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# Relationship between Nusselt and Rayleigh Numbers in Natural Convection

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**Abstract**: When studying heat transfer in laminar or turbulent flow, the aim is to maximize heat transfer, using for example corrugated channels; nanofluids or hybrid nanofluids, etc.; to determine the factors that influence the thermal and hydraulic performance of the used equipment and to determine the relationships that may exist between heat transfer efficiency and these various variables; hence the study of the relationships between these various factors. The findings can be analyzed by using tables, graphs or correlations. The objective of the current paper is focused on the last approach. In addition, we are attempting to determine the influence of the physical parameter Rayleigh number (Ra) on thermal and hydraulic behavior, using the FLUENT calculation code, on the one hand, and to validate the correlation obtained, based on a statistical model, on the other hand. We have also validated that our findings are in excellent accord with those of prior work involving this statistical method.

**Keywords:** Correlations, Convection, Nusselt (Nu), Rayleigh (Ra), Statistical model

## Introduction

In industrial companies, heat exchangers are an important element in energy management, since a large proportion of thermal energy passes through a heat exchanger. These heat exchangers find their application in the food industry, HVAC engineering, petrochemicals, etc. Many heat exchangers in a wide variety of processes are equipped with exchange surfaces specially designed to offer high heat exchange coefficients, notably higher than those of smooth surfaces. Among the techniques used in exchanger design, the use of corrugations increases turbulence and therefore heat transfer. Many researchers have been motived in recent years to study the heat transfer by natural convection in different enclosures and the correlations between the Nu and the Ra.

A numerical study of the natural convection of air (a compressible, Newtonian fluid) in a square enclosure was conducted by Prasopchingchana et al. (2009). The left and right walls of the enclosure were heated and cooled respectively, while the top and bottom sides were adiabatic. In addition, the equations governing the flow of the fluid under consideration were discretized using the finite volume method. The angles of inclination of the square cavity yielding the mean Nu are 110° for $Ra = 10^3$ and 130° for $Ra = 3.10^3$ and $Ra = 10^4$.

Simulations of natural convection flow in a square cavity were undertaken by Barakos et al. (1994). The control volume method was used to solve the governing equations for Ra up to $10^{10}$. The k-ϵ model was used to model turbulence. Numerous correlations of Nusselt and Rayleigh numbers with experimental results have been given in this study.

In (Fusegi et al., 1991), transient three-dimensional calculations are performed for Ra of $10^6$ and for $Pr = 0.71$. The numerical resolution used in this work is compared with the maximum precisions that have been reached in two-dimensional steady-state situations. The equations governing the flow are solved using the finite difference method. In addition, the current investigation is a natural continuation of the three-dimensional steady-state analysis. The authors have studied the flow properties and proposed a correlation between Nu and Ra.

A computational method used to obtain an analytical solution of the flow equations for two-dimensional natural convection in an air-filled square enclosure is given by De Vahl Davis (1983). The mean Nu are given for Ra varying from $10^3$ to $10^6$. A numerical study concerning two-dimensional laminar natural convection for rectangular enclosures filled with differentially heated air is carried out by Chang (2014). The author studied the cavity tilt angle, which varies from 0° to 180°, and the effect of tilt on heat transfer and flow field in the range $10^3 \leq Ra \leq 10^6$ . Mean Nusselt correlations are also reported. Marcatos and Pericleous, (1984) investigated turbulent and laminar flows numerically. A turbulence model was used for Ra higher than $10^6$. The findings are presented in the form of tables, graphs and correlations between Nu and Ra. These results are also compared with those of (De Vahl Davis, 1983).

In the present work, we are attempting to determine the influence of the physical parameter Rayleigh number (Ra) on thermal and hydraulic behavior, using the FLUENT calculation code, on the one hand; and examine a possible relationship between Nu and Ra, in the other hand. Findings can be presented in tables, graphs or correlations. The objective of the current study is to use the correlation method (Draper & Smith, 1981; D'agostino et al., 1986; Chang et al., 2008; Yoshitsugu et al., 2015). The equations of these correlations are used to predict the average Nu. We have also established that our findings are in excellent accord with those of prior work involving a statistical method.

## Physical and Mathematical Model Formulation

### Geometric Configuration

The used geometry (figure.1) is a square, differentially-heated enclosure. It consists of two active vertical sides, held at temperatures $T_h$ and $T_c$ with $T_h > T_c$, while the other sides are assumed to be adiabatic. The zone under study is long in both x and y directions.

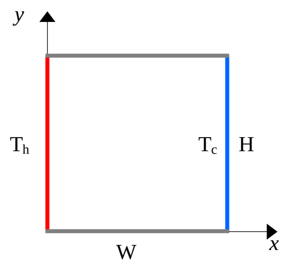


Figure 1. The studied configuration

### Governing Equations

The equations governing the flow are:
Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

x – Momentum:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \vartheta\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] \quad (2)$$

y – Momentum:

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \vartheta\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right] - g\beta(T - T_0) \quad (3)$$

Energy equation:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] \quad (4)$$

## Solution Procedure

In our previous work, a numerical study of laminar natural convection inside an air-filled, differentially heated square enclosure, on horizontal walls subjected to zero flow, was carried out using the CFD program FLUENT. Numerical computations were conducted on a fine square mesh near the walls. The finite volume approach was used to discretize the equations governing natural convection flow. The calculation validation code was run and good agreement was observed between numerical and experimental results. Figure 2 illustrates the increase in mean Nu numbers with increasing Raleigh numbers.

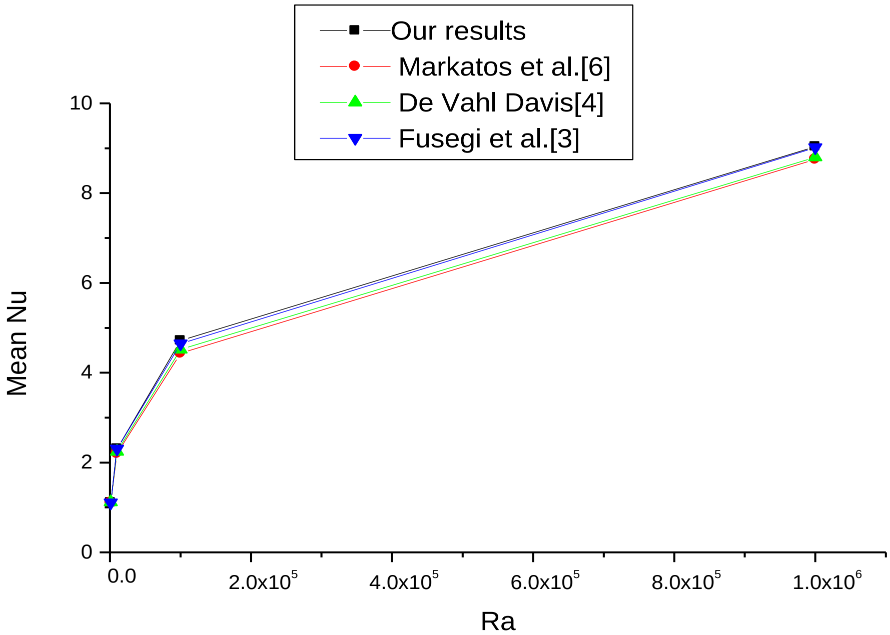


Figure 2. Change of the mean Nu vs Ra

### Statistical Method

This approach uses a statistical model. The employed approach is a power function. This method allows us to predict the average Nusselt number as a function of the Rayleigh number. It also provides us the relationship between these numbers. To adjust the various data and calculate the correlation between the average Nu and the Ra number, we use the following equation:

$$\overline{N_u} = B(Ra)^a \quad (5)$$

B and a: constants to be calculated.
The correlation coefficient is a very significant factor, used to determine the relationship between the mean Nusselt and the Rayleigh.

## Results Discussion

### Experimental Results and Simulation

Table1 shows the findings of a prior investigation. Table 1 illustrates that our findings are in excellent accord with those of the cited authors.The findings of the current research are summarized in Table 2.

Table1. A comparison of average Nu

| Mean Nu numbers | Current results | Markatos & pericleous | De Vahl Davis | Fusegie et al. |
|---|---|---|---|---|
| $Ra=10^3$ | 1.1073 | 1.108 | 1.118 | 1.105 |
| $Ra=10^4$ | 2.3057 | 2.201 | 2.243 | 2.302 |
| $Ra=10^5$ | 4.7175 | 4.430 | 4.519 | 4.646 |
| $Ra=10^6$ | 9.0339 | 8.754 | 8.799 | 9.012 |

### Statistical Method

Using equation (5), we found the different values of B and those of parameter (a) applying the linear regression method. In addition, the correlation coefficient (ρ) between mean Nu and Ra is calculated.

Table 2. B value; (a) and the coefficient of correlation (ρ)

| | B | a | $\rho$ |
|---|---|---|---|
| Present work | 0.136 | 0.306 | 0.997 |
| Markos & Pericleous | 0.141 | 0.299 | 0.997 |
| DeVahl Davis | 0.141 | 0.298 | 0.997 |
| Fusegie et al. | 0.139 | 0.303 | 0.992 |

Table 2 shows that the correlations obtained in our study and those of previous authors demonstrates the appropriateness of the used power model, on the one hand and that, our findings are in excellent accord with those obtained by the authors cited in Table 1, on the other hand.

## Conclusions

In this study, we first took a series of Nusselt and Rayleigh numbers ranging from $10^3$ to $10^6$ corresponding to laminar natural convection; after studying these data, we deduced the following results:

1- We have validated the association between mean Nu and Ra. There is a strong correlation between these numbers. In addition, there is an increasing Nu as the Ra rises.
2- We found our results to be in good accord with those of the previous mentioned authors using a statistical approach.

## Scientific Ethics Declaration

The authors declare that the scientific ethical and legal responsibility of this article published in EPSTEM journal belongs to the authors.

## Acknowledgements or Notes

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