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An Analysis of a Numerical Solution of the Lotka-Volterra Predator-Prey Model

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Abstract: This paper explores numerical solutions of the Lotka-Volterra predator-prey model with specific initial conditions. We analysis the behavior of the model under different initial conditions to investigate the sensitivity of the solutions and their interdependence. This work demonstrates the application of numerical techniques in understanding ecological systems and the efficiency of the computational methods.

Keywords: Differential equations, Predator-prey system, Lotka-Volterra model, Ecological system

Introduction

No species can live in complete isolation. All species must eat to stay alive, hence they must interact. In this paper, we will consider a model where one population serves as food for another, which is called predator-prey model of two interacting species. Lotka-Volterra equations, (Volterra, 1926; Lotka, 1925), well recognized system presenting cyclical nature of such relationships.

For our purposes, we consider the following Lotka-Volterra predator-prey model

$$\acute{x_1}(t) = ax_1(1 - bx_2) \ , x_1(0) = x_{10}$$

$$\acute{x_2}(t) = cx_2(dx_1 - 1) \ , x_2(0) = x_{20} \tag{1}$$

where t: denote the time, $x_1(t)$: the size of the preys population at time t. $x_2(t)$: the size of the predators population at time t , x_{10} and x_{20} are the initial populations of x_1 and x_2, respectively, and a, b, c, d are positive constants describing the interaction of the two species. Solving (1) analytically is often infeasible because of their nonlinearity nature, so the use of numerical method to find an approximate solution to help us to understand the sensitivity of the model and the relationship between predator and prey populations, would be an excellent approach. Recently, several methods were proposed for the solution of (1), for example; He et al. (2024), proposed a new approach to solve the Lotka–Volterra predator–prey models approximately The method called Extreme Learning Machine. The method transformed the system of differential equations into a set of nonlinear equations. The authors show that their proposed method produced high-precision results compared with other numerical methods (Ripoll & Font,2023), discussed the impact of age-dependent interaction. They presented two approaches, the partial differential equation and the other is the renewal equation. They develop a numerical technique to analyze the instability of the steady-states and the time-evolution of the populations to test the efficiency of the proposed methods. Boulnois (2023), drove an exact, closed-form solution to the prey-predator model when the growth rate of the prey and decay rate of the predators are equal in magnitude. Cherniha and Davydovych (2022), reviewed a known methods of integrating Lotka–Volterra equations with diffusion and presented a range of exact solutions, the important of this summary, may serve as references for validating numerical approaches. In addition, for further readings, see the rest of references.

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In this paper, we aim to see how numerical techniques and computational tools can provide deep insights into real-world problems. We consider three cases with different initial populations of the prey and predator sizes.

Numerical Demonstrations and Analysis

In this section, we consider the Lotka-Volterra predator-prey model (1) with the following different initial population sizes. Also, for calculations purposes, we assume constant interaction coefficients $\boldsymbol{a} = \mathbf{4}$: Growth rate of the prey, $\boldsymbol{b} = \mathbf{0.5}$: Rate at which predators consume prey, $\boldsymbol{c} = \mathbf{3}$: Growth rate of the predator due to predation, $\boldsymbol{d} = \mathbf{1/3}$: Efficiency of converting consumed prey into predator reproduction.

Case 1: First, we solve (1) with the initial populations $x_1(0) = 3\,, x_2(0) = 5$ over $0 \le t \le 2$ numerically using Mathematica. The following table and figures demonstrate the findings

Table 1. Shows the solution with$x_1(0) = 3\,, x_2(0) = 5$

t	Prey Population	Predator Population	t	Prey Population	Predator Population
0.0000	3.0000	5.0000	1.1000	1.8309	0.5990
0.1000	1.6900	4.6467	1.2000	2.4368	0.5485
0.2000	1.0664	3.9348	1.3000	3.2631	0.5392
0.3000	0.7808	3.1910	1.4000	4.3557	0.5833
0.4000	0.6580	2.5378	1.5000	5.7185	0.7137
0.5000	0.6246	2.0034	1.6000	7.2084	1.0095
0.6000	0.6522	1.5813	1.7000	8.3161	1.6362
0.7000	0.7337	1.2549	1.8000	8.0434	2.7891
0.8000	0.8740	1.0070	1.9000	5.9444	4.2050
0.9000	1.0869	0.8223	2.0000	3.4833	4.9690
1.0000	1.3951	0.6890			

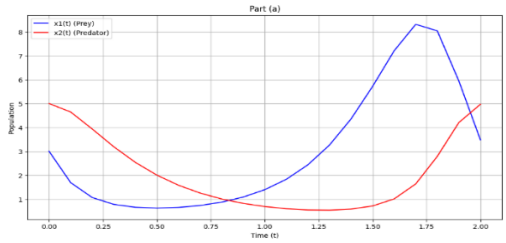

Figure 1. $x_1(t)$ and $x_2(t)$ vs. t with $x_1(0) = 3, x_2(0) = 5$

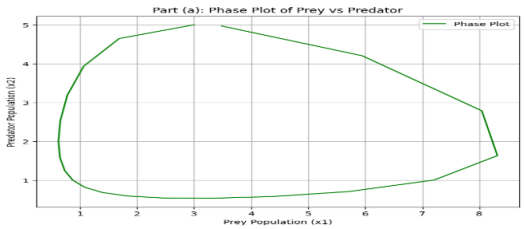

Figure 2. $x_1(t)$ vs. $x_2(t)$ with $x_1(0) = 3\,, x_2(0) = 5$

From Table 1 and Figures 1 and 2 we conclude that

1. the prey population starts at 3 but decreases rapidly due to the high number of predators, between t=0 and t=0.75, the prey population decreases steadily, reaching its minimum as predator pressure is strong, after t=0.75 , the prey population begins to recover as conditions likely improve (e.g., predator numbers decrease or prey reproduction exceeds predation), finally, the prey population reaches a maximum around t≈1.75 , before beginning to decline again
2. the predator population decreases steadily up to t =1.25 because the prey population deckine during this period which leads to food scarcity for predators, and may be due to the predator death rate which greater than its reproduction rate.

3. The closed curve , Fig. 2, indicates that the cyclic nature of the predator-prey relationship; As prey increases, predators increase shortly after and as predators peak, prey decreases due to higher predation rates. With fewer prey, predators decline, which allows prey to recover, restarting the cycle. Finally, the system oscillates around a central point (an unstable equilibrium), but it does not settle into a steady-state.

Case 2: in this case we consider the initial populations $x_1(0) = 3$ and $x_2(0) = 1$ over $0 \leq t \leq 2$. The following tables and figures demonstrate the findings:

Table 2. Shows the solution with $x_1(0) = 3\,, x_2(0) = 1$

t	Prey Population	Predator Population	t	Prey Population	Predator Population
0.0000	3.0000	1.0000	1.1000	1.4800	2.6464
0.1000	3.6565	1.0325	1.2000	1.3529	2.2565
0.2000	4.3948	1.1436	1.3000	1.3317	1.9103
0.3000	5.1132	1.3640	1.4000	1.3969	1.6210
0.4000	5.6120	1.7325	1.5000	1.5436	1.3901
0.5000	5.6281	2.2625	1.6000	1.7762	1.2149
0.6000	5.0247	2.8688	1.7000	2.1057	1.0918
0.7000	4.0105	3.3447	1.8000	2.5456	1.0197
0.8000	2.9969	3.5125	1.9000	3.1057	1.0011
0.9000	2.2366	3.3722	2.0000	3.7801	1.0455
1.0000	1.7531	3.0434			

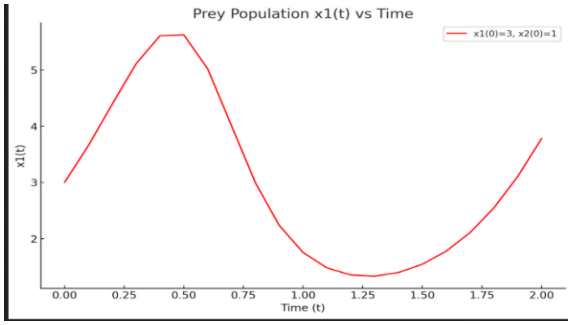

Figure 3. $x_1(t)$ vs. t with $x_1(0) = 3\,, x_2(0) = 1$

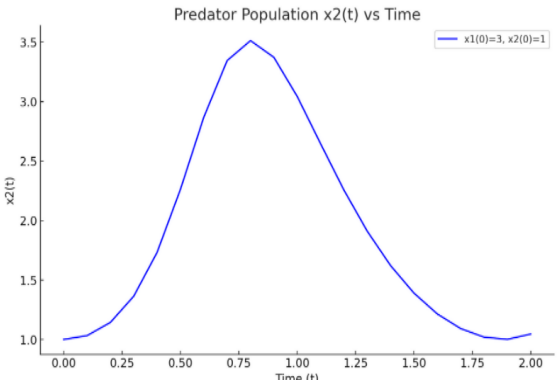

Figure 4. $x_2(t)$ vs. t with $x_1(0) = 3\,, x_2(0) = 1$

From Table 2 and Figures 3, 4, and 5, we conclude that

1. From t=0 , t≈0.5 the prey population increases due to the low initial number of predators, allowing the prey to reproduce without significant predation pressure. This leads to a peak in the prey population at t≈0.5. As time progresses, t≈0.5 to t≈1.25, the predator population likely grows in response to the earlier increase in prey, causing the prey population to decline. the decline continues until it reaches a minimum at t≈1.25, after that it starts to recover as the predator population potentially decreases or stabilizes, the prey population steadily rises.

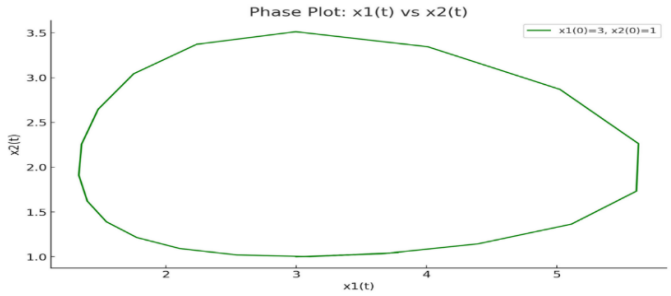

Figure 5. $x_1(t)$ vs. $x_2(t)$ with $x_1(0) = 3\,, x_2(0) = 1$

2. At the beginning, the predator population $x_2(t)$ is relatively low, while the prey population $x_1(t)$ is abundant. This abundance provides enough resources for predators to grow and reproduce, causing their numbers to increase rapidly over time. The predator population eventually reaches its maximum level when the prey population starts to decline due to overconsumption. At this point, resources for predators become scarcer, signaling the start of their decline. After the peak, the predator population $x_2(t)$ decreases as prey resources become insufficient to sustain their growth. Finally, over time, the system may stabilize as predator and prey populations balance each other, leading to cyclical patterns of rise and fall in population sizes.
3. The curve in Fig. 5 still forms a closed loop, indicating periodic oscillations between prey and predator populations. However, the trajectory is smaller and more compact compared to Fig. 2 reflecting the reduced initial predator population. Also, as you can see that the curve moves leftward, with both $x_1(t)$ and $x_2(t)$ decreasing.

Case 3: finally, in this case we consider the initial populations $x_1(0) = 3$ and $x_2(0) = 1.9$ over $0 \leq t \leq 2$. The following tables and figures demonstrate the findings

Table 3. The solution with $x_1(0) = 3\,, x_2(0) = 1.9$

t	Prey Population	Predator Population	t	Prey Population	Predator Population
0.0000	3.0000	1.9000	1.1000	2.8862	2.0779
0.1000	3.0594	1.9057	1.2000	2.8491	2.0503
0.2000	3.1130	1.223	1.3000	2.8297	2.0174
0.3000	3.1540	1.9484	1.4000	2.8297	1.9830
0.4000	3.1765	1.9812	1.5000	2.8486	1.9510
0.5000	3.1771	2.0169	1.6000	2.8845	1.9249
0.6000	3.1553	2.0511	1.7000	2.9338	1.9074
0.7000	3.1141	2.0792	1.8000	2.9913	1.9001
0.8000	3.0589	2.0975	1.9000	3.0510	1.9042
0.9000	2.9975	2.1034	2.0000	3.1058	1.9193
1.0000	2.9375	2.0965			

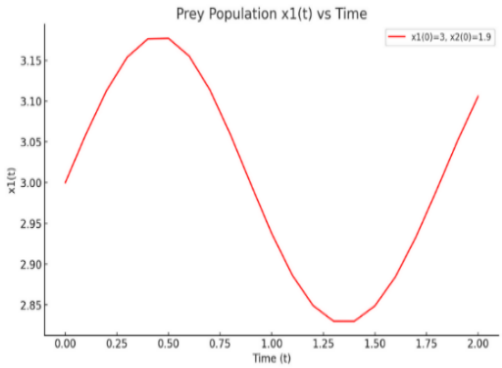

Figure 6. $x_1(t)$ vs. t with $x_1(0) = 3\,, x_2(0) = 1.9$

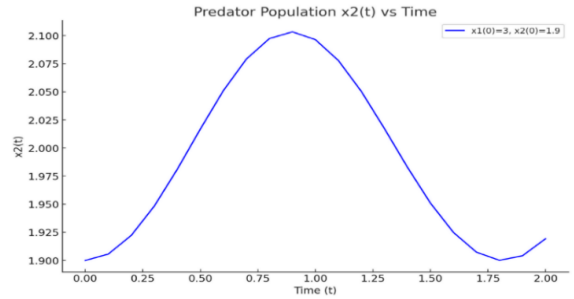

Figure 7. $x_2(t)$ vs. t with $x_1(0) = 3\,, x_2(0) = 1.9$

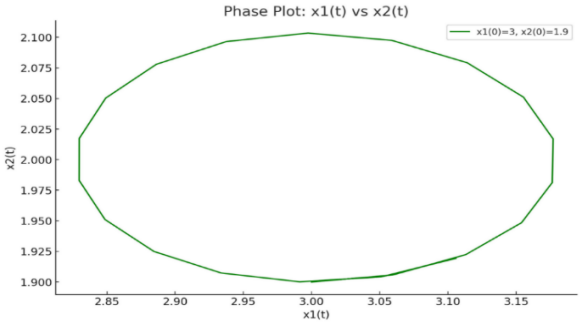

Figure 8. $x_1(t)$ vs. $x_2(t)$ with $x_1(0) = 3\,, x_2(0) = 1.9$

From Table 3 and Figures 6, 7, and 8, we conclude that

1- Initially, the predator population is moderate, allowing a slight increase in the prey population. So, the prey population reaches a peak of approximately 3.15 within a short period. After the peak, the population gradually decreases due to an increase in predator numbers. The prey population decreases to its lowest point of around 2.8 near t=1.5. After reaching the minimum, the prey population begins to recover as predator numbers decline. The predator numbers are insufficient to limit the growth of the prey population, allowing a slight increase. As predator numbers rise, the prey population declines.
2- The predator population reaches a peak at around 2.1, which is lower than in the earlier cases. This smaller peak indicates that the higher initial predator population reduces the overall growth potential. After reaching the peak, the predator population declines and stabilizes at a lower level, similar to previous behaviors but starting from a higher baseline. With $x_2(0)$=1.9, the predator population starts closer to its peak, leaving less room for growth. The higher initial predator count increases competition for prey, reducing the predators' ability to sustain higher population growth.
3- The closed loop indicates that the populations of prey (x_1) and predators (x_2) follow a periodic cycle. This behavior is typical of predator-prey systems modeled by the Lotka-Volterra equations. Finally, neither population grows unbounded nor collapses to zero, maintaining an oscillatory balance in the ecosystem.

Conclusion

In this paper, we aim to see how numerical techniques and computational tools can provide deep insights into real-world problems. Hence, we use Mathematica to examine the system's oscillatory and the relationship between the two populations by varying initial population sizes. The results illustrate the balance between predator and prey populations by revealing their cyclical nature and how the system is sensitive to the initial conditions. In addition, the solutions we obtained guaranteed accuracy and stability during the simulation over a period, $0 \leq t \leq 2$ with step size $h = 0.1$.

Future work

- To make the model more realistic for future research, include external variables such as environmental variability migration, or disease or include new species.

- For increased accuracy over longer simulation timeframes, smaller step sizes or adaptive techniques can be investigated.

Scientific Ethics Declaration

* The author declares that the scientific ethical and legal responsibility of this article published in EPSTEM journal belongs to the author.

Conflict of Interest

* The author declares no conflict of interest

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