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Reasserting the Epistemic Role of Non-Parallel Line Contexts in Introducing Angle Relationships

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Abstract: Angle relationships are an important topic in Euclidean plane geometry and form a conceptual basis for understanding line parallelism. In scholarly geometry, these relationships were introduced through configurations of two non-parallel lines intersected by a transversal. However, in several instructional contexts, angle relationships were presented first through the context of parallel lines. This practice risked obscuring the epistemic status of angle relationships within the structure of Euclidean geometry. This study aimed to reaffirm the epistemic role of non-parallel line contexts as the conceptual foundation for introducing angle relationships. A qualitative study was adopted. Data were collected through both of the analysis of mathematical documents such as geometry textbooks and Focus Group Discussion (FGD) involving scholars in geometry, pure mathematics, and advanced mathematics education. The analysis focused on logical coherence and structural relationships among concepts. The results showed that the configuration of non-parallel lines provided the most conceptually neutral framework for introducing angular relationships. This can be seen, among other things, from the use of the term special pair of angles (in this case same-side interior angles) in Euclid's parallel postulate. Consequently, angle congruences or supplementary were interpreted as consequences of Euclid's fifth postulate rather than as their conceptual origin. These findings highlight the epistemic necessity of introducing angle relationships through non-parallel contexts to preserve coherence and conceptual hierarchy in Euclidean geometry.

Keywords: Angle relationships, Euclidean plane geometry, Non-parallel line, Parallelism

Introduction

The angle relationships formed when two lines are intersected by another line, referred to as a transversal, constitute a fundamental structure in plane geometry, particularly within the framework of Euclidean Geometry. The relationships between specific pairs of angles that arise serve as the foundation for understanding the concept of parallelism as well as theorems related to Euclid's fifth postulate, such as the criteria for parallel lines and the properties of parallel lines (Lewis, 1968; Prenowitz & Jordan, 1965). Various geometry literatures note that these angle relationships do not emerge as a consequence of line parallelism; rather, they originate from the intersection of any two lines with a transversal in a plane (Leonard et al., 2014; Lewis, 1968).

In the study of scholarly knowledge in geometry, the introduction of angle relationships begins with the configuration of two non-parallel lines intersected by a transversal (Lewis, 1968; Moise, 1990). This context represents the general setting in plane geometry, where angle relationships can be analyzed without additional assumptions. Parallelism is then introduced as a special condition that constrains the line configuration so that certain angle relationships become invariant and can serve as criteria for parallelism (Lewis, 1968). Thus, from a geometrical hierarchy perspective, angle relationships precede the concept of parallelism.

In the practice of presenting geometry content, however, angle relationships are often introduced directly through the context of parallel lines. This approach has the potential to obscure the conceptual structure of geometry, as angle relationships may appear to depend on parallelism. Consequently, it can lead to implicit assumptions regarding the congruence of certain angle pairs, such as alternate interior angles, or the supplementary nature of other pairs, for instance, same-side interior angles (Herizal et al., 2025). This situation risks inverting the conceptual hierarchy in Euclidean geometry, where special conditions are treated as foundational. Moreover, it may foster misconceptions regarding the measures of certain angle pairs, for example, the assumption that corresponding angles are necessarily congruent (Herizal & Priatna, 2024).

From a mathematical epistemology perspective, the introduction of concepts should follow a general-to-special principle, moving from the most general structures to specific cases through the addition of assumptions or postulates (Carter, 2024; Morales-Carballo et al., 2024). From a philosophical perspective, this reflects Hilbert's axiomatic method, which organizes mathematical theories from basic propositions and examines how different parts, such as geometry, are interrelated (Hilbert, 1918). The configuration of two non-parallel lines intersected by a transversal provides the fundamental framework for understanding angle relationships as intrinsic properties of line intersections. Conversely, introducing them in the context of parallel lines makes it impossible to distinguish whether certain angle relationships arise from the intersection itself or are merely consequences of parallelism.

Although this distinction is conceptually crucial, the mathematical justification for introducing angle relationships through non-parallel line contexts is rarely discussed explicitly. Considering the potential disruption of the epistemological structure of angle concepts when using parallel line contexts, this study aimed to reaffirm the epistemic role of non-parallel line contexts as a conceptual foundation for introducing angular relationships in Euclidean geometry, ensuring that such relationships were understood as a geometric structure that preceded and was independent of the concept of parallelism. In line with this objective, the study was designed to address the research question: What is the epistemic role of non-parallel line contexts in introducing angle relationships in Euclidean geometry?

Method

This study employed a qualitative approach. The investigation was directed toward examining the role of two non-parallel lines in establishing specific angle relationships within the structure of Euclidean geometry, prior to the introduction of the concept of parallelism. The qualitative approach enabled an in-depth exploration (Creswell, 2014). In this study was the exploration of the mathematical justifications and conceptual considerations regarding the object of study, non-parallel lines and their angle relationships, through various documents and expert perspectives.

Data were obtained from two primary sources. First, a documentary analysis of mathematical texts, including geometry textbooks representing scholarly knowledge in geometry. This analysis aimed to identify the conceptual sequence and structural relationships between angle relationships and parallelism in plane geometry. Second, a Focus Group Discussion (FGD) involving academics with expertise in geometry, pure mathematics, and advanced mathematics education. The FGD focused on epistemic discussions, particularly regarding the status of angle relationships, the generality of non-parallel line contexts, and the conceptual implications of introducing angular relationships directly through parallel line configurations. The FGD was employed to explore and test arguments concerning the necessity of introducing angular relationships using non-parallel line contexts.

Data from the documentary analysis and FGD transcripts were analyzed with an emphasis on the coherence of the material's structure and its mathematical justification. Emergent themes were synthesized to address the research question concerning the epistemic necessity of introducing angular relationships through non-parallel line contexts. Analytical validity was ensured through source triangulation and internal consistency between the study's objectives, the research question, and the resulting conceptual conclusions. Figure 1 showed the process.

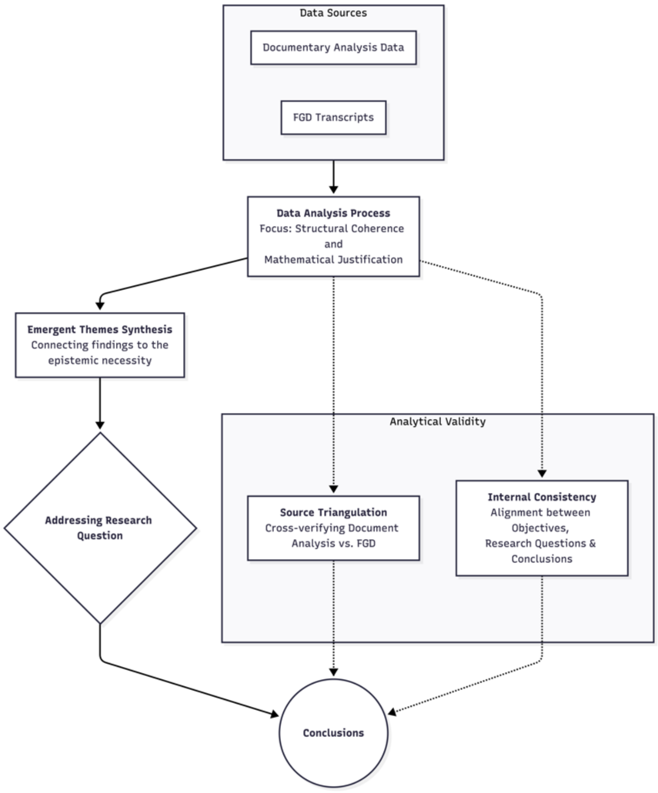

Figure 1. Data analysis process

Results and Discussion

This section presents the findings, accompanied by a discussion based on the analysis of mathematical documents and the arguments of participants during the Focus Group Discussion (FGD), with a focus on the epistemic role of the two non-parallel line context in the introduction of angle relationships.

Scholarly Knowledge of Angle Relationships

All mathematical concepts originally stem from knowledge developed by mathematicians. In the framework of didactic transposition, such knowledge is referred to as scholarly knowledge (Bosch & Gascón, 2006; Chevallard & Bosch, 2020b) and is considered a priori. The concept of forming angle pairs when two lines are intersected by a transversal represents an initial study prior to discussing parallelism. To arrive at the concept of angle relationships, two fundamental topics must first be discussed, as they serve as prerequisites for defining each specific type of angle pair formed. These two topics are transversal, and interior and exterior angles.

Transversal

A transversal is defined as a line that intersects two (or more) other lines at two distinct points, with all lines lying in the same plane (coplanar) (Alexander & Koeberlein, 2020; Clemens et al., 1990). Meanwhile, Lewis (1968) defined a transversal as a line that intersects two other lines at two different points. From these definitions, the characteristics of a transversal can be summarized as follows: (1) it intersects at least two lines, (2) the lines being intersected are arbitrary (not necessarily parallel), (3) it intersects the lines at distinct points (if three lines are involved, there are three distinct points of intersection), and (4) it lies in the same plane. Examples of a transversal and a non-transversal are shown in Figure 2, where in (i), line n is a transversal of lines m and l. In contrast, in (ii), line n is not a transversal of lines l and m because it does not intersect both lines at two distinct points.

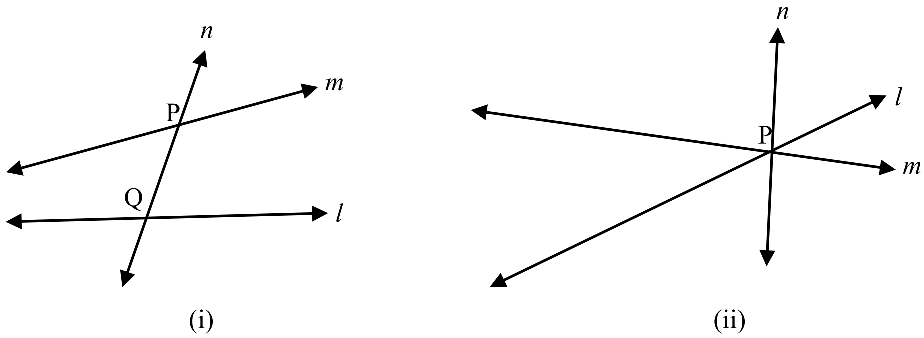

Figure 2. Illustration of transversal and non-transversal examples

Interior and Exterior Angles

When two lines are intersected by a transversal, four angles are formed at each point of intersection. Considering their positions relative to the two lines, some angles lie outside the lines while others lie between them. This distinction gives rise to the terms interior angles and exterior angles. Formally, if two lines *r* and *s* are intersected by a transversal *t*, the angles located between lines *r* and *s* are referred to as interior angles, whereas the angles outside lines *r* and *s* are called exterior angles (Alexander & Koeberlein, 2020; Lewis, 1968).

When any two lines are intersected by a transversal, eight angles are formed in total: four interior angles and four exterior angles. As illustrated in Figure 3, the exterior angles are ∠1, ∠2, ∠7, and ∠8, while the interior angles are ∠3, ∠4, ∠5, and ∠6. Identifying interior and exterior angles is crucial, as this concept forms the foundation for defining various specific angle pairs that arise when two lines are intersected by a transversal. From this explanation, it is clear that interior and exterior angles are formed whenever any two lines (not necessarily parallel) are intersected by a transversal.

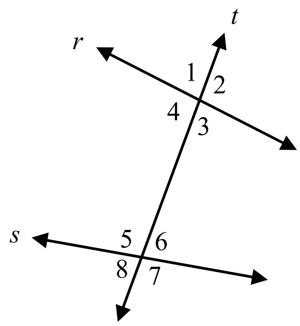

Figure 3. Illustration of two lines intersected by a transversal

Angle Relationships

The angle pairs formed, as shown in Figure 3, have specific names as follows:

1. Alternate interior angles: Two non-adjacent interior angles located on opposite sides of the transversal. There are two pairs of alternate interior angles: ∠3 and ∠5, ∠4 and ∠6.
2. Alternate exterior angles: Two non-adjacent exterior angles located on opposite sides of the transversal. There are two pairs of alternate exterior angles: ∠1 and ∠7, ∠2 and ∠8.
3. Same-side interior angles: Two interior angles located on the same side of the transversal. There are two pairs of same-side interior angles: ∠3 and ∠6, ∠4 and ∠5.
4. Same-side exterior angles: Two exterior angles located on the same side of the transversal. There are two pairs of same-side exterior angles: ∠1 and ∠8, ∠2 and ∠7.
5. Corresponding angles: Two non-adjacent angles located on the same side of the transversal such that one is an interior angle and the other is an exterior angle. There are four pairs of corresponding angles: ∠1 and ∠5, ∠3 and ∠7, ∠2 and ∠6, ∠4 and ∠8 (Alexander & Koeberlein, 2020; Clemens et al., 1990; Leonard et al., 2014; Moise, 1990).

Based on praxeology within the framework of the Anthropological Theory of Didactic (ATD) (Chevallard & Bosch, 2020a), the sequence of material described above can be organized into praxis and logos blocks. The praxis block includes tasks (T) and techniques (τ), while the logos block consists of technology (θ) and theory (Θ). Tasks (T) refer to the problems or situations presented, whereas techniques (τ) describe the methods or steps used to accomplish these tasks. Technology (θ) provides the rationale behind the choice of techniques employed, and theory (Θ) serves to justify or explain the technology, that is, the reasoning underlying the techniques used (Chevallard, 2019). Table 1 presents the arrangement of the aforementioned material organized within a mathematical praxeology framework.

Table 1. The mathematical praxeological organization of angle relationships

Task (T)	Techniques (τ)	Technology (θ)	Theory (Θ)
T_1: Explaining the Concept of a Transversal	τ_1: Identifying lines that intersect two or more other lines at two or more distinct points	θ_1: The conditions for a line to be a transversal are that it: (i) intersects two (or more) other lines, (ii) intersects them at two (or more) distinct points, and (iii) lies in the same plane	Θ_1: Formal definition of transversal
T_2: Explaining the Concept of Interior and Exterior Angles	τ_2: Identifying angles located between two lines intersected by a transversal and angles located outside the two lines intersected by a transversal	θ_2: The condition for an interior angle is that it lies between two lines intersected by a transversal, and the condition for an exterior angle is that it lies outside the two lines intersected by a transversal	Θ_2: Formal definition of interior and exterior angles when two lines intersected by a transversal
T_3: Identifying Specific Angle Pairs Formed When Any Two Lines Are Intersected by a Transversal	τ_3: Identifying: (i) non-adjacent angles on the same side of a transversal, where one is an interior angle and the other is an exterior angle; (ii) two non-adjacent interior angles on opposite sides of the transversal; (iii) two non-adjacent exterior angles on opposite sides of the transversal; (iv) interior angles on the same side of the transversal; (v) two exterior angles on the same side of the transversal	θ_3: The conditions for corresponding angles, alternate interior angles, alternate exterior angles, same-side interior angles, and same-side exterior angles	Θ_3: Formal definition of corresponding angles, alternate interior angles, alternate exterior angles, same-side interior angles, and same-side exterior angles .

The Context of Non-Parallel Lines as the Epistemic Foundation of Angle Relationships

Analysis of geometry textbooks indicated that the configuration of two non-parallel lines intersected by a transversal constituted the most fundamental conceptual context for understanding angle relationships in plane geometry. These angle pairs were formed whenever "any" two lines were intersected by a transversal, without the necessity of the lines being "parallel" (Leonard et al., 2014; Lewis, 1968; Moise, 1990). In this configuration, angle relationships did not arise as a consequence of any special condition but rather as a direct result of the intersection structure of two lines with a third line (the transversal) in the plane.

In other words, the configuration of two non-parallel lines intersected by a transversal provided the most natural framework for understanding angle relationships. In this context, the angles formed did not depend on additional assumptions such as parallelism but emerge directly from line intersections in the plane. This approach emphasizes that angle relationships precede and are independent of the concept of parallelism, allowing them to be understood as a universal geometric phenomenon. Furthermore, introducing angle relationships through non-parallel line contexts helps preserve logical clarity and the conceptual hierarchy in Euclidean geometry, where based on Lewis (1968), parallelism is understood as an additional condition that constrains line configurations and produces specific angle equalities, rather than serving as the primary source of angular relationships. In this way, the understanding of angle relationships becomes more conceptually and epistemically transparent, providing a solid foundation before studying the special case of parallel lines.

The Relationship Between Angle Relationships and Euclid's Parallel Postulate

Euclid, often referred to as the "Father of Geometry," was a Greek mathematician who lived around 325–265 BCE. He authored thirteen volumes, with Volume I, titled Elements*,* containing 23 definitions, five axioms, and five postulates (Boyer, 1968; Burton, 2011). One of Euclid's greatest contributions in Elements was the formulation of the five geometric postulates, which served as the foundation of the entire Euclidean geometry system. The parallel postulate originates from Euclid's fifth postulate, later known as Euclid's Parallel Postulate. Its statement is: "If a straight line intersects two other lines such that the sum of the interior angles on one side is less than two right angles (180°), then the two lines will eventually meet on that side if extended indefinitely" (Burton, 2011; Byrne, 1847).

The discussions from the FGD indicated that introducing the concept of angle pairs in the context of two non-parallel lines intersected by a transversal serves as a bridge to understanding Euclid's fifth postulate as shown in Figure 4. When Euclid's Fifth Postulate is linked to the angles formed by two non-parallel lines intersected by a transversal, the focus naturally falls on the same-side interior angles. The postulate states that if the sum of the interior angles on one side of the transversal is less than two right angles, then the lines will meet on that side if extended indefinitely. Within this framework, introducing angles through the context of two non-parallel lines is both appropriate and conceptual: the same-side interior angles central to the postulate emerge naturally from the line intersections without assuming parallelism. In other words, the non-parallel context provides a neutral foundation for understanding the parallel postulate, as the relevant angle relationships are already present and can be analyzed directly. This emphasizes that introducing angle relationships through non-parallel lines is not merely a visual representation but an epistemic necessity to maintain logical order and conceptual coherence in Euclidean geometry. Furthermore, it facilitates the comprehension of the parallel postulate.

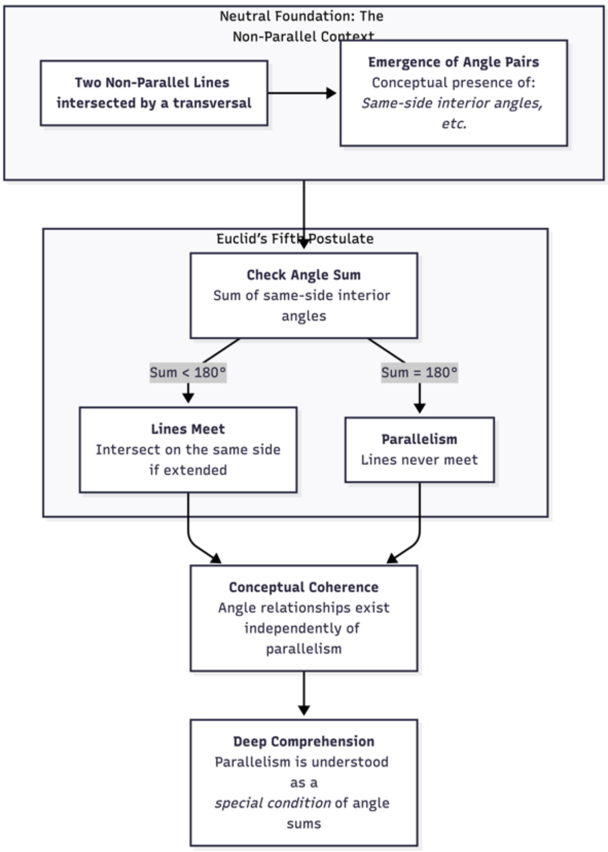

Figure 4. The epistemic foundation of non-parallel lines for Euclid's fifth postulate

When angle relationships are introduced first through non-parallel contexts, the application of the parallel postulate can be understood as a subsequent conceptual step that adds a structure to the pre-existing angle relationships. In this framework, parallelism does not serve as the source of angle relationships; rather, it acts as a condition that constrains the geometric configuration so that certain angle pairs become congruent or supplementary (Leonard et al., 2014; Lewis, 1968). This approach clarifies that angle equalities in parallel lines are an additional consequence, not the origin of the angle relationships themselves.

Non-Parallel Contexts to Avoid Potential Misconceptions

One important reason for introducing specific angle pairs through the context of non-parallel lines is to prevent misleading understandings. Figure 5 shows the logical reasoning behind this approach. If angle relationships are introduced directly in the context of parallel lines, learner or readers may mistakenly assume that angle congruences and certain relationships exist solely because the lines are parallel. In other words, parallelism might appear to be a necessary condition for the formation of specific angle pairs. This is not an exaggeration, as several studies have shown this (Herizal et al., 2025; Herizal & Priatna, 2024). In reality, angle relationships such as corresponding angles, alternate interior angles, or same-side interior angles are inherently established by the intersection of two lines by a transversal, without requiring the lines to be parallel.

Moreover, a parallel-line-based approach may lead to generalization errors, such as the belief that "corresponding angles are always congruent" or that other angle pairs, like alternate interior angles, must always be equal, without recognizing that such equalities are in fact specific consequences of the parallel postulate (Euclid's Postulate V). A study by Herizal et al. (2025) showed these results. This phenomenon is related to learning obstacles, which Brousseau (1997) refers to as epistemological and didactical obstacles. By introducing angle relationships through a non-parallel context, it is emphasized that these relationships are universal and independent of parallelism. The non-parallel context separates general phenomena from special cases, resulting in a mathematical understanding that is logical, coherent, and hierarchical: angles emerge as local consequences of line intersections, while parallelism imposes additional constraints that produce specific congruences.

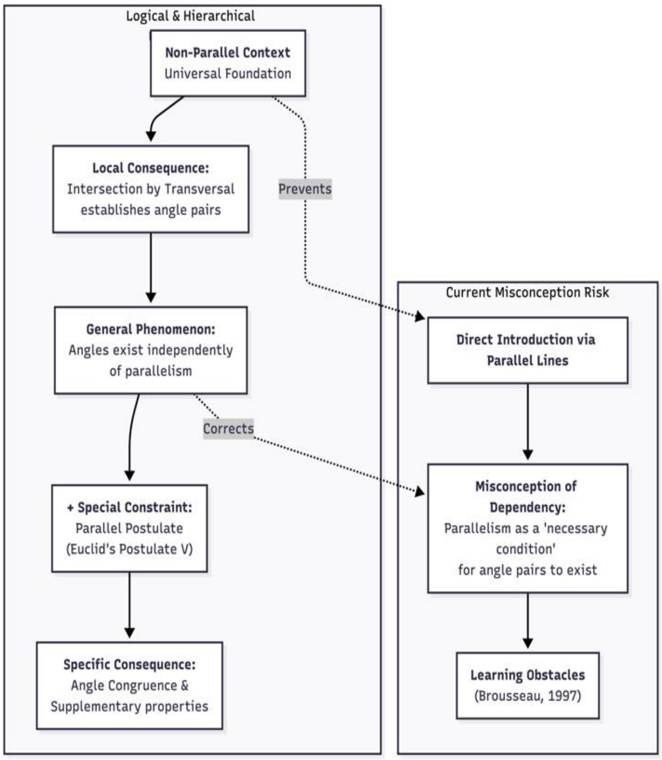

Figure 5. The role of non-parallel lines for supporting understanding and preventing misconception

Conclusion

This study reaffirms that the context of two non-parallel lines intersected by a transversal plays a fundamental epistemic role in the introduction of angle relationships in Euclidean geometry. Through the analysis of mathematical documents and expert discourse in Focus Group Discussions, it is shown that angle relationships constitute a geometric structure emerging directly from line intersections, independent of any assumption of parallelism. Thus, angle relationships conceptually precede and are independent of the concept of parallelism.

The findings indicate that the non-parallel line context provides the neutral conceptual framework for developing an understanding of angle relationships. In this context, angle relationships can be derived solely using basic geometric postulates, ensuring that their origin remains epistemically transparent. In contrast, introducing angle relationships directly through parallel line contexts may obscure the dependence of certain angle relationships on line parallelism and, potentially, on Euclid's Fifth Postulate, while also reversing the internal hierarchy of concepts within the geometric structure. By establishing the non-parallel context as the starting point, parallelism can be properly understood as a subsequent axiomatic assumption that produces specific angle equalities, rather than serving as the primary source of angle relationships.

Recommendations

Based on these findings, it is recommended that the conceptual introduction of angle relationships in the study of geometry begin with the context of two non-parallel lines intersected by a transversal. This approach allows angle relationships to be understood as a general and independent geometric structure before the additional condition of parallelism is introduced.

Scientific Ethics Declaration

* The authors declare that the scientific ethical and legal responsibility of this article published in EPSTEM journal belongs to the authors.

Conflict of Interest

* The authors declare that they have no conflicts of interest.

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