system := its semi trajectory L^s_p ($p \neq O$, $s \in \{+,-\}$), adjoining to a point *O* under a condition that $st \to +\infty$.
$O^{+(-)}$- curve of a system := its *O*-curve $L_p^{+(-)}$.
$O_{+(-)}$-curve of a system := its *O*-curve, adjoining to a point *O* from a domain $x > 0$ ($x < 0$).
TO-curve of a system := its *O*-curve, which, being supplemented by a point *O*, touches some ray in it.
A nodal bundle of *NO*-curves of a system := an open continuous family of its *TO*-curves L^s_p, where $s \in \{+,-\}$ is a fixed index, $p \in \Lambda$, Λ - a simple open arc, $L^s_p \cap \Lambda = \{p\}$.
A saddle bundle of *SO*-curves of a system, a separatrix of the point *O*:= a fixed *TO* -curve, which is not included into some bundle of *NO*-curves of a system.

E, H, P - O-sectors of a system: an elliptical, a hyperbolic, a parabolic ones.
A topological type (T-type) of a singular point *O* of a system := a word A_O consisting of letters *N,S* (a word B_O consisting of letters *E,H,P*), which describes a circular order of bundles *N, S* of its *O*-curves (of its *O*-sectors *E, H, P*) when traversing the point *O* in the «+»-direction, i. e. counterclockwise, starting with some of them.

$$P(u) \coloneqq X(1,u) \equiv p_0 + p_1 u + p_2 u^2 + p_3 u^3,$$
$$Q(u) \coloneqq Y(1,u) \equiv a + bu + cu^2$$

Note 1. For each Eq.(1) system:

1) a topological type (T-type) of a singular point *O* in its form B_O is naturally to obtain from its T-type in the form A_O , and backwords (we have to find both forms);
2) the real roots of a polynomial $P(u)$ (polynomial $Q(u)$) are actually the angular coefficients of isoclines of infinity (isoclines of a zero);
3) while listing the real roots of the system's polynomials $P(u)$, $Q(u)$, alltogether or separately, we number the roots of each polynomial in an ascending order.

Investigation of a Singular Point O (0, 0). Topological types of it

For finding of all *O*-curves and splitting of their totality into the bundles *N*, *S*, we apply the method of exceptional directions of a system in the point O (A. Andronov and other authors, 1973). In accordance with this manuscript, an equation of the exceptional directions for the point O of the Eq.(1) system must be written as

$$xY(x,y) \equiv x(ax^2 + bxy + cy^2) = 0.$$

The following cases can be implemented for it:
1) if $d \equiv b^2 - 4ac > 0$ this equation defines simple straight lines $x = 0$ and $y = q_i x$, $i = 1,2$, $q_1 < q_2$,
2) if $d = 0$ this equation defines the straight line $x = 0$ and the double straight line $y = qx$, $q = -\frac{b}{2c}$,
3) if $d < 0$ – only a straight line $x = 0$.
Theorem 1. Words A_O and B_O defining a T-type of a singular point O(0, 0) of the Eq.(1) system:
1) if $d > 0$ depending on signs of values $P(q_i)$, $i = 1,2$, have forms, indicated in a Table 1,
2) if $d = 0$ depending on signs of values q and $P(q)$ - forms, indicated in a Table 2,
3) if $d < 0$ a form: $A_O = S_0S^0$, $B_O = HH$
(A. Andreev, I. Andreeva, 2007, 2008, 2009, 2010, 2017).

Table 1. T-type of a singular point in the case of $d > 0$ $(r = \overline{1,6})$.

r	$P(q_1)$	$P(q_2)$	A_0	B_0
1, 4	+	+	$S_0S_+^1N_+^1S^0N_-^1S_-^2 = S_0S_+^1NS_-^2$	PH^2
2	–	–	$S_0N_+^1S_+^1S^0S_-^1N_-^2 = NS_+^2S^0S_+^1$	PH^2
3, 6	–	+	$S_0N_+^1N_+^1S^0S_-^1S_-^2$	$PEPH^3$
5	+	–	$S_0S_+^1S_+^2S^0N_-^1N_-^2$	H^3PEP

Table 2. T-type of the singular point O(0, 0) in the case of $d = 0$.

q	$P(q)$	A_0	B_0
+	+	$S_0S_+N_+S^0$	H^2P
–	–	$S_0N_+S_+S^0$	PH^2
+	–	$S_0S^0S_-N_-$	H^2P
–	+	$S_0S^0N_-S_-$	PH^2
0	+	$S_0S_+NS_-$	H^2P
0	–	$NS_+S^0S_-$	PH^2

Note 2. It is necessary to explain the meaning of some new symbols appeared in the statement of the Theorem 1. A symbol S_0 (a symbol S^0) means a bundle S, adjoining to the point $O(0, 0)$ from the domain $x > 0$ along a semi axis $x = 0, y < 0$, when $t \to +\infty$ (along a semi axis $x = 0$, $y > 0$, when $t \to -\infty$).

A lower sign index «+» or «-» of every bundle N or S, different from S_0 and S^0, indicates wheather the bundle consists of O_+-curves or of O_--curves.

Upper index 1 or 2 of every bundle indicates wheather its O-curves adjoining to the point O along a straight line $y = q_1 x$ or along a straight line $y = q_2 x$.
In the Table 2, lines 5, 6, a bundle N doesn`t have a lower sign index, because it contains both O_+ -curves and O_- -curves simultaneously.
Corollary 1. It's possible to deduce from the Theorem 1, that Eq.(1) systems on the $R^2_{x;y}$ plane do not have limit cycles.

Really, a limit cycle could surround a singular point O (0,0), and in that case the Poincare index of such a singular point should equal to 1 (A. Andronov and other authors, 1973). But let us use the Bendixon's formula for the index of an isolated singular point of a smooth dynamic system. It looks like the follows:

$$I(O) = 1 + \frac{e-h}{2},$$

where e (h) is a number of elliptical (hyperbolic) O-sectors of the system. Bendixon's formula and our Theorem 1 say, that Poincare index $I(O) = 0$ for the singular point O (0, 0) of every Eq.(1) system.

Corollary 2. For the singular point O (0, 0) of the Eq.(1) system eleven different topological types are possible. Their further analysis give: for every Eq.(1) system a singular point O(0, 0) has not more than four separatrices (in fact 2, 3 or 4 of them).

Infinitely Remote Singular Points (IR-points)

Further we study infinitely remote singular points of Eq.(1) system.
Topological types of IR-points $O_i\,(u_i, 0) \neq O_0\,(0, 0), i = \overline{1, m},$ of Eq.(1) systems are given in the
Theorem 2. Let a real number $u_i\,(\neq 0)$ be a root having the multiplicity $k_i \in \{1,2,3\}$ of a polynomial $P(u)$ of the Eq.(1) system. In this case a value $g_i = P^{(ki)}(u_i)Q(u_i) \neq 0$ and words $A_i^{\pm}$, which determine theT-types of IR-points $O_i^{\pm}\,(u_i, 0)$ of such a system, depending on the value of k_i and signs of numbers u_i and g_i, have forms indicated in the Table 3**.**

Table 3**.** T-types of IR-points $O_i^{\pm}\,(u_i, 0),\ i \in \{1, \ldots, m\}$.

u_i	k_i	g_i	A_i^+	A_i^-
+(-)	1, 3	+	$N_+(N_-)$	$S_-(S_+)$
+(-)	1, 3	_	$S_-(S_+)$	$N_+(N_-)$
+(-)	2	+	$S_-N_+(\emptyset)$	$\emptyset\,(N_-S_+)$
+(-)	2	_	$\emptyset\,(N_-S_+)$	$S_-N_+(\emptyset)$

Corollary 3. For the IR-points of the Eq.(1) systems only finite number, namely 13 of different T-types can appear. The further study of them indicates, that IR-points of each Eq.(1) system may have only *m* separatrices: one separatrice per each singular point $O_i\,(u_i, 0),\quad i = \overline{1, m}.$
Note 3. In the tables 3 and 4 a lower sign index «+» or «-» of every bundle *N* or *S*, indicates wheather the bundle adjusts to the point O_i^+ (or to the point O_i^-) from the side $u > u_i$ or from the side $u < u_i$ of the isocline $u = u_i$.

Different Subfamilies of an Eq.(1) Family of Dynamic Systems

Further we shall discuss special subfamilies of different order, which must be naturally distinguished depending on particular form of the decompositions of their polynomial right parts into multipliers of lower degrees. They need individual investigation and show different results of it, surely having also some common features.

Systems Containing 3 and 2 Multipliers in their Right Parts

Here we discuss Eq. (1) systems with decompositions of forms *X (x, y), Y (x, y)* into real forms of lower degrees including 3 and 2 multipliers correspondingly:

$$X(x, y) = p_3(y - u_1 x)(y - u_2 x)(y - u_3 x),\ Y(x, y) = c(y - q_1 x)(y - q_2 x), \qquad (2)$$

where $p_3 > 0,\ c > 0,\ u_1 < u_2 < u_3,\ q_1 < q_2,\ u_i \neq q_j$ for each *i* and *j*.

The investigation method demands the following actions.

Basic notation and main concepts

P(u), Q(u) – system's polynomials *P, Q:*
$P(u) := X(1, u) \equiv p_3(u - u_1)(u - u_2)(u - u_3),\ Q(u) := Y(1, u) \equiv c(u - q_1)(u - q_2),$

RSP (*RSQ*) – be an ascending sequence of all real roots of the polynomial *P(u) (Q(u)), RSPQ* – be an ascending sequence of all real roots of both polynomials *P(u), Q(u).*

A Double Change (DC)-transformation let's call a double change of variables: *(t, y)*→ *(-t, -y).* A Double Change transformation converts a system into another dynamic system, where signs and numberings of roots of *P(u),* *Q(u)*, and also the direction of motion upon trajectories with the increasing of *t* are reversed. We call a pair of different Eq. (2) systems mutually inversed in relation to a DC-transformation, if such a transformation converts one system in this pair into another member of the given pair, and call them independent of a DC-transformation otherwise.

Naturally, 10 diverse types of *RSPQ* may appear for an arbitrary Eq. (2) system, due to $C_5^2 = \frac{5!}{3!2!} = 10$.

Using the DC-transformation of Eq. (2) systems we can conclude: six of them appear to be independent in pairs. But each of the rest four systems has the mutually inversed one among the first six dynamic systems.

Now we assign a specific number $r \in \{1,\ldots,10\}$ to each one of different *RSPQ's* of the Eq. (2) systems in such a way that *RSPQ* $r = \overline{1,6}$ will be independent in pairs, but *RSPQ* sequences numbered with $r = \overline{7,10}$ will be mutually inversed to *RSPQ*'s having numbers $r = \overline{1,4}$.

The following notion is important.
An *r*-family of the Eq. (2) systems $:=$ the totality of systems (belonging to the Eq. (2) family) having the *RSPQ* number *r* .

Via the common plan we research families of Eq. (2) systems with numbers $r = \overline{1,6}$. For families numbered with $r = \overline{7,10}$, we receive data using the DC-transformation of families, $r = \overline{1,4}$.

Here is a sequential plan of investigations for every given Eq. (2) family.

Step 1. We draw up a list of singular points of systems in a Poincare circle $\overline{\Omega}$. They are: a point $O\ (0, 0) \in \Omega$ and points $O_i^{\pm}$ $(u_i, 0) \in \Gamma$, $i = \overline{0,3}$, $u_0 = 0$. For every certain point in the list we apply the notions of a saddle (S) and node (N) bundles of adjacent to this point semi trajectories, of a separatrix of this singular point, and of its topodynamical type (TD – type).

Step 2. Divide the whole family into subfamilies with numbers $s = \overline{1,7}$. For each given subfamily find topodynamical types of singular points as well as separatrices of them.

Step 3. Reveal the separatrices' behavior for all singular points of systems $\forall\ s \in \{1, \ldots, 7\}$. The important questions are: a question of uniqueness of continuation of every given separatrix from a small neighborhood of a singular point to all the lengths of it; a question of a mutual arrangement of all separatrices in a Poincare circle Ω. We have answered this questions for all families of considered systems.

Step 4. Depict phase portraits belonging to systems of a given family and describe criteria of each portrait existence.

Investigation results for these subfamilies looks like the follows.

Systems belonging to the family number *r*=1 have 25 different types of phase portraits.
Families number 2 and 3: there exist 9 types of phase portraits per each family.
Families 4 and 5: there are 7 types of phase portraits per each one.
Systems belonging to the family number *r*=6 show us 36 different topological types of phase portraits.
Thus we conclude, that 93 different types of phase portraits in a Poincare circle exist in a total for the Eq. (2) systems. At the first glance there are lots of possibilities. But keep in mind, please: every chosen subfamily includes an uncountable number of differential dynamic systems.

Other Possible Types of Right Parts

Here we enlist other variants of decompositions of polynomials in right parts of our dynamic systems. All those types we've fully investigated and described.

1. Systems, for which decompositions of forms *X (x, y), Y (x, y)* into real multipliers of lowest degrees contain two multipliers each:

$X(x, y) = p(y - u_1 x)^{k_1} (y - u_2 x)^{k_2}$, $Y(x, y) = q(y - q_1 x)(y - q_2 x)$, (3)
where $p, q, u_1, u_2, q_1, q_2 \in R$, $p > 0$, $q > 0$, $u_1 < u_2$, $q_1 < q_2$, $u_i \neq q_j$ for each i, $j \in \{1, 2\}$, $k_1, k_2 \in N$, $k_1 + k_2 = 3$.

It's naturally to distinguish two classes of Eq. (3) -systems. The A class contains systems with $k_1 = 1$, $k_2 = 2$, and the B class contains systems with $k_1 = 2, k_2 = 1$.

2. Systems containing 3 and 1 different multipliers in right parts.

$\frac{dx}{dt} = p_3(y - u_1 x)(y - u_2 x)(y - u_3 x)$, $\frac{dy}{dt} = c(y - q_1 x)^2$, (4)
$p_3 > 0$, $c > 0$, $u_1 < u_2 < u_3$, $q\ (\in R) \neq u_i$, $i = \overline{1,3}$.

3. Systems containing 2 and 1 different multipliers in right parts.

$\dot{x} = p_0 x^3 + p_1 x^2 y + p_2 x y^2 + p_3 y^3 \equiv p_3 (y - u_1 x)^2 (y - u_2 x)$, (5)

$\dot{y} = x^2 + bxy + cy^2 \equiv c(y - qx)^2$,
where $p_3 > 0$, $c > 0$, $u_1 < u_2$, $q\ (\in R) \neq u_{1,2}$.

Conclusion

This paper presents the results of the original investigation project. Its global aim is to reveal and describe all phase portraits in a Poincare circle different in the topological sense, which are possible for the broad and extended family of dynamic differential systems. All those portraits have been successfully constructed in the two forms - in the descriptive (as a table) and in the graphical ones.

The second aim of this work was to develop, successfully apply and describe certain new effective methods of investigation (A. Andreev, I. Andreeva, 2007, 2010, 2017).

Recommendations

The article presents a theoretical work, but above mentioned new methods of investigation may be useful for applied studies of dynamic systems of the second order with polynomial right parts. The work may be interesting for researchers as well as for students and post-graduate students.

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