The Eurasia Proceedings of Science, Technology, Engineering & Mathematics (EPSTEM)
ISSN: 2602-3199

The Eurasia Proceedings of Science, Technology, Engineering & Mathematics (EPSTEM), 2019

Volume 6, Pages 148-150

ICRES 2019: International Conference on Research in Education and Science

A Perturbative Approach in the Minimal Length of Quantum Mechanics

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Abstract: There are many pieces of evidence for a minimal length of the order of Planck length in the problems in quantum gravity, string theory, and black-hole physics etc. Existing of such a minimal length description modifies the traditional Heisenberg uncertainty principle. The novel form is called "the generalized uncertainty principle" in the jargon. Such a deformation in the uncertainty relation changes the corresponding wave equation. The latter Schrodinger equation is now no more a second-order differential equation. Consequently, this causes a great difficulty to obtain the analytic solutions. In this study, we propose a perturbative approach to the bound state solutions of the Woods-Saxon potential in the Schrodinger equation by adopting the minimal length. Here, we take the extra term as a perturbative term to the Hamiltonian. Then, we calculate the first order corrections of the energy spectrum for a confined particle in a well by a Woods-Saxon potential energy.

Keywords: Schrödinger equation, Generalized uncertainty principle, Perturbation theory

Introduction

Since the foundation of quantum mechanics, there has been a growing interest in investigating exact solutions in the relativistic and non-relativistic regime. Especially, in the ordinary non-relativistic quantum mechanics examination of the analytical solutions of the Schrödinger equation with various potential energies is an ongoing research area in molecular physics [Eshghi et al 2019].

Recently, a new concept, namely the Generalized Uncertainty Principle (GUP), is being a subject to the studies on quantum mechanics [Chung et al 2019], quantum gravity [Villalpando et al 2019], quantum cosmology [Bosso et al 2019] and black-hole physics [Xiang et al 2018]. The novelty in the GUP is the modification of the commutation relation among the position and the momentum operator.

$$[\hat{x}, \hat{p}] = i\hbar\delta_{ij}(1 + \beta\, p^2).$$

Here, β is the deformation parameter and usually known as the minimal length (ML) parameter. Since p represents the momentum, the unit of the ML parameter is the inverse momentum square. Consequently, the Heisenberg Uncertainty Principle is modified with

$$\Delta x\, \Delta p \geq i\hbar(1 + \beta\, (\Delta p)^2).$$

In 2012, Hassanbadi et al. examined the Woods-Saxon potential energy in the existence of the ML [Hassanbadi et al 2012]. In that paper, the authors made an approach to obtain the solutions. In this text, we emphasize the author's misleading approach and give the true treatment of the problem in the contents of the perturbative quantum mechanics.

Method

In one dimension the Schrödinger equation

$$\hat{\mathrm{H}}\,\Psi(\mathrm{x},\mathrm{t})=\hat{\mathrm{E}}\Psi(\mathrm{x},\mathrm{t}).$$

is given with Hamilton and Energy operators. It is a very well-known result that when the potential energy does not depend on time, Schrödinger equation turns to a time-independent eigenfunction problem of the form of

$$\left[\frac{\hat{\mathrm{p}}\hat{\mathrm{p}}}{2m}+\mathrm{V}(\mathrm{x})\right]\,\psi(\mathrm{x})=\mathrm{E}\psi(\mathrm{x}).$$

In presence of the GUP, one dimensional time-independent Schrödinger equation becomes

$$\left[\frac{(\hat{\mathrm{p}}+\beta\hat{\mathrm{p}}^3)^2}{2m}+\mathrm{V}(\mathrm{x})\right]\,\psi(\mathrm{x})=\mathrm{E}\psi(\mathrm{x}).$$

Then,

$$\left[\frac{\hat{\mathrm{p}}^2+2\beta\hat{\mathrm{p}}^2\hat{\mathrm{p}}^2+\beta^2\hat{\mathrm{p}}^2\hat{\mathrm{p}}^2\hat{\mathrm{p}}^2}{2m}+\mathrm{V}(\mathrm{x})\right]\,\psi(\mathrm{x})=\mathrm{E}\psi(\mathrm{x}).$$

Since the ML parameter is a very small parameter, we ignore the second-order ML parameter terms. After simple algebra we get

$$\hat{\mathrm{p}}^2\psi(\mathrm{x})=2\mathrm{m}[\mathrm{E}-\mathrm{V}(\mathrm{x})]\psi(\mathrm{x})-2\beta\hat{\mathrm{p}}^2\hat{\mathrm{p}}^2\psi(\mathrm{x}).$$

At this point, Hassanabadi et al. made an approximation

$$\hat{\mathrm{p}}^2(\hat{\mathrm{p}}^2\psi(\mathrm{x}))=\hat{\mathrm{p}}^2(2\mathrm{m}[\mathrm{E}-\mathrm{V}(\mathrm{x})]\psi(\mathrm{x})-2\beta\hat{\mathrm{p}}^2\hat{\mathrm{p}}^2\psi(\mathrm{x})),$$

$$\hat{\mathrm{p}}^2\big(2\mathrm{m}[\mathrm{E}-\mathrm{V}(\mathrm{x})]\psi(\mathrm{x})-2\beta\hat{\mathrm{p}}^2\hat{\mathrm{p}}^2\psi(\mathrm{x})\big)=2\mathrm{m}[\mathrm{E}-\mathrm{V}(\mathrm{x})]\big(\hat{\mathrm{p}}^2\psi(\mathrm{x})\big)-2\beta\hat{\mathrm{p}}^2\hat{\mathrm{p}}^2\hat{\mathrm{p}}^2\psi(\mathrm{x}).$$

This approximation is not valid since

$$\hat{\mathrm{p}}^2\big(\mathrm{V}(\mathrm{x})\psi(\mathrm{x})\big)\neq\mathrm{V}(\mathrm{x})\big(\hat{\mathrm{p}}^2\psi(\mathrm{x})\big).$$

Instead, the correct form should be

$$\hat{\mathrm{p}}^2\big(\hat{\mathrm{p}}^2\psi(\mathrm{x})\big)=2\mathrm{m}[\mathrm{E}-\mathrm{V}(\mathrm{x})]\big(\hat{\mathrm{p}}^2\psi(\mathrm{x})\big)-2m[(\hat{\mathrm{p}}^2\mathrm{V}(\mathrm{x}))\psi(\mathrm{x})+2(\hat{\mathrm{p}}\mathrm{V}(\mathrm{x}))(\hat{\mathrm{p}}\psi(\mathrm{x}))]-2\beta\hat{\mathrm{p}}^2\hat{\mathrm{p}}^2\psi(\mathrm{x}).$$

Therefore, the final expression becomes

$$\left[\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+\left(E-4m\beta\big(\mathrm{E}-\mathrm{V}(\mathrm{x})\big)^2-\mathrm{V}(\mathrm{x})-2\beta\left(\frac{d^2\mathrm{V}(\mathrm{x})}{dx^2}+2\frac{d\mathrm{V}(\mathrm{x})}{dx}\frac{d}{dx}\right)\right)\right]\psi(\mathrm{x})=0.$$

This result differs from the one that is given in the article of Hassanabadi et al. For the solution of this differential equation, we suggest the non-degenerate perturbation theory can be employed. We briefly discuss the non-degenerate perturbation theory in the next section.

Results and Discussion

In a standard text book of quantum mechanics, for example [Flügge 1974], the non-degenerate perturbation theory is given with the serial expansion of the Hamilton operator, energy eigenvalue of the nth state, and the nth stationary state function $|\mathrm{n}>$ via a small perturbation constant as follows

$$\hat{\mathrm{H}}=\widehat{H}_0+\beta\widehat{H}_1,$$
$$E_n=E_n^0+\beta E_n^1+\cdots,$$
$$|\mathrm{n}>=|\mathrm{n}^0>+\beta|\mathrm{n}^1>+\cdots,$$

When these expansion are inserted in the time-independent Schrödinger equation, we get

$$E_n^0 = < \mathrm{n}^0 | \hat{H}_0 | \mathrm{n}^0 >,$$
$$E_n^1 = < \mathrm{n}^0 | \hat{H}_1 | \mathrm{n}^0 >,$$
$$| \mathrm{n}^0 > = \frac{|\mathrm{m}^0><\mathrm{m}^0|\hat{H}_1|\mathrm{n}^1>}{E_n^0 - E_m^0}.$$

Since, the exact solution of the Woods-Saxon potential is given in the textbook of Flügge [Flügge 1974], we conclude that the formalism can be applied.

Conclusion

In this work, we discuss a perturbative approach to a non-relativistic problem in the presence of a minimal length formalism. Initially, we first showed the improper approach which exists in the literature. Then, we derived the correct approach to the problem, and then, point out the formalism where it can be solved perturbatively.

Acknowledgements

The author thanks to the organization committee for being very kind and helpful not only during the meeting ICRES 2019 but after too. This work was supported by The Scientific Research Projects Coordination Unit of Akdeniz University. Project Number is FBA-2018-3471.

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