The Eurasia Proceedings of Science, Technology, Engineering & Mathematics (EPSTEM)
ISSN: 2602-3199

The Eurasia Proceedings of Science, Technology, Engineering and Mathematics (EPSTEM), 2025

Volume 36, Pages 141-150

ICBAST 2025: International Conference on Basic Sciences and Technology

A Modified Peak Load Method Based on the Two-Parameter Model in Concrete Fracture

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Abstract: The experimental studies on fracture mechanics of concrete until the 1970s revealed that classical linear elastic fracture mechanics (LEFM) was invalid for cementitious materials such as mortar and concrete. This inapplicability of LEFM is due to the existence of an inelastic zone with large-scale and full cracks in front of the crack tip in concrete. This so-called fracture process zone is ignored by LEFM. For this reason, several investigators have developed nonlinear fracture mechanics approaches to characterize failure of concrete structures. According to the two-parameter model (TPM) in concrete fracture, a free crack propagates unstably whenever the stress intensity factor and the crack tip opening displacement are equal to their threshold values, such as Mode I fracture toughness and critical crack tip opening displacement, respectively. This methodology derives fracture parameters from one of two experimental techniques: the compliance method and the peak load method. In the compliance method, fracture parameters are ascertained from the correlation between the load and the crack mouth opening displacement of beams, utilizing closed-loop testing apparatus, while the peak-load method, which does not necessitate complex testing apparatus, is more straightforward than the compliance method for determining fracture parameters within TPM. In this study, a modified peak load method based on an optimization procedure was proposed to determine fracture parameters of TPM instead of the statistical procedure based on the peak load method.

Keywords: Beam test, Concrete, Fracture mechanics, Semi-circular bending test, Two-parameter model.

Introduction

Linear elastic fracture mechanics (LEFM) were first utilized for cement-based materials by Kaplan (1961), followed by Kesler and his colleagues (1972). The latter researchers determined that LEFM was insufficient for concrete. This inadequacy arises from the relatively inelastic region present in quasi-brittle materials, such as concrete mixtures, rocks, and asphalt concretes, which is extensive and fully fractures ahead of the crack tip. Consequently, numerous researchers have created nonlinear fracture mechanics models to characterize, effectively, fracture-dominated failures in concrete structures. These ways include the fictitious crack model by Hillerborg et al. (1976), the crack band model by Bazant and Oh (1983), the two-parameter model (TPM) by Jenq and Shah (1985), the effective crack model by Nallathambi and Karihaloo (1986), the size effect model (SEM) by Bazant and Kazemi (1990), the double-K model by Xu and Reinhardt (1999), and the boundary effect method by Hu and Duan (2008).

To assess the fracture characteristics of cementitious materials, certain models, such as SEM, rely solely on the peak loads of test specimens of varying dimensions. Conversely, models like TPM use the load-displacement relationships of test samples of a single size but require closed-loop testing apparatus, often referred to as compliance. Simplified compliance-based methods that rely only on peak loads have also been proposed by researchers, as detailed below.

The compliance approach of TPM requires complex experimental equipment. In contrast, the peak-load method, which is also based on TPM, only uses the peak loads of quasi-brittle samples with varying notch lengths to

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calculate the relevant fracture parameters. Similarly, SEM requires the peak loads of at least three geometrically similar specimens to determine the fracture parameters. Notably, the variable-notch one-size method eliminates the need for geometrically similar samples. Unlike the compliance approach of TPM, the double-K model does not require complex experimental equipment. Consequently, the simplified approaches of TPM and SEM models provide significant practical convenience, as they do not rely on complex experimental equipment and only require the peak loads of a single-sized structure with varying notch lengths to determine the relevant fracture parameters.

The solution based on the peak load method of TPM was initially modified by using an optimization technique in this study. To verify this modified formulation, split-tension cylinders and beams by Tang et al. (1996), who initially proposed the peak-load method, were simulated according to the modified way. Consequently, this investigation indicated that the results of the modified approach based on the peak-load method look viable and very promising.

The Two-Parameter Model (TPM) in Concrete Fracture

In the 1970s, experimental studies on the fracture mechanics of cement-based materials, including paste, mortar, and concrete, demonstrated that classical Linear Elastic Fracture Mechanics (LEFM) were no longer applicable to quasi-brittle materials Kesler et al. (1972). This inapplicability arises from the relatively extensive inelastic zone present in concrete, known as the fracture process zone (FPZ), which forms ahead of and around the tips of significant cracks. As a result, various non-linear fracture mechanics models have been created to describe the FPZ.

These models can be grouped as the cohesive crack models and the effective crack models, such as the two-parameter fracture model (TPM) proposed by Jenq and Shah (1985). The main aim of any approach is to determine the critical crack extension, defined as $\Delta a = a_c - a_0$, where a_c and a_0 are the critical crack length at the peak load and the initial crack length, respectively, during the load-displacement response or the load-crack mouth opening displacement response of the structure.

A concrete structure fails according to the TPM when the stress intensity factor K_I and the crack opening displacement $CTOD$ reach their critical values, K^s_{Ic} and $CTOD_c$, respectively. These fracture parameters can be determined with the following LEFM equations:

$$K_{Ic}^{s} = \sigma_{Nc}\sqrt{\pi a_c}\,Y\left(g,l\right) \tag{1}$$

$$CTOD_c = \frac{\gamma\sigma_{Nc}a_c}{E}V_1\left(g,l\right)M\left(g,l\right) \tag{2}$$

here σ_{Nc} is the nominal failure stress; E is Young's modulus; and Y, V_1, and M are dimensionless functions that depend on the geometry of the structure (g) and the load type (l). The factor of γ in Eq. (2) can be taken as π and 4 for splitting specimens and beams, respectively. The parameter Y is also called the geometry factor. The function M is derived from the ratio of $COD(a_c)/CMOD_c$, where $CMOD_c$ is the critical crack mouth opening displacement. TPM can easily be used in structural analysis since the Y, V_1, and M functions can be obtained in LEFM handbooks (Tada et al., 2000).

This methodology derives fracture parameters from one of two experimental techniques: the compliance method by Jenq and Shah (1985) and the peak load method, proposed by Tang et al. (1996). In the compliance method, fracture parameters are ascertained from the correlation between the load and the crack mouth opening displacement (P-CMOD) of the three-point bending sample of width b, depth d, and span S, utilizing a closed-loop testing apparatus, as illustrated schematically in Figure 1a (Ince et al., 2024). In the TPM, a_c is calculated by using unloading compliance (C_u) at 95% of the ultimate load, as shown in Figure 1b.

The peak-load method, which does not necessitate complex testing apparatus, is more straightforward than the RILEM method for determining fracture parameters within TPM. However, it necessitates three or more distinct specimens due to the inherent variability in concrete properties. This requirement holds true for both methodologies. The specimens may be uniform in size, yet their initial crack lengths may vary. Conversely, the initial crack lengths may be consistent, while the dimensions of the specimens may differ. For each specimen evaluated, the following equations can be formulated in accordance with TPM (Tang et al., 1992):

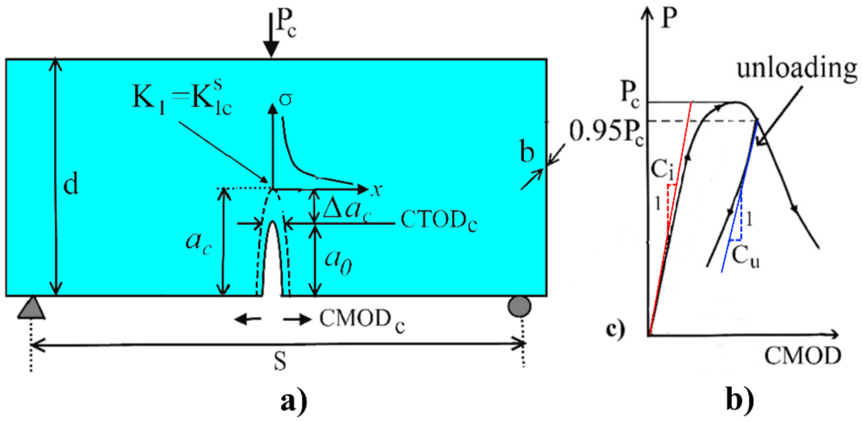


Figure 1. Modeling based on TPM a) notched beam b) typical load-CMOD curve

$$\begin{aligned} K_I^i\left(\sigma_{Nc}^i, a_c^i\right) &= K_{Ic}^S \qquad , i=1,2 \\ CTOD^i\left(\sigma_{Nc}^i, a_c^i\right) &= CTOD_c \end{aligned} \tag{3}$$

here i denotes the ith specimen. Consequently, the fracture parameters can be found by simultaneously solving four non-linear equations. However, three or more distinct specimens must be tested to ensure statistically valid results because random errors always exist in measured values of σ^1_{Nc} and σ^2_{Nc}.

In addition to the above approaches based on TPM, Eq. (1) can be simplified to the following form for geometrically similar structures (a_0/d = constant) by Bazant (1994):

$$\sigma_{Nc} = \frac{K_{Ic}^S}{\left[Y\left(\alpha_0\right) + Y'\left(\alpha_0\right)\dfrac{\Delta a_\infty}{d}\right]\sqrt{\pi\left(a_0 + \Delta a_\infty\right)}} \tag{4}$$

here, Δa_∞ is the critical crack extension for an infinitely large structure and Y' is the derivation of the Y function concerning α_0. On the other hand, it is possible to transform the fracture parameter, Δa_∞, into the fundamental parameter of TPM, $CTOD_c$, by employing the well-known LEFM relationship below:

$$CTOD_c = \frac{K_{Ic}^S}{E}\sqrt{\frac{32\Delta a_\infty}{\pi}} \tag{5}$$

Method

In this study, a modified peak load method based on an optimization procedure was proposed to calculate K^s_{Ic} and $CTOD_c$ instead of the statistical procedure based on the peak load method proposed by Tang et al. (1996). The basis of the way proposed in this study is based on the simultaneous solution of equation (3) as the failure criterion of a concrete structure. As mentioned above, the main aim of any fracture model is to evaluate the critical crack extension (Δa) at the peak load. Accordingly, for this problem, it may be sufficient to find the nominal strength value (σ_{Nc}) corresponding to the peak load and the Δa value for the initial crack length (a_0) for each sample. These Δa values calculated for each sample should be such that the same fracture parameters (K^s_{Ic} and $CTOD_c$) are met for all tested samples. However, it is impossible to obtain an exact solution for a heterogeneous material like concrete. Consequently, such a problem can only be approached using optimization techniques. In this study, to find the fracture parameters of TPM, the following two expressions were initially minimized by utilizing the least squares error criterion:

$$f\left(K_{Ic}^S\right) = \sum_{i=1}^{n} \Delta^2\left(K_{Ic}^S\right) = \sum_{i=1}^{n}\left(K_{Ic,i}^S - \overline{K_{Ic,i}^S}\right)^2 \tag{6}$$

$$f\left(CTOD_c\right) = \sum_{i=1}^{n} \Delta^2\left(CTOD_c\right) = \sum_{i=1}^{n}\left(CTOD_{c,i} - \overline{CTOD_{c,i}}\right)^2 \tag{7}$$

Here n is the number of samples tested, $\overline{K_{Ic,i}^{s}}$ is the average value of K^s_{Ic} and $K^s_{Ic,i}$ is the value of K^s_{Ic} or the i^{th} sample in Equation 6 while $\overline{CTOD_{c,i}}$ is the average value of $CTOD_c$ and $CTOD_{ci}$ is the value of $CTOD_c$ for the i^{th} sample in Equation 7. On the other hand, to provide the simultaneous solution of the above two minimization equations, the root sum squared (RSS) method, which is also called the statistical tolerance analysis method, was used as follows:

$$RSS = \sqrt{\left(f\left(K_{Ic}^{s}\right)\right)^2 + \left(f\left(CTOD_c\right)\right)^2} \tag{8}$$

Note that although, in practice, K^s_{Ic} and $CTOD_c$ are commonly used in terms of MPa√m and mm, respectively, it is recommended to choose MPa√mm and μm because the quantities in these units are very different from each other. The procedures described above, which form the basis of the modified peak load method, can be easily implemented using a spreadsheet-based program. However, as will be explained later, the procedures of the classical peak load method are quite difficult to implement in a computer environment. In the following descriptions, this novel method is explained using the MS-EXCEL-based SOLVER toolkit. Note that the generalized reduced gradient (GRG) algorithm was used in the SOLVER toolkit.

Using of the MS-EXCEL-Based SOLVER Toolkit for the Novel Approach

The previous work by the author is used as an instance (Ince, 2012). In the previous study, a concrete mixture with a maximum aggregate diameter of 16 mm was prepared. The CEM I 42.5N cement type was used. The water/cement ratio was 0.54 in the mixture. The beams of width b=50 mm were conducted by geometrically similar specimens with three different sizes: d=50, 100, and 200 mm. The notch was precast in all of the beams so that α_0=0.2. The samples were loaded at three points with span/depth=2.5. Fracture tests of beams were tested in a compression machine with a maximum capacity of 100 kN (precision ±0.1 kN/s). In the fracture tests of the samples, the time to reach the peak load was set as 4 minutes (± 30 seconds). Young's modulus of concrete was determined as 32.5 GPa. According to the size effect formula based on TPM described in Equation 4, the fracture parameters were obtained as K^s_{Ic} =40.83 MPa√mm and $CTODc$=19.8 μm, as shown in Figure 2.

An alternative solution for these test results is outlined using the modified peak-load method as detailed in Figure 3. The test data were summarized for each sample in columns A-F of the spreadsheet as shown in Figure 3. Note that although a total of nine beams were cast, three from each sample size in the above study, average values for each sample size were used to explain the theory in this study. For each sample, the crack extensions (Δa), initially selected as 5 mm as shown in column G, were taken as variables of the problem. The normalized functions of the cracked structure, such as Y and V_1, are defined in columns J and L, respectively, while the function of M in Equation 2 is presented as embedded in $CTOD_c$ in column M. The following formulas were applied to beams with a span/depth ratio of 2.5 in this study (Yang et al., 1997):

$$Y\left(\alpha\right) = \frac{1.83 - 1.65\alpha + 4.76\alpha^2 - 5.3\alpha^3 + 2.51\alpha^4}{\sqrt{\pi}\left(1+2\alpha\right)\left(1-\alpha\right)^{3/2}} \tag{9}$$

$$V_1\left(\alpha\right) = 0.65 - 1.88\alpha + 3.02\alpha^2 - 2.69\alpha^3 + \frac{0.68}{\left(1-\alpha\right)^2} \tag{10}$$

$$M\left(\alpha_0, \alpha_c\right) = \sqrt{\left(1 - \frac{\alpha_0}{\alpha_c}\right)^2 + \left(1.081 - 1.149\alpha_c\right)\left[\frac{\alpha_0}{\alpha_c} - \left(\frac{\alpha_0}{\alpha_c}\right)^2\right]} \tag{11}$$

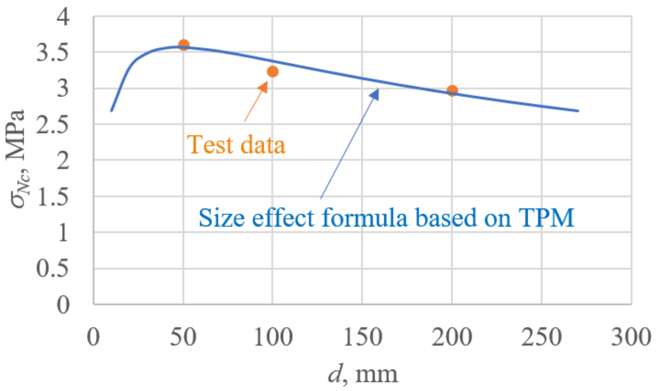
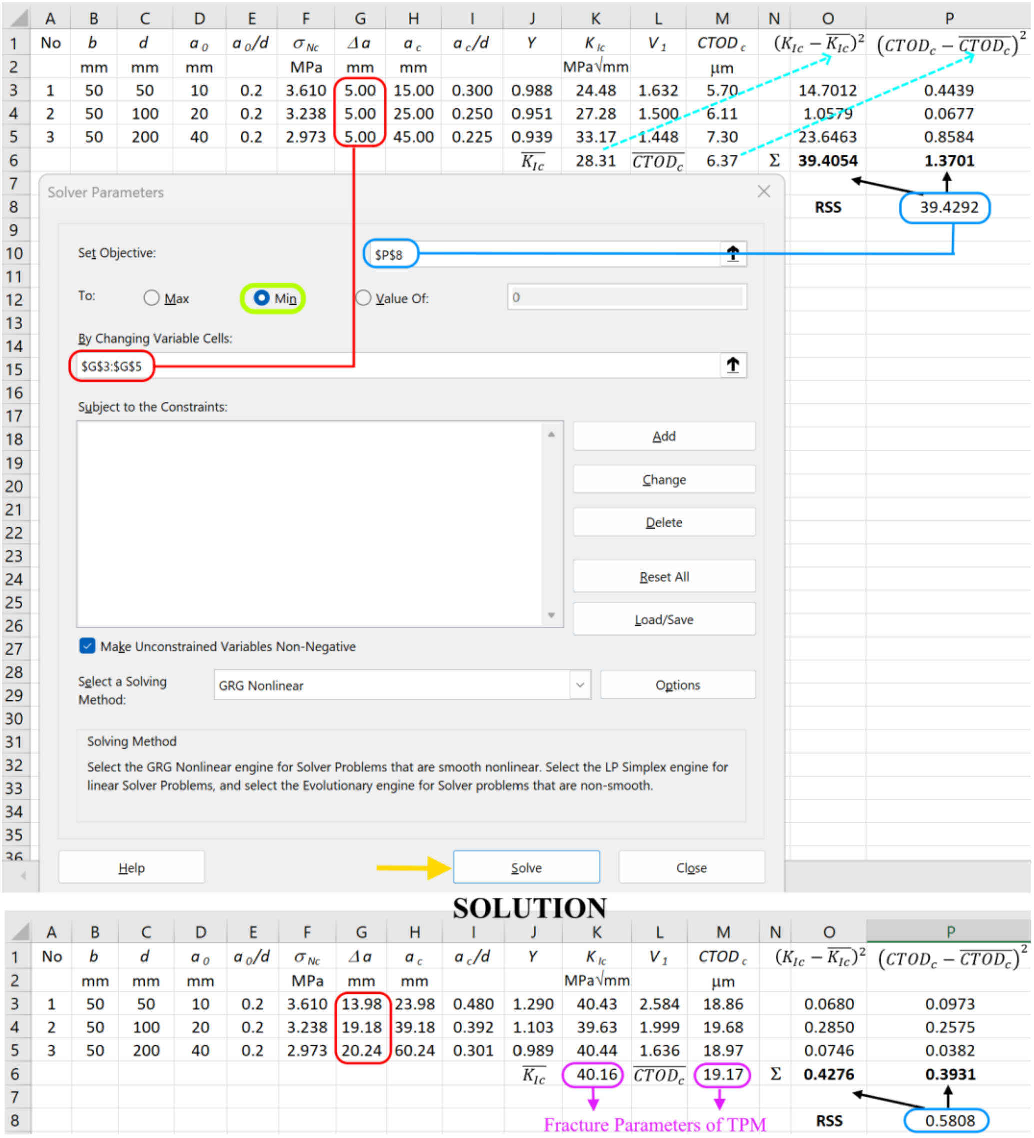


Figure 2. Application of Equation 3 to beams conducted by Ince (2012)



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	No	b	d	a_0	a_0/d	σ_{Nc}	Δa	a_c	a_c/d	Y	K_{Ic}	V_1	$CTOD_c$		$(K_{Ic}-\overline{K_{Ic}})^2$	$(CTOD_c-\overline{CTOD_c})^2$
2		mm	mm	mm		MPa	mm	mm			MPa√mm		μm			
3	1	50	50	10	0.2	3.610	5.00	15.00	0.300	0.988	24.48	1.632	5.70		14.7012	0.4439
4	2	50	100	20	0.2	3.238	5.00	25.00	0.250	0.951	27.28	1.500	6.11		1.0579	0.0677
5	3	50	200	40	0.2	2.973	5.00	45.00	0.225	0.939	33.17	1.448	7.30		23.6463	0.8584
6										$\overline{K_{Ic}}$	28.31	$\overline{CTOD_c}$	6.37	Σ	**39.4054**	**1.3701**
8															RSS	39.4292

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	No	b	d	a_0	a_0/d	σ_{Nc}	Δa	a_c	a_c/d	Y	K_{Ic}	V_1	$CTOD_c$		$(K_{Ic}-\overline{K_{Ic}})^2$	$(CTOD_c-\overline{CTOD_c})^2$
2		mm	mm	mm		MPa	mm	mm			MPa√mm		μm			
3	1	50	50	10	0.2	3.610	13.98	23.98	0.480	1.290	40.43	2.584	18.86		0.0680	0.0973
4	2	50	100	20	0.2	3.238	19.18	39.18	0.392	1.103	39.63	1.999	19.68		0.2850	0.2575
5	3	50	200	40	0.2	2.973	20.24	60.24	0.301	0.989	40.44	1.636	18.97		0.0746	0.0382
6										$\overline{K_{Ic}}$	40.16	$\overline{CTOD_c}$	19.17	Σ	**0.4276**	**0.3931**
8											Fracture Parameters of TPM				RSS	0.5808

Figure 3. Application of the modified peak load method to beams conducted by Ince (2012)

The least squares error criteria, Equations 6 and 7, are given in columns O and P, respectively. The root sum squared (RSS) defined in Equation 8 was chosen as the objective function for the minimization problem, and it was defined in the cell P8. Consequently, fracture parameters of TPM were obtained as K^s_{Ic} =40.16 MPa√mm and $CTOD_c$=19.17 μm, corresponding to the value of the minimum RSS (0.5808), as shown at the bottom of Figure 4. As a result, the modified peak-load method-based solution proposed in this study are in good agreement with the Equation 4-based solution. In this figure, the critical crack extensions, which were chosen as non-negative in the SOLVER parameters dialog box (Figure 3), the determination of which is the main aim of any fracture model, are also shown for each sample in column G.

The Equation 3-based solution for the beams with three different sizes is presented in Figure 4, in which the fracture parameters computed according to the modified peak-load method are employed. As shown in Figure 4, the critical nominal stress decreases with increasing crack length for either fracture quantity (K_I and *CTOD*). On the other hand, Figure 4 clearly demonstrates that the critical nominal stress based on *CTOD* decreases much faster than that based on K_I (Tang et al., 1992). The intersection of the two curves yields the simultaneous solutions. Accordingly, it should be emphasized that the nominal strengths computed in Figure 4 agree well with the strength values based on Equation 4.

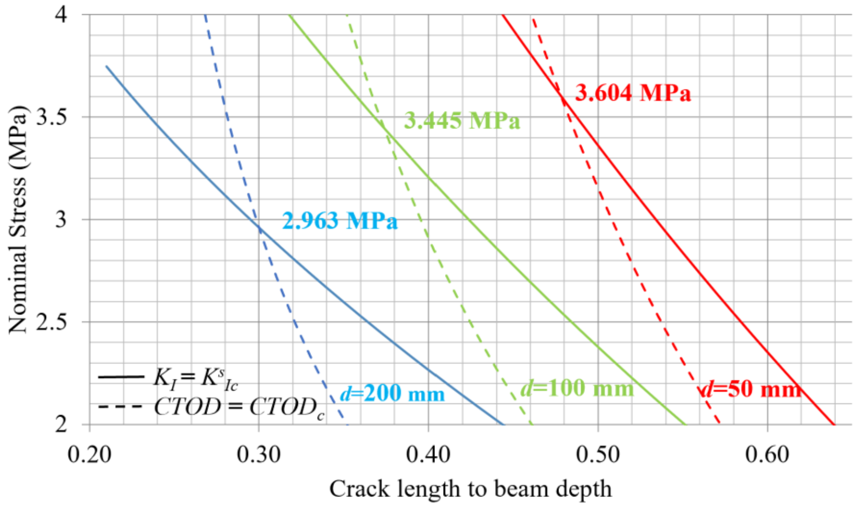


Figure 4. Geometric interpretation of the modified peak load method for beams conducted by Ince (2012)

Applications of the Modified Peak Load Method

Tang and co-workers (1996) tested notched beams and notched splitting cylinders to compare the compliance method with the peak-load method. Samples were prepared from mortar with a maximum aggregate diameter of 3 mm. While beams with width × depth × length = 25.4×50.8×203 mm were loaded at three points, cylindrical samples with width = 31.8 mm and radius = 51 mm were tested under splitting load. According to results of the compliance method applied to beams, the fracture parameters of TPM were determined as K^s_{Ic} =27.36 MPa√mm, $CTOD_c$=6.9 μm, and E=27.8 GPa, on average, by Tang et al. (1996).

Tables 1 and 2 show the applications of the modified peak load method to the study by Tang et al. (1996). The test data were summarized for each sample in the first two columns of these tables. For each sample, the crack extensions (Δa), were initially selected as 5 mm. The values in the other columns are arranged as detailed in Figure 3. All analyses were conducted by using the MS-EXCEL-based SOLVER toolkit. The root sum squared (RSS) values in these tables are the minimized values. Tables 1 and 2 show a strong correlation between beams and splitting cylinders according to the fracture parameters of TPM.

Table 1. Application of the modified peak load method to mortar beams by Tang et al. (1996)

No	a_0/d	σ_{Nc} MPa	Δa mm	a_c mm	a_c/d	Y	K^s_{Ic} MPa√mm	V_1	$CTOD_c$ μm		$\Delta^2(K^s_{Ic})$	$\Delta^2(CTOD_c)$
1	0.2	3.646	6.72	5.88	0.316	1.061	27.49	1.774	8.19		0.8010	0.8606
2	0.2	3.335	5.14	7.07	0.340	1.088	26.73	1.867	8.95		2.7375	2.8424
3	0.35	2.546	6.53	5.06	0.450	1.281	27.64	2.513	8.04		0.5555	0.6037
4	0.35	2.488	5.30	5.33	0.455	1.294	27.44	2.556	8.25		0.8929	0.9692
5	0.35	3.038	4.70	3.00	0.409	1.196	29.37	2.228	6.28		0.9722	0.9691
6	0.5	2.004	4.29	2.41	0.547	1.583	29.65	3.558	5.93		1.5857	1.7833
7	0.5	1.800	5.07	3.44	0.568	1.670	28.60	3.871	7.03		0.0440	0.0545
8	0.5	2.096	5.30	2.00	0.539	1.552	30.17	3.447	5.44		3.1854	3.3191
						Mean	28.39	Mean	7.27	Σ	10.774	11.402
											RSS	15.687

Table 2. Application of the modified peak load method to mortar splitting cylinders by Tang et al. (1996)

No	a_0/d	σ_{Nc} MPa	Δa mm	a_c mm	a_c/d	Y	K^s_{Ic} MPa√mm	V_1	$CTOD_c$ μm		$\Delta^2(K^s_{Ic})$	$\Delta^2(CTOD_c)$
1	0.100	4.632	6.88	11.98	0.235	1.010	28.69	1.298	7.34		0.0127	0.0250
2	0.175	4.573	3.94	12.84	0.252	1.020	29.62	1.313	6.25		0.6681	0.8767
3	0.175	3.886	6.53	15.43	0.303	1.055	28.53	1.366	7.51		0.0742	0.1062
4	0.249	3.533	5.17	17.87	0.350	1.092	28.90	1.426	7.09		0.0093	0.0098
5	0.249	3.395	5.84	18.54	0.364	1.103	28.58	1.445	7.41		0.0483	0.0524
6	0.249	3.356	6.05	18.75	0.368	1.106	28.50	1.451	7.51		0.0953	0.1039
						Mean	28.80	Mean	7.19	Σ	0.9079	1.1740
											RSS	1.4841

The following formulas were applied to beams with a span/depth ratio of 4 in Table 1 (Tada et al., 2000):

$$Y(\alpha)=\frac{1.99-\alpha(1-\alpha)\left(2.15-3.93\alpha+2.7\alpha^2\right)}{\sqrt{\pi}(1+2\alpha)(1-\alpha)^{3/2}} \tag{12}$$

$$V_1(\alpha)=0.76-2.28\alpha+3.87\alpha^2-2.04\alpha^3+\frac{0.66}{(1-\alpha)^2} \tag{13}$$

Note that Equation 11, $M(\alpha_0, \alpha_c)$, is valid for a beam with any span/depth ratio. For the split-cylinder specimens used in Table 2, the LEFM formulas are given by (Tang, 1994):

$$Y(\alpha)=0.94496-0.06851\alpha+1.6235\alpha^2-0.66022\alpha^3 \tag{14}$$

$$V_1(\alpha)=1.1886+0.15419\alpha+0.97529\alpha^2+1.4825\alpha^3 \tag{15}$$

$$M(\alpha_0,\alpha_c)=\sqrt{\left(1-\frac{\alpha_0}{\alpha_c}\right)^2+\left(2.048-0.27\alpha_c\right)\left[\frac{\alpha_0}{\alpha_c}-\left(\frac{\alpha_0}{\alpha_c}\right)^2\right]} \tag{16}$$

The following statistical procedures were used to evaluate TPM, K^s_{Ic} and $CTOD_c$, according to this method (Tang et al., 1996; Yang et al., 1997; Ince, 2025). Samples were grouped with reference to their relative initial crack length α_0, as shown in Tables 1 and 2. The average initial crack length and the average nominal strength were then calculated for each group. The K^s_{Ic} - $CTOD_c$ relationships were determined for each group; an example for mortars is shown in Figure 5. Subsequently, the average K^s_{Ic} - $CTOD_c$ curve was obtained for these groups, as indicated by the curved solid line in Figure 5. The sample standard deviation of the groups was then calculated as follows:

$$s\left(K^s_{Ic}\right)=\sqrt{\sum_{i=1}^{n}\left(\overline{CTOD_c}-CTOD_{ci}\right)^2 \Big/ n-1} \tag{17}$$

here n is the number of groups (n=3 in this study); $\overline{CTOD_c}$ is the average value of $CTOD_c$ for all groups; and $CTOD_{ci}$ is the value of $CTOD_c$ for the i^{th} group. As shown in Figure 5, the value of K^s_{Ic} corresponding to the minimum value of s was taken as the fracture parameter K^s_{Ic}. The fracture parameter $CTOD_c$ was obtained by substituting this value of K^s_{Ic} into the average K^s_{Ic} - $CTOD_c$ curve. In Figure 5, the K^s_{Ic} and $CTOD_c$ relations for the beams are shown at the top, while those for the splitting cylinders are shown at the bottom.

The minimization curves, based on Equation 17, are presented in the middle. In conclusion, the fracture parameters of TPM were determined from the s_{min} values corresponding to minimum standard deviations, as detailed in Figure 5. This figure shows a strong correlation between beams and splitting cylinders. Similarly, it can also be concluded that the parameters derived from the classical peak-load method are in good agreement with the parameters obtained from the modified peak-load method.

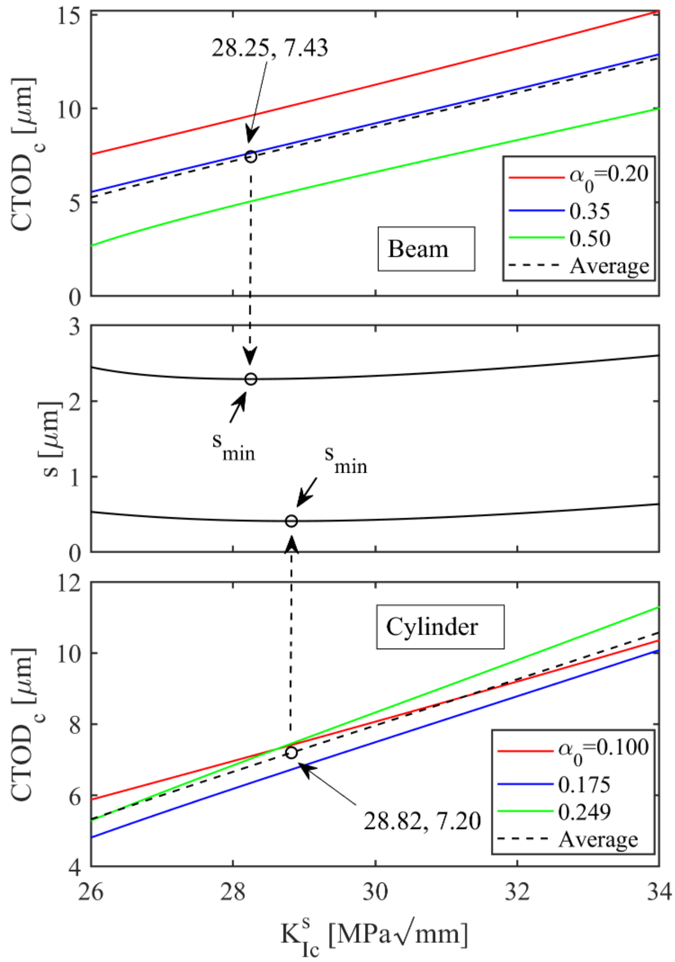


Figure 5. Application of the classical peak load method to cylinder and beams by Tang et al. (1996)

Conclusion

In the presented work, the peak-load method based on the two-parameter model, which is commonly employed in concrete fracture, was modified to model quasi-brittle materials. The key findings are outlined below:

The simplified approaches of concrete fracture models provide significant practical convenience, as they do not rely on complex experimental equipment and only require the peak loads of a single-sized structure with varying notch lengths to determine the relevant fracture parameters. However, it is not possible to obtain an exact solution in modeling fracture mechanics for a heterogeneous material such as mortar and concrete. For this reason, in this study, this difficult problem was approached by using optimization techniques.

The modified peak load method proposed in this study can be easily implemented using a spreadsheet-based program. Therefore, all analyses in the presented work were conducted by using the MS-EXCEL-based SOLVER toolkit, which is a supreme tool that can be used to investigate numerous problems in the field of civil engineering. However, the procedures of the classical peak load method, which are based on some statistical computations, are quite difficult to implement in a computer environment. Moreover, while the classical peak load method only minimizes the errors in the fracture toughness parameter of the material, the way proposed in this study also minimizes the errors in the critical crack tip opening displacement parameter of the material.

Recommendations

A simplified way was proposed to simulate quasi-brittle materials such as mortar, concrete, asphalt concrete, and rock and two successful applications were conducted. However, future studies can achieve more reliable results by comparing different types of samples.

Scientific Ethics Declaration

*The author declares that the scientific ethical and legal responsibility of this article published in EPSTEM Journal belongs to the author.

Funding

* This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

Conflict of Interest

*The author declares that they have no conflicts of interest

Acknowledgements or Notes

*This article was presented as an oral presentation at the International Conference on Basic Sciences and Technology (www.icbast.net) held in Budapest/Hungary on August 28-31, 2025.

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To cite this article:

Ince, R. (2025). A modified peak load method based on the two-parameter fracture model in concrete fracture. *The Eurasia Proceedings of Science, Technology, Engineering and Mathematics (EPSTEM), 36,* 141-150.