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Central Automorphism Groups for Semidirect Product of p-Groups

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Abstract: Let $\mathbb{Z}_{p^2}\rtimes_\phi\mathbb{Z}_p$ be the semi-direct product of $\mathbb{Z}_{p^2}$ and $\mathbb{Z}_p$ with respect to ϕ, (ϕ is homomorphism from $\mathbb{Z}_p$ to automorphisms group of $\mathbb{Z}_{p^2}$). In this work, the group $Aut_C(\mathbb{Z}_{3^2}\rtimes_\phi\mathbb{Z}_3)$ of all central automorphisms of $\mathbb{Z}_{3^2}\rtimes_\phi\mathbb{Z}_3$ is studied and we determine the form of central automorphisms of $\mathbb{Z}_{3^2}\rtimes_\phi\mathbb{Z}_3$.

Keywords: P-group, Semi-direct product, Central aoutomorphism

Introduction

Let G be a group. By C(G),AutG and Inn(G)we denote the center, the group of all automorphisms and the group of all inner automorphisms of G, respectively. An automorphism θ of G is called central automorphisms if θ commutes with every inner automorphism, or equivalently, if $g^{-1}\theta(g)$ lies in the center of G for all g in G. The central automorphisms form a normal subgroup of Aut(G) and we denote this group with $Aut_C(G)$. Also $Aut_C(G)$ is the subgroup of Inn(G).

A p-group is a group in which every element has finite order, and the orderof every element is a power of prime number p. The term p-group is typically used for a finite p-group, which is equivalent to a group of prime power order.

In literature, there are important studies about central automorphisms of p-groups [1], [2]. In [1] Adney and Yen has shown that if G is a finite purely non abelian group then $|Aut_C(G)| = |Hom(G/G'), Z(G)|$. The automorphisms of direct and semidirect product of p-groups was given by Stahl in [3].

In this work our goal is to determine the central automorphisms of $\mathbb{Z}_{p^2}\rtimes_\phi\mathbb{Z}_p$ where p=3 and ϕ is homomorphism from $\mathbb{Z}_p$ to automorphisms group of $\mathbb{Z}_{p^2}$.

Preliminaries

Definition. Let H and K be non-trivial finite groups and $\phi:K\rightarrow Aut(H)$ be a homomorphism. We define the operation $\rtimes_\phi$ as the following: Let $H\rtimes_\phi K$ be the set $\{(h,k):h\in H,k\in K\}$ on which it acts an operation $*$ as

$$(h_1,k_1)*(h_2,k_2)=(h_1\cdot\phi(k_1)(h_2),(k_1\cdot k_2)$$

We define $G\triangleq H\rtimes_\phi K$ as the semi-direct product of H and K with respect to ϕ.

Definition. An automorphism θ of G is called central automorphism if θ commutes with every inner automorphism, or equivalently, if $g^{-1}\theta(g)$ lies in the C(G). The central automorhisms form a normal subgroup of Aut(G).

Main Results

Theorem. Let φ be an automorphism of $\mathbb{Z}_{p^2} \rtimes_\phi \mathbb{Z}_p$ (p is odd number) where $\phi: \mathbb{Z}_p \to Aut(\mathbb{Z}_{p^2})$ and $\phi(a)=1+pa$ then φ is defined by

$$\varphi(a,b)=(a^i b^j, a^{pm} b)$$

where $i \in \mathbb{Z}_{p^2}$, $j,m \in \mathbb{Z}_p$ and $i \not\equiv 0 (mod p)$

Proof. REF.[3]

Theorem. $|Aut(\mathbb{Z}_{p^2} \rtimes_\phi \mathbb{Z}_p)|=p^3(p-1)$.

Proof. REF.[3]

For determining the central automorphisms of $\mathbb{Z}_{3^2} \rtimes_\phi \mathbb{Z}_3$, first we find the $C(\mathbb{Z}_{3^2} \rtimes_\phi \mathbb{Z}_3)$.

Lemma. $C(\mathbb{Z}_{3^2} \rtimes_\phi \mathbb{Z}_3)=\{(0,0),(3,0),(6,0)\}$.

Proof. If $(a,b) \in C(\mathbb{Z}_{3^2} \rtimes_\phi \mathbb{Z}_3)$ then for every $(c,d) \in (\mathbb{Z}_{3^2} \rtimes_\phi \mathbb{Z}_3$,

$$(a,b)*(c,d)=(c,d)*(a,b)$$

from this we get

$$(a+(1+3b)c, b+d)=(c+(1+3d)a, d+b).$$

a must be 0,3 or 6 and b must be 0 for the last equation to be provided for every $(c,d) \in (\mathbb{Z}_{3^2} \rtimes_\phi \mathbb{Z}_3)$. Therefore $C(\mathbb{Z}_{3^2} \rtimes_\phi \mathbb{Z}_3)=\{(a,0)|a=0,3,6\}$

Corollary. $C(\mathbb{Z}_{3^2} \rtimes_\phi \mathbb{Z}_3)=<(3,0)>$ and the order of $C(\mathbb{Z}_{3^2} \rtimes_\phi \mathbb{Z}_3)$ is 3.

Theorem. Let θ be an automorphism of $\mathbb{Z}_{3^2} \rtimes_\phi \mathbb{Z}_3$..If θ is central then it has the form

$$\theta(a,b)=(a \to a^{3k+1}, a^{3m} b)$$

where $k,m \in \mathbb{Z}_3$.

Proof. Let θ be an automorphism of $\mathbb{Z}_{3^2} \rtimes_\phi \mathbb{Z}_3$.Then it has the form

$$\theta(a,b)=(a^i b^j, a^{pm} b) \qquad (1)$$

where $i \in \mathbb{Z}_{3^2}$, $j,m \in \mathbb{Z}_3$ and $i \not\equiv 0 (mod 3)$

If $\theta \in Aut_C(\mathbb{Z}_{3^2} \rtimes_\phi \mathbb{Z}_3)$ then for all $g=(a,b) \in \mathbb{Z}_{3^2} \rtimes_\phi \mathbb{Z}_3$, θ satisfy $g^{-1}*\theta(g) \in C(\mathbb{Z}_{3^2} \rtimes_\phi \mathbb{Z}_3)$

By using the operation $*$ rule we get

$$g^{-1}*\theta(g)=(i(a+3ab)+j(b+3b^2)-a, 0).$$

For $(i(a+3ab)+j(b+3b^2)-a,0) \in C(\mathbb{Z}_{3^2} \rtimes_\phi \mathbb{Z}_3)$

$(i(a+3ab)+j(b+3b^2)-a,0)=0,3,6$

Therefore the conditions (i=1,j=0), (i=4,j=0) and (i=7,j=0) satisfy this equation for all g. We put this conditions at (1) we get the general form of central automorphisms as:

$\theta(a,b)=(a \to a^{3k+1}, a^{3m} b)$.

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