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A New Potential Model for Cluster Decay of the Heavy Nuclei

Orhan BAYRAK
Akdeniz University

Abstract: We calculate cluster decay half-life of the heavy nuclei by using the WKB barrier potential model. In this model, the cluster is already preformed in the parent nucleus and orbiting around the daughter nucleus before tunneling through a potential barrier between the cluster and daughter nuclei. In this paper, we purposethe Morse potential describing interactions between the cluster and daughter nuclei in order to calculate the cluster decay half-life of the heavy nuclei. The Morse potential model is used frequently the vibrational and rotational energy spectrum of diatomic molecules and atomic scattering process. Similar calculation for Morse potential is used the vibrational and rotational energy spectrum of the nucleus and there is also experimental evidence that the nucleus has a molecular structure. Since the nucleus has a molecular structure, we use the Morse potential in order to describe the interactions between the cluster and daughter nuclei and show that the Morse potential is a convenient model in order to explain cluster decay half-life of the heavy nuclei.

Keywords: Cluster decay, Morse potential, WKB method, Decay half-life

Introduction

Investigating the cluster decay of a heavy nucleus is a convenient way in order to probe the experimental observable of the nucleus for both nuclear structure and reactions. In literature, some studies have been carried out for the idea that light or heavy nuclei are clustered as alpha or heavier cluster. Pei and Xu (2007) used a mean-field-type cluster potential to model decay of alpha particle from ^{10}Be, ^{20}Ne and ^{12}C parent nuclei by using the WKB (Wentzel-Kramers-Brillouin) method. It was calculated the alpha decay widths for the excited states of ^{10}Be, ^{20}Ne and ^{12}C parent nuclei and compared the experimental data. Xu et al. (2010) analyzed the decay mechanisms of the ^{8}Be and ^{12}C cluster from the ^{24}Mg nucleus within the framework of the Woods-Saxon potential model and calculated the decay widths for the excited rotational band levels of the ^{24}Mg nucleus. Another phenomenological model is the proximity potential and has extensive application in the explaining the cluster decay mechanism of the heavy nuclei (Santhosh, 2012; Zheng, 2013; Zhang, 2016). The proximity potential model was also applied to the alpha and clustering mechanisms of super-heavy nuclei ($^{294-326}$122) (Santhosh, 2009). Soylu et al. (2012) examined the effect of deformation on decay half-lives in the cluster decay processes of parent nuclei by using Woods-Saxon square potential with deforming nuclear radius. The decay half-life of ^{14}C, ^{20}O, ^{23}F, $^{24\text{-}26}$Ne, $^{28\text{-}30}$Mg and ^{32}Si cluster from heavy nuclei were investigated by using the generalized liquid drop model formula for heavy nuclei (Royer, 2001). Another model in the explaining nuclear decay observable is the microscopic model. Microscopic density-dependent double folding potential model, which takes into account the M3Y nucleon-nucleon interaction, was used in order to investigate the clustering mechanisms of heavy nuclei (Ren, 2004; Delion, 2007; Adel, 2017).

In this paper, taking into account that the nucleus is a molecular structure (Oertzen et. al. (2006), we suggest the Morse potential in order to explain the cluster decay observable of heavy nuclei. In next section, we present model and calculation procedure. The brief discussion for obtained numerical results are given in Results and Discussion section. Then, we give a conclusion.

Method

According to the cluster model, the cluster is already preformed in the parent nucleus and orbiting around the daughter nucleus before tunneling through a potential barrier constituted between the cluster and daughter nuclei. The effective potential between the cluster and daughter nuclei is

$$V_{eff}(r) = V_L(r) + V_N(r) + V_C(r) ,$$

where $V_L(r), V_N(r)$ and $V_C(r)$ are the centrifugal, nuclear and Coulomb potentials, respectively. In the calculation, modified version of the centrifugal potential is used (Langer, 1937) and the Coulomb potential consists of the point charged cluster and uniformly charged spherical daughter nucleus with radius R_C (Satchler, 1983). The nuclear potential is the Morse potential and given as follows (Morse, 1929; Bayrak et. al., 2006),

$$V_N(r) = D_e(e^{-a(r-re)} - 2e^{-a(r-re)}) ,$$

where D_e, a and r_e are dissociation energy, a control parameter and the equilibrium distance (bound length) between nuclei.

The Alpha decay width is

$$\Gamma = P_c F \frac{\hbar^2}{4\mu} T ,$$

where P_c, F and T are the form factor, normalization factor and decay probability of the cluster nucleus from the parent nucleus respectively (Xu et. al., 2008). In literature, the form factor P_c is found to vary in the range of $P_c = 0,005 - 1$ (Bai et. al., 2019). In the calculation, we take $P_c = 1$. Normalization factor F (Xu et. al., 2008) is,

$$F = (\int_{r_1}^{r_2} dr \frac{1}{k(r)})^{-1} ,$$

where r_1 and r_2 are the inner and outer turning points and calculated from root of equation $V_{eff}(r) = Q$ and $k(r) = \sqrt{\frac{2\mu}{\hbar^2} |V_{eff}(r) - Q|}$. Here μ and Q are the reduced mass of the cluster and daughter nucleus and reaction energy, respectively (Xu et. al., 2008). Decay probability can be calculated by WKB methods as follows,

$$T = \exp(-2\int_{r_2}^{r_3} k(r)dr) ,$$

where r_3 is the outermost turning point. The cluster decay half-life can be calculated from $T_{1/2} = \hbar \ln 2 / \Gamma$ (Xu et. al., 2008). In order to eliminate the uncertainty in the depth of the potential we can use the Bohr-Sommerfeld formula in following form (Xu et. al., 2008),

$$\int_{r_1}^{r_2} k(r)dr = (G - L + 1)\frac{\pi}{2}$$

where G and L are the global and angular momentum quantum numbers. In the calculation we take $G = 5Ac$ and $L = 0$ (Buck et. al., 1996). Here Ac is atomic mass number of cluster nucleus.

Results and Discussion

We systematically calculate the cluster decay half-life of Ra, Th, U and Pu nuclei by using WKB method within the framework of the Bohr-Sommerfeld quantization formula. In the calculation we use the Morse potential for the nuclear potential. In order to eliminate uncertainty in depth of the nuclear potential, we take into account the clustering effect by using the Bohr-Sommerfeld quantization formula. In the calculation the equilibrium distance is taken as $r_e = 1.1 fm$. In order to obtain a best fit for the experimental data, we search optimum control parameter and find in the range of 0.59-0.61 fm for variation of atomic mass number of parent nucleus A. After considering variation of parameter a with A, we develop a formula as follow,

$$a = 8618.71 - 6.24252\,A + 990966.93/A - 174267.854/\sqrt{A}$$

We calculate decay half- life of the Ra, Th, U and Pu parent nuclei with the same potential parameters.

Table 1. Comparison of theoretical and experimental half-lives (Buck et. al., 1996)

Decay	Q(MeV)	$T_{1/2}$(Square Well) (s)	$T_{1/2}$(cosh)	$T_{1/2}$(Mixed Woods-Saxon) (s)	$T_{1/2}$(Present Study) (s)	$T_{1/2}$(Experimental Data) (s)
${}^{222}_{88}Ra \rightarrow {}^{208}_{82}Pb + {}^{14}_{6}C$	33.158	1.28x10^{11}	1.38x10^{11}	1.93x10^{11}	0.98×10^{11}	(1.01±0.14) x10^{11}
${}^{224}_{88}Ra \rightarrow {}^{210}_{82}Pb + {}^{14}_{6}C$	30.639	5.31x10^{15}	7.36x10^{15}	1.30x10^{16}	10.83×10^{15}	(8.25±2.22) x10^{15}
${}^{226}_{88}Ra \rightarrow {}^{212}_{82}Pb + {}^{14}_{6}C$	28.316	4.07x10^{20}	7.06x10^{20}	1.21x10^{21}	2.77 x10^{21}	(2.21±0.96) x10^{21}
${}^{228}_{90}Th \rightarrow {}^{208}_{82}Pb + {}^{20}_{8}O$	44.867	3.95x10^{21}	5.64x10^{21}	5.78x10^{21}	6.95 x10^{20}	(5.29±1.01) x10^{20}
${}^{230}_{90}Th \rightarrow {}^{206}_{80}Hg + {}^{24}_{10}Ne$	57.954	3.73x10^{24}	5.16x10^{24}	4.83x10^{24}	1.44 x10^{24}	(4.10±0.95) x10^{24}
${}^{232}_{92}U \rightarrow {}^{208}_{82}Pb + {}^{24}_{10}Ne$	62.492	5.77x10^{20}	5.68x10^{20}	4.75x10^{20}	5.79 x10^{20}	(2.50±0.30) x10^{20}
${}^{234}_{92}U \rightarrow {}^{206}_{80}Hg + {}^{28}_{12}Mg$	74.349	2.30x10^{25}	2.51x10^{25}	2.13x10^{25}	8.84 x10^{25}	(5.50±1.00) x10^{25}
${}^{236}_{94}Pu \rightarrow {}^{208}_{82}Pb + {}^{28}_{12}Mg$	79.896	2.72x10^{21}	1.95x10^{21}	1.42x10^{21}	4.46 x10^{21}	4.7x10^{21}
${}^{238}_{94}Pu \rightarrow {}^{206}_{80}Hg + {}^{32}_{14}Si$	91.474	6.79x10^{25}	5.64x10^{25}	4.19x10^{25}	2.26 x10^{25}	(1.89±0.68) x10^{25}

In Table 1, we compare the cluster decay half-lives obtained the Morse potential with the experimental data. We also compare other potential models (the square well, cosh potentials) in literature (Buck et. al., 1996). In the calculation, we see that the Morse potential is also a convenient model in the explaining cluster decay half-lives of heavy nuclei (Table 1).

Conclusion

We systematically explain the cluster decay half-lives of Ra, Th, U and Pu nuclei within the framework of the Bohr-Sommerfeld quantization by using WKB method. We compare our result with experimental data and obtain good agreement between our result and experimental data. We see that the Morse potential is also good model in the explaining the decay mechanism of heavy nuclei. But, in order to increase the reliability of our model potential, we have to extent the calculations for the Morse potential to all cluster decay experimental half-lives data of heavy nuclei. These calculations are in progress.

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Author Information

Orhan Bayrak
Akdeniz University
Akdeniz University, Dumlupınar Boulevard,
07058 Campus, Antalya / TURKEY
Contact E-mail: *bayrak@akdeniz.edu.tr*