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Inverse Kinematics for a Walking in-Pipe Robot Based on Linearization of Small Rotations

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Abstract: The paper considers walking in-pipe robots, which represent a novel class of in-pipe robots, with better agility but also a more complicated control compared with other, more prevalent in-pipe robot types. The focus of the paper is on the inverse kinematics (IK) of these robots. IK for walking in-pipe robots is a difficult problem due to a combination of factors, such as joint limits, multiple possible kinematic singularities, as well as a significant number of joints that these robots have. All this requires the use of an algorithm that could take into account multiple objectives and constraints when solving the problem, and provide a solution in real time using on-board computers. Existing approaches can achieve this with local linearization of both the objective function and the constraints; alternatively they do it by taking the constraints into account. In this work, the IK is transformed into a quadratic program. Instead of linearizing the objective function, here the orientations of the robot's links are approximated by convex combinations of rotation matrices. This allows relaxing the constraints associated with the special orthogonal group, placed on the matrices describing the links' orientation. The paper shows the form of the resulting quadratic program, discusses the practical aspects of using this approach and lists its limitations.

Keywords: In-pipe walking robot, Inverse kinematics, Orthogonal matrices, Constraint relaxation

Introduction

In-pipe robots are an important research direction in the modern robotics. Their tight connection to other types of robots, such as snake-like and caterpillar-like robots, wheeled and tracked robots, as well as walking robots, makes the in-pipe robotics an especially interesting field from both the theoretical and practical perspectives.

Most popular types of in-pipe robots include wheeled and tracked robots with parallel structures, which allow the robot to adapt to the changing geometry of the pipeline (Brown et al., 2018). Some examples of these robots can be found in (Roh et al., 2009; Jun et al., 2004). However, the ability of these robots to adapt to the varied and changing geometries of the pipelines is limited, due to their locomotion type. In particular, the pipes with significant changes in diameter and with sharp bends could present difficulties for a number of prototypes of tracked and wheeled in-pipe robots. Thus, the use of these robots is limited by the pipe types they can navigate.

In order to solve tasks that require navigating more challenging pipelines, walking in-pipe robots are proposed. Some of the original designs for in-pipe walking robots can be found in (Zagler & Pfeiffer, 2003; Pfeiffer, 2007; Gálvez et al., 2001). Those designs feature 8 legs with point-like contact pads. In (Savin & Vorochaeva, 2017a, 2017b, 2017c) in-pipe robots with 6 and 4 legs are studied, including a design with the robot's body split into two sections. The papers provided a computationally efficient approach to generating step sequences for these robots, taking into account the geometry of the pipeline. These methods require pipeline maps in order to work.

Papers (Thielemann et al., 2008; Savin, 2017; Tsubouchi et al., 2000) provide some tools for generating and working with pipeline maps, however this task remains open for further research.

The control of walking robots, including in-pipe robots, is based on the separation of control tasks. It is typical to separate out inverse kinematics, state estimation and feedback control. Some approaches to feedback control for walking robots can be found in (Mason et al., 2014; Savin et al., 2017b). State estimation for in-pipe walking robots was studied in (Savin et al., 2018) and for on-ground walking robots in (Bloesch et al., 2013). Analytic approaches to the inverse kinematics for some types of in-pipe walkers were proposed in (Savin et al., 2017a). However, the analytic solutions for the inverse kinematics are limited and do not allow to include inequality constraints associated with joint limits and mechanism self-intersections without reworking the original solution. This motivates the use of numeric inverse kinematics methods.

Numeric Inverse Kinematics, State of the Art

There are a number of numeric inverse kinematics algorithms proposed for mobile robots. These include a variety of so-called Jacobian methods (Buss, 2004). Walking robots can be viewed as systems with redundancy, hence the inverse kinematics methods developed for such systems are applicable for them as well (Chang, 1987). One of the popular approaches for redundant inverse kinematics tasks had been the division of tasks into a hierarchy and assigning the tasks different priorities (Sentis & Khatib, 2005). It can be effectively used with walking robots, as shown in (Jatsun et al., 2016). This can be achieved by a null space projection. Alternative approaches include learning methods (D'Souza, 2001).

There had been some efforts to formulate inverse kinematics as an optimization problem. In (Dai et al., 2017) inverse kinematics had been formulated as a mixed integer convex program, which can be solved with a branch and bound algorithm. It is also possible to formulate the inverse kinematics as a quadratic program, which can be directly solved by an interior point algorithm. This can be done with a local linearization of the inverse kinematics task.

This paper focuses on an alternative approach. Instead of linearizing the task, we propose to linearize the rotations (the rotation matrices in particular) that are included in the robot kinematics. This is similar to what is done in (Dai et al., 2017), where sine functions are replaced by piece-wise linear functions, suited for use in a mixed integer convex program. However, our approach is based on local linear approximation which does not require integer variables. Additionally, our approach does not require working with individual harmonic functions and allows to work directly with rotation operators.

Rotation Linearization

In order to linearize rotations, we use the following representation of the rigid body orientation:

$$\mathbf{T} = \mathbf{T}_d \mathbf{T}_0 \tag{1}$$

where $\mathbf{T}$ is a matrix defining the orientation of a rigid body, $\mathbf{T}_0$ is a matrix, that defines the initial position of the rigid body (or equivalently, an initial guess for the rigid body orientation), and $\mathbf{T}_d$ is a rotation matrix that defines the change in the body's orientation. The idea behind this representation is to allow us to tune the matrix $\mathbf{T}_d$, which would be close enough to an identity matrix, instead of tuning matrix $\mathbf{T}$, which is an arbitrary element of the SO(3) group.

The matrix $\mathbf{T}_d$ is an element of the SO(3) group, and its elements are constrained to lie on a three dimentional surface in the ambient 9-dimensional space $\mathbb{R}^{3\times3}$. However, because $\mathbf{T}_d$ is close to an identity matrix, we can approximate it as a convex combination of rotation matrices, each of which is close enough to an identity:

$$\begin{cases} \mathbf{T}_d = \alpha_1 \mathbf{I} + \alpha_2 \mathbf{T}_x^\varphi + \alpha_3 \mathbf{T}_y^\varphi + \alpha_4 \mathbf{T}_z^\varphi + \alpha_5 (\mathbf{T}_x^\varphi)^\mathrm{T} + \alpha_6 (\mathbf{T}_y^\varphi)^\mathrm{T} + \alpha_7 (\mathbf{T}_z^\varphi)^\mathrm{T} \\ \sum_{i=1}^{7} \alpha_i = 1 \\ \alpha_i \geq 0 \end{cases}, \quad (2)$$

where $\mathbf{T}_x^\varphi$, $\mathbf{T}_y^\varphi$, $\mathbf{T}_z^\varphi$ are rotation matrices around axes x, y and z axes by angle φ, $\mathbf{I}$ is an identity matrix and α_i are positive scalar coeffitients. The transposed matrices serve as inverses of the rotation matrices $\mathbf{T}_x^\varphi$, $\mathbf{T}_y^\varphi$, $\mathbf{T}_z^\varphi$, allowing $\mathbf{T}_d$ to represent all possible rotations, rather than only the ones in the clock-wise direction.

The downside of the proposed linearization is that it relies on the fact that $\mathbf{T}_d$ is close enough to $\mathbf{I}$, so that φ can be chosen to be small. The larger the φ is, the further the convex combination (2) can drift from the set of special orthogonal matrices. This motivates the use of this linearization in iterative algorithms, where on each iteration only a slight change in the orientation needs to be calculated.

Inverse Kinematics as a Quadratic Program

Let G be a point that needs to be moved to its desired position $\mathbf{r}_G^*$ (expressed in the world frame). Its actual position in the body frame is $\mathbf{r}_G^0$. The error $\mathbf{e}$ of the inverse kinematics task can then be defined as follows:

$$\mathbf{e} = \mathbf{T}_d \mathbf{T}_0 \mathbf{r}_G^0 + \mathbf{r}_0 - \mathbf{r}_G^* \quad (3)$$

where $\mathbf{r}_0$ is the position of the origin of the body frame, expressed in the world frame.

Using expressions (2) and (3) it is possible to formulate inverse kinematics as a quadratic program. The condition (2) is further relaxed by the introduction of a slack variable $\mathbf{S}$:

$$\begin{aligned} &\text{minimize:} \quad w_1 \|\mathbf{T}_d - \mathbf{S}\|_2^2 + w_2 \|\mathbf{r}_0\|^2 + w_3 \|\mathbf{T}_d\|_2^2 + w_4 \|\mathbf{S}\|_2^2 + w_5 \|\mathbf{e}\|^2 + w_6 \sum_{i=1}^{7} \alpha_i^2 \\ &\text{subject to:} \\ &\begin{cases} \mathbf{e} = \mathbf{T}_d \mathbf{T}_0 \mathbf{r}_G^0 + \mathbf{r}_0 - \mathbf{r}_G^* \\ \mathbf{S} = \alpha_1 \mathbf{I} + \alpha_2 \mathbf{T}_x^\varphi + \alpha_3 \mathbf{T}_y^\varphi + \alpha_4 \mathbf{T}_z^\varphi + \alpha_5 (\mathbf{T}_x^\varphi)^\mathrm{T} + \alpha_6 (\mathbf{T}_y^\varphi)^\mathrm{T} + \alpha_7 (\mathbf{T}_z^\varphi)^\mathrm{T} \\ \sum_{i=1}^{7} \alpha_i = 1 \\ \alpha_i \geq 0 \end{cases} \end{aligned}, \quad (4)$$

where $\|\cdot\|_2$ is a Frobenius matrix norm, and $\|\cdot\|$ is a Euclidean vector norm. In this optimization problem, variables $\mathbf{S}$, $\mathbf{T}_d$, α_i, $\mathbf{r}_0$ and $\mathbf{e}$ serve as decision variables (optimization parameters), and the rest are constants. It is possible to add any linear constraints (equalities or inequalities), which can be expressed as linear functions of these variables.

We can observe that the problem (4) includes 31 decision variables. If the inverse kinematics is solved for a robot with *n* links connected via rotary joints, then the number of decision variables is $25n+6$. This number does not depend on the degrees of freedom of the joints; they will however affect the form of the additional constraints imposed on the problem. The proposed algorithm includes a forward kinematics expression (3). In order to generalize the algorithm to work for multilink mechanisms, this expression needs to be changed accordingly.

Additionally, the resulting matrix $\mathbf{T}_d$ might require orthogonalization, in order to avoid having the matrices $\mathbf{T}$ and $\mathbf{T}_0$ drift from the set of orthogonal matrices. This can be accomplished by the Gram-Schmidt procedure.

Simulation Results

Let us consider the case when the robot stands on two legs on the supporting surface. The typical structures of walking in-pipe robots include four and more legs, and the structure presented in this section can be viewed as a simplified model.

Figure 1 shows the robot's pose generated by the proposed algorithm. The inverse kinematic task in this case includes contact elements G_1 and G_2, as well as the robot's center of mass. In the picture, the red stars show the desired position of the points in the inverse kinematics task, while green dots show their position, corresponding to the found solution.

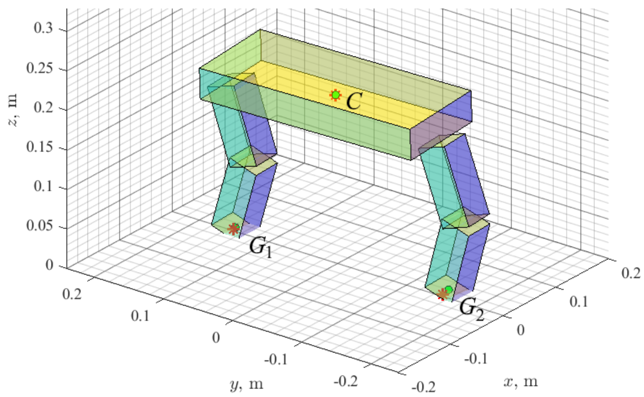

Figure 1. The robot pose, found by solving inverse kinematics

In the previous section, it was discussed that the chosen value of φ in the linear approximation of the rotation matrices, can influence the quality of the resulting approximation, and hence influence the drift of the approximation away from the set of orthogonal matrices. In order to measure this drift, we introduce the following cost function:

$$g = \sqrt{\sum_{i=1}^{n} (\det(\mathbf{T}_{d,i}) - 1)^2}\ , \tag{5}$$

where $\mathbf{T}_{d,i}$ are the matrices, obtained by solving equations (4). The motivation for this choice of the cost function is that the determinant of the orthogonal matrices in the SO(3) group must be equal to 1. Figure 2 shows how g depends on the choice of the parameter φ.

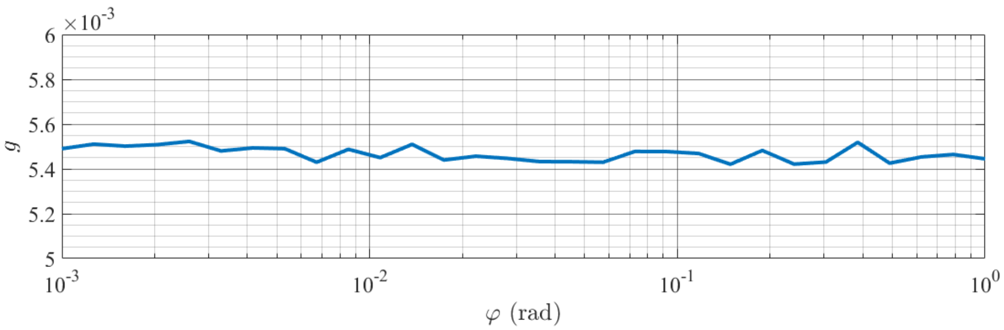

Figure 2. The dependence of g on the choice of φ; the horizontal axis is logarithmic

Figure 2 demonstrates that in this case the drift, as characterized by the cost function (5), stays the same, regardless of the choice of φ. This can be viewed as robustness of the algorithm.

Conclusion

In this work, it was shown that a linearization of the rotation matrix can be used to represent the inverse kinematics as a quadratic program, which allows the use of effective numeric methods to solve it. The proposed solution relied on approximating small angle rotations as convex combination of seven base matrices. This relaxes the constraints associated with the special orthogonal group, but makes it possible for the solution (and in particular, for the resulting linear transformation representing the orientation of the body) to drift, violating the orthogonality constraints.

The paper showed that the violation of the orthogonality constraints does not strongly depend on the chosen approximation parameter. Whether or not different parametrizations of the linear approximation could allow to better control this drift requires further study.

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References

Bloesch, M., Hutter, M., Hoepflinger, M. A., Leutenegger, S., Gehring, C., Remy, C. D., & Siegwart, R. (2013). State estimation for legged robots-consistent fusion of leg kinematics and IMU. *Robotics*, *17*, 17-24.

Brown, L., Carrasco, J., Watson, S., & Lennox, B. (2018). Elbow Detection in Pipes for Autonomous Navigation of Inspection Robots. *Journal of Intelligent & Robotic Systems*, 1-15.

Buss, S. R. (2004). Introduction to inverse kinematics with jacobian transpose, pseudoinverse and damped least squares methods. *IEEE Journal of Robotics and Automation*, *17*(1-19), 16.

Chang, P. (1987). A closed-form solution for inverse kinematics of robot manipulators with redundancy. *IEEE Journal on Robotics and Automation*, *3*(5), 393-403.

Dai, H., Izatt, G., & Tedrake, R. (2017). Global inverse kinematics via mixed-integer convex optimization. In *International Symposium on Robotics Research, Puerto Varas, Chile* (pp. 1-16).

D'Souza, A., Vijayakumar, S., & Schaal, S. (2001). Learning inverse kinematics. In *Intelligent Robots and Systems, 2001. Proceedings. 2001 IEEE/RSJ International Conference on*(Vol. 1, pp. 298-303). IEEE.

Gálvez, J. A., De Santos, P. G., & Pfeiffer, F. (2001). Intrinsic tactile sensing for the optimization of force distribution in a pipe crawling robot. *IEEE/ASME Transactions on mechatronics*, *6*(1), 26-35.

Jatsun, S., Savin, S., Lushnikov, B., & Yatsun, A. (2016). Algorithm for motion control of an exoskeleton during verticalization. In *ITM Web of Conferences* (Vol. 6). EDP Sciences.

Jun, C., Deng, Z., & Jiang, S. (2004, August). Study of locomotion control characteristics for six wheels driven in-pipe robot. In *Robotics and Biomimetics, 2004. ROBIO 2004. IEEE International Conference on* (pp. 119-124). IEEE.

Mason, S., Righetti, L., & Schaal, S. (2014, November). Full dynamics LQR control of a humanoid robot: An experimental study on balancing and squatting. In *Humanoid Robots (Humanoids), 2014 14th IEEE-RAS International Conference on* (pp. 374-379). IEEE.

Pfeiffer, F. (2007). The TUM walking machines. *Philosophical Transactions of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, *365*(1850), 109-131.

Roh, S. G., Lee, J. S., Moon, H., & Choi, H. R. (2009). In-pipe robot based on selective drive mechanism. *International Journal of Control, Automation and Systems*, *7*(1), 105-112.

Savin, S. (2017, June). An algorithm for generating convex obstacle-free regions based on stereographic projection. In *Control and Communications (SIBCON), 2017 International Siberian Conference on* (pp. 1-6). IEEE.

Savin, S., Jatsun, S., & Vorochaeva, L. (2017). Trajectory generation for a walking in-pipe robot moving through spatially curved pipes. In *MATEC Web of Conferences* (Vol. 113, p. 02016). EDP Sciences.

Savin, S., Jatsun, S., & Vorochaeva, L. (2017, November). Modification of Constrained LQR for Control of Walking in-pipe Robots. In *Dynamics of Systems, Mechanisms and Machines (Dynamics), 2017* (pp. 1-6). IEEE.

Savin, S., Jatsun, S., & Vorochaeva, L. (2018). State observer design for a walking in-pipe robot. In *MATEC Web of Conferences* (Vol. 161, p. 03012). EDP Sciences.

Savin, S., & Vorochaeva, L. (2017, June). Footstep planning for a six-legged in-pipe robot moving in spatially curved pipes. In *Control and Communications (SIBCON), 2017 International Siberian Conference on* (pp. 1-6). IEEE.

Savin, S., & Vorochaeva, L. (2017, May). Pace pattern generation for a pipeline robot. In *Industrial Engineering, Applications and Manufacturing (ICIEAM), 2017 International Conference on* (pp. 1-6). IEEE.

Savin, S., & Vorochaeva, L. (2017, May). Nested quadratic programming-based controller for pipeline robots. In *Industrial Engineering, Applications and Manufacturing (ICIEAM), 2017 International Conference on* (pp. 1-6). IEEE.

Sentis, L., & Khatib, O. (2005). Synthesis of whole-body behaviors through hierarchical control of behavioral primitives. *International Journal of Humanoid Robotics*, *2*(04), 505-518.

Thielemann, J. T., Breivik, G. M., & Berge, A. (2008, June). Pipeline landmark detection for autonomous robot navigation using time-of-flight imagery. In *Computer Vision and Pattern Recognition Workshops, 2008. CVPRW'08. IEEE Computer Society Conference on* (pp. 1-7). IEEE.

Tsubouchi, T., Takaki, S., Kawaguchi, Y., & Yuta, S. I. (2000). A straight pipe observation from the inside by laser spot array and a TV camera. In *Intelligent Robots and Systems, 2000.(IROS 2000). Proceedings. 2000 IEEE/RSJ International Conference on* (Vol. 1, pp. 82-87). IEEE.

Zagler, A., & Pfeiffer, F. (2003, September). "MORITZ" a pipe crawler for tube junctions. In *Robotics and Automation, 2003. Proceedings. ICRA'03. IEEE International Conference on* (Vol. 3, pp. 2954-2959). IEEE.

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