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Common Fixed-Point Results for Geraghty-Ciric Contraction in Fuzzy Metric Space

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Abstract: The theory of fixed points has undergone significant development over the years through the generalization of contraction conditions and the modification of the axioms of metric spaces. These advancements have broadened the applicability of fixed-point results across various branches of mathematics and applied sciences. In particular, fuzzy metric spaces have provided a flexible and effective framework for dealing with uncertainty, leading to a growing interest in the study of contractive mappings within this setting. In this paper, we investigate common fixed-point results for a pair of weakly compatible mappings satisfying a Geraghty–Ćirić-type contraction condition in the context of complete fuzzy metric spaces. By using the properties of Cauchy sequences and convergence in fuzzy metric spaces, we establish sufficient conditions for the existence and uniqueness of a common fixed point for the mappings under consideration. The results obtained in this work extend and generalize several known theorems in the existing literature, especially those related to Geraghty- and Ćirić-type contractions. To demonstrate the applicability of the main theorem, an illustrative example is provided. Furthermore, a corollary is derived as a special case of the main result, highlighting its versatility and potential for further generalizations.

Keywords: Fixed point, Fuzzy metric space, Geraghty-Ciric contraction, Cauchy sequence, convergent sequence

Introduction

Fixed point theory has played a fundamental role in nonlinear analysis since Banach contraction principle was stated (Banach, 1922). Over the years, numerous generalizations of contractive conditions and metric spaces have been introduced to broaden the applicability of fixed-point results. In this direction, the development of fuzzy metric spaces brought a new perspective to fixed point theory.

The Theory of Fuzzy Sets was introduced by Zadeh (1965), marking a revolution in mathematics Kramosil and Michalek (1975) gave the definition of a fuzzy metric space by changing the metric axioms using a third variable time, which measures the distance between points (Kramosil & Michalek, 1975). Later, George and Veeramani generalized the axioms of a fuzzy metric space and studied fixed-point theorems.Nowadays, their definition is widely used because it relates to convergence and Cauchy sequences (George and Veeramani, 1994). Many authors have worked in this direction, giving their contribution to fixed point theory in fuzzy metric spaces (Moussaoui, 2025; Gerbeti et al., 2025; Ghods, 2022; Huang, et al, 2021).

Shamas et al. (2021) introduced the notion of Ćirić-type fuzzy contraction mappings and established a fixed point theorem in the setting of fuzzy metric spaces (Building on these developments, Shu-Fang Li et al. proposed new Geraghty- and Ćirić-type contractive mappings and derived a corresponding fixed-point result in the framework of a complete metric space (Li, 2022). Inspired by the aforementioned authors, in this paper, we introduce a pair of Geraghty–Ćirić-type functions in fuzzy metric spaces and establish the existence and uniqueness of a fixed point for this pair of mappings. Our result generalizes the theorem of Li et al. (2022) as we establish the existence and uniqueness of a common fixed point for a pair of weakly compatible functions.

Preliminaries

Definition 1. (Zadeh, 1965) A binary operation $*: [0,1] \times [0,1] \to [0,1]$ is a continuous triangular norm or t-norm if it satisfies the following conditions:

1. $a * 1$ for all $a \in [0,1]$;
2. $*$ is an associative operation.
3. $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ and $a, b, c, d \in [0,1]$;
4. $*$ is continuous.

Definition 2. (Zadeh, 1965) The 3-triple $(X, M, *)$ called a fuzzy metric space if X is an arbitrary nonempty set, $*$ is a continuous t-norm and M is a fuzzy set on $X \times X \times (0, \infty)$ satisfying the following conditions

1. $M(x, y, t) > 0$ for every $x, y \in X, t \in (0, \infty)$;
2. $M(x, y, t) = 1$ if and only if $x = y$;
3. $M(x, y, t) = M(y, x, t)$;
4. $M(x, y, t + s) \geq \mathrm{M}(y, x, t) + \mathrm{M}(y, x, s)$;
5. $M(x, y, t + s): (0, \infty) \to (0, 1]$ is continuous for $x, y \in X, t, s \in (0, \infty)$.

Definition 3. (Zadeh, 1965) Let $(X, M, *)$ be a fuzzy metric space and let $\{x_n\}$ be a sequence in X. Then

1. $\{x_n\}$ is convergent, if there exist $x \in X$ such that $\lim_{n \to \infty} M(x_n, n, t) = 0$ for every $t > 0$.
2. $\{x_n\}$ *is* a Cauchy sequence if for every $\varepsilon \in (0,1)$ and $t > 0$, there exist $n_0 \in N$ such that for every $n, m, \geq n_0$, $M(x_n, x_m, t) > 1 - \varepsilon$;
3. $(X, M, *)$ is complete if every Cauchy sequence convergent.

Shamas et al. (2021) defined Ciric-type fuzzy contraction mappings and proved a fixed point theorem on a fuzzy metric space.

Definition 4. (Shamas et al. 2021) Let $(X, M, *)$ be a complete fuzzy metric space. A mapping $f: X \to X$ is said to be a Ciric contraction if there exist $\alpha \in (0, 1)$ such that

$$\frac{1}{\mathrm{M}(fx, fy, t)} - 1 \leq \alpha \max\left\{\frac{1}{\mathrm{M}(x, y, t)} - 1, \frac{1}{\mathrm{M}(x, fy, t)} - 1, \frac{1}{\mathrm{M}(fx, y, t)} - 1, \frac{1}{\mathrm{M}(x, fx, t)} - 1, \frac{1}{\mathrm{M}(y, fy, t)} - 1\right\}$$

for each $x, y \in X, t \in (0, \infty)$.

Li et al. (2022) present Geraghty and Ciric type contractive mapping and gave a fixed point result related to them in a complete metric space.

Definition 5. (Li et al. 2022) Let (X, d) be a complete metric space. The map $f: X \to X$ is called a Geragthy-Ciric-type contraction mapping if there exist

$$\beta \in S = \left\{R^+ \to [0, 1), \lim_{n \to \infty} \beta(s_n) = 1 \Rightarrow \lim_{n \to \infty} s_n = 0\right\}$$

such that for each $x, y \in X$ and

$M(x,y) =$
$max\{\beta(d(x,y))d(x,y), \beta(d(x,Tx))d(x,Tx), \beta(d(y,Ty))d(y,Ty), \beta(d(x,Ty))d(x,Ty), \beta(M(Tx,y))\, d(y,T$

We extend this definition to Geraghty-Ciric fuzzy contraction mapping.

Definition 6. Let $(X,M,*)$ be a complete fuzzy metric space. The function $f: M \to M$is said to be a Geraghty-Ciric fuzzy contraction if there exists $\beta \in S = \left\{R^+ \to [0,1), \lim_{n\to\infty} \beta(s_n) = 1 \Rightarrow \lim_{n\to\infty} s_n = 0\right\}$ such that

$$\frac{1}{M(fx,fy,t)} - 1 \le N(x,y)$$

for each $x, y \in X, t \in (0,\infty)$ and

$$N(x,y) = max\left\{\beta\left(\frac{1}{M(x,y,t)} - 1\right)(\frac{1}{M(x,y,t)} - 1), \beta\left(\frac{1}{M(fx,y,t)} - 1\right)(\frac{1}{M(fx,y,t)} - 1), \beta\left(\frac{1}{M(x,fy,t)} - 1\right)(\frac{1}{M(x,fy,t)} - 1), \beta\left(\frac{1}{M(x,fx,t)} - 1\right)(\frac{1}{M(x,fx,t)} - 1), \beta\left(\frac{1}{M(y,fy,t)} - 1\right)(\frac{1}{M(y,fy,t)} - 1)\right\}$$

Definition 7. Let $(X,M,*)$ be a complete fuzzy metric space. A pair of mappings $f,g: X \to X$ is called a Geraghty-Ciric fuzzy couple if there exists $\beta \in S = \left\{R^+ \to [0,1), \lim_{n\to\infty} \beta(s_n) = 1 \Rightarrow \lim_{n\to\infty} s_n = 0\right\}$ such that $\frac{1}{M(fx,gy,t)} - 1 \le \tilde{N}(x,y)$.

for each $x, y \in X, t \in (0,\infty)$ and

$$\tilde{N}(x,y) = max\left\{\beta\left(\frac{1}{M(x,y,t)} - 1\right)(\frac{1}{M(x,y,t)} - 1), \beta\left(\frac{1}{M(fx,y,t)} - 1\right)(\frac{1}{M(fx,y,t)} - 1), \beta\left(\frac{1}{M(x,gy,t)} - 1\right)(\frac{1}{M(x,gy,t)} - 1), \beta\left(\frac{1}{M(fx,x,t)} - 1\right)(\frac{1}{M(fx,x,t)} - 1), \beta\left(\frac{1}{M(y,gy,t)} - 1\right)(\frac{1}{M(y,gy,t)} - 1)\right\}$$

Results and Discussion

Theorem 1. Let $(X,M,*)$ be a complete fuzzy metric space and $f,g: X \to X$ a Geraghty-Ciric fuzzy couple. Then f,g have a unique fixed point in $(X,M,*)$.

Proof. Let $x_0 \in X$ be an arbitrary point in X. Putting $x_1 = f(x_0), x_2 = g(x_1), x_3 = f(x_2), \ldots, x_{2n+1} = f(x_{2n}), x_{2n+2} = g(x_{2n+1})$, and so on, we construct a sequence $\{x_{2n}\}_{n\in N}$.

Firstly, we show that the sequence

$P_n = max\left\{\frac{1}{M(x_{2k+1},x_{2s+2},t)} - 1,\ 0 \le k,s \le n,\ k,s \in N \cup \{0\}, t > 0\right\}$ is bounded in X.

Indeed, taking $n \in N$ and $1 \le k,s \le n$ we have

$$\frac{1}{M(x_{2k+1},x_{2s+2},t)} - 1 = \frac{1}{M(fx_{2k},gx_{2s+1},t)} - 1 \le \tilde{N}(x_{2k},x_{2s+1}) = max\left\{\beta\left(\frac{1}{M(x_{2k},x_{2s+1},t)} - 1\right)(\frac{1}{M(x_{2k},x_{2s+1},t)} - 1), \beta\left(\frac{1}{M(x_{2k+1},x_{2s+1},t)} - 1\right)(\frac{1}{M(x_{2k+1},x_{2s+1},t)} - 1), \beta\left(\frac{1}{M(x_{2k},x_{2s+2},t)} - 1\right)(\frac{1}{M(x_{2k},x_{2s+2},t)} - 1), \beta\left(\frac{1}{M(x_{2k+1},x_{2s},t)} - 1\right)(\frac{1}{M(x_{2k+1},x_{2k},t)} - 1),\ \beta\left(\frac{1}{M(x_{2s+2},x_{2s+1},t)} - 1\right)(\frac{1}{M(x_{2s+2},x_{2s+1},t)} - 1)\right\} < max\left\{\frac{1}{M(x_{2k},x_{2s+1},t)} - 1, \frac{1}{M(x_{2k+1},x_{2s+1},t)} - 1,\ \frac{1}{M(x_{2k},x_{2s+2},t)} - 1,\ \frac{1}{M(x_{2k+1},x_{2k},t)} - 1,\ \frac{1}{M(x_{2s+2},x_{2s+1},t)} - 1\right\} \le P_n$$

Consequently, $max\left\{\left(\frac{1}{\mathrm{M}(x_{2k+1},x_{2s+2},t)}-1\right), 0 \le k, s \le n \quad \right\} < P_n$

As a result, there is r_n, $1 \le r_n \le n$ such that

$$P_n = \frac{1}{\mathrm{M}(x_0, x_{2r_n+2}, t)} - 1$$

We have that $0 \le P_{n-1} \le P_n$. So the sequence $\{P_n\}_{n\in N}$ is non-decreasing.

Let us show that this sequence is also bounded.

Suppose that the sequence $\{P_n\}_{n\in N}$ is unbounded. As a result, since it is increasing, we have

$$\lim_{n\to\infty} P_n = +\infty$$

Since the fuzzy space is triangular, we have

$$P_n = \frac{1}{\mathrm{M}(x_0,x_{2r_{n+2}},t)} - 1 \le \frac{1}{\mathrm{M}(x_0,x_1,t)} - 1 + \frac{1}{\mathrm{M}(x_1,x_{2r_n+2},t)} - 1 \quad \le \frac{1}{\mathrm{M}(x_0,x_1,t)} - 1 + \tilde{N}\left(x_{0,}, x_{2r_n+2}\right)$$

Such that

$$\tilde{N}\left(x_{0,}, x_{2r_n+2}\right) = max\left\{\beta\left(\frac{1}{\mathrm{M}(x_0,x_{2r_n+1},t)} - 1\right)\left(\frac{1}{\mathrm{M}(x_0,x_{2r_n+1},t)} - 1\right),\ \beta\left(\frac{1}{\mathrm{M}(x_1,x_{2r_n+1},t)} - 1\right)\left(\frac{1}{\mathrm{M}(x_1,x_{2r_n+1},t)} - 1\right),\right.$$
$$\beta\left(\frac{1}{\mathrm{M}(x_0,x_{2r_n+2},t)} - 1\right)\left(\frac{1}{\mathrm{M}(x_0,x_{2r_n+2},t)} - 1\right),\ \beta\left(\frac{1}{\mathrm{M}(x_0,x_1,t)} - 1\right)\left(\frac{1}{\mathrm{M}(x_1,x_0,t)} - 1\right),\ \beta\left(\frac{1}{\mathrm{M}(x_{2r_n+2},x_{2r_n+1},t)} - \right.$$
$$\left.\left.1\right)\frac{1}{\mathrm{M}(x_{2r_n+2},x_{2r_n+1},t)} - 1\right\}$$

Depending on the above inequality, we consider the following cases.

Case 1. We take $\tilde{N}\left(x_{0,}, x_{2r_n+2}\right) = \beta\left(\frac{1}{\mathrm{M}(x_0,x_{2r_n+1},t)} - 1\right) \cdot \left(\frac{1}{\mathrm{M}(x_0,x_{2r_n+1},t)} - 1\right)$. By replacing this expression, we have

$$\frac{1}{\mathrm{M}(x_0,x_{2r_n+2},t)} - 1 \le \frac{1}{\mathrm{M}(x_0,x_1,t)} - 1 + \beta\left(\frac{1}{\mathrm{M}(x_0,x_{2r_n+1},t)} - 1\right) \cdot \left(\frac{1}{\mathrm{M}(x_0,x_{2r_n+1},t)} - 1\right)$$

$$P_n < \frac{1}{\mathrm{M}(x_0, x_1, t)} - 1 + \beta\left(\frac{1}{\mathrm{M}(x_0, x_{2r_n+1}, t)} - 1\right) \cdot P_n$$

$$\frac{1 - \left(\frac{1}{\mathrm{M}(x_0, x_1, t)} - P_n\right)}{P_n} \le \beta\left(\frac{1}{\mathrm{M}(x_0, x_{2r_n+1}, t)} - 1\right) < 1$$

$$1 - \frac{\left(\frac{1}{\mathrm{M}(x_0, x_1, t)} - 1\right)}{P_n} \le \beta\left(\frac{1}{\mathrm{M}(x_0, x_{2r_n+1}, t)} - 1\right) < 1$$

Taking the limit $\lim_{n\to\infty} 1 - \frac{\left(\frac{1}{\mathrm{M}(x_0,x_1,t)}-1\right)}{P_n} = 1$, we have $\lim_{n\to\infty} \beta\left(\frac{1}{\mathrm{M}(x_0,x_{2r_n+1},t)} - 1\right) = 1$ and $\lim_{n\to\infty} \frac{1}{\mathrm{M}(x_0,x_{2r_{n+1}},t)} - 1 = 0$

Consequently $\lim_{n\to\infty} P_n = \frac{1}{\mathrm{M}(x_0,x_1,t)} - 1$ which is a contradiction because $\lim_{n\to\infty} P_n = +\infty$.

Case 2. If we choose $\tilde{N}(x_0,, x_{2r_n+2}) = \beta\left(\frac{1}{M(x_1,x_{2r_n+1},t)} - 1\right)(\frac{1}{M(x_1,x_{2r_n+1},t)} - 1)$

$$P_n \leq \frac{1}{M(x_0,x_1,t)} - 1 + \beta\left(\frac{1}{M(x_1,x_{2r_n+1},t)} - 1\right)(\frac{1}{M(x_1,x_{2r_n+1},t)} - 1)$$

$$P_n < \frac{1}{M(x_0,x_1,t)} - 1 + \beta\left(\frac{1}{M(x_0,x_{2r_n+1},t)} - 1\right) \cdot P_n$$

$$\frac{1 - \left(\frac{1}{M(x_0,x_1,t)} - P_n\right)}{P_n} \leq \beta\left(\frac{1}{M(x_1,x_{2r_n+1},t)} - 1\right) < 1$$

$$1 - \frac{\left(\frac{1}{M(x_0,x_1,t)} - 1\right)}{P_n} \leq \beta\left(\frac{1}{M(x_1,x_{2r_n+1},t)} - 1\right) < 1$$

Knowing that limit $\lim\limits_{n\to\infty} 1 - \frac{\left(\frac{1}{M(x_0,x_1,t)}-1\right)}{P_n} = 1$, it results $\lim\limits_{n\to\infty} \beta\left(\frac{1}{M(x_1,x_{2r_n+1},t)} - 1\right) = 1$ and $\lim\limits_{n\to\infty} \frac{1}{M(x_1,x_{2r_n+1},t)} - 1 = 0$

So $\lim\limits_{n\to\infty} P_n = \frac{1}{M(x_0,x_1,t)} - 1$ which is a contradiction because $\lim\limits_{n\to\infty} P_n = +\infty$.

Case 3. In this case, we take $\tilde{N}(x_0,, x_{2r_n+2}) = \beta\left(\frac{1}{M(x_0,x_{2r_n+2},t)} - 1\right)(\frac{1}{M(x_0,x_{2r_n+2},t)} - 1)$

$$P_n \leq \frac{1}{M(x_0,x_1,t)} - 1 + \beta\left(\frac{1}{M(x_0,x_{2r_n+2},t)} - 1\right) \cdot \left(\frac{1}{M(x_0,x_{2r_n+2},t)} - 1\right)$$

$$P_n < \frac{1}{M(x_0,x_1,t)} - 1 + \beta\left(\frac{1}{M(x_0,x_{2r_n+2},t)} - 1\right) \cdot P_n$$

From the above inequality, we have $1 - \frac{\left(\frac{1}{M(x_0,x_1,t)}-1\right)}{P_n} \leq \beta\left(\frac{1}{M(x_0,x_{2r_n+2},t)} - 1\right) < 1$.

Taking the limit of three sides and using squeezing theorem, it yields $\lim\limits_{n\to\infty} \beta\left(\frac{1}{M(x_0,x_{2r_n+2},t)} - 1\right) = 1$ and $\lim\limits_{n\to\infty} \frac{1}{M(x_0,x_{2r_n+2},t)} - 1 = 0$.

This leads to $\lim\limits_{n\to\infty} P_n = \frac{1}{M(x_0,x_1,t)} - 1$ which contradicts our assumption.

Case 4. Taking now $\tilde{N}(x_0,, x_{2r_n+2}) = \beta\left(\frac{1}{M(x_0,x_1,t)} - 1\right)\left(\frac{1}{M(x_1,x_0,t)} - 1\right)$

we have

$$P_n \leq \frac{1}{M(x_0,x_1,t)} - 1 + \beta\left(\frac{1}{M(x_0,x_1,t)} - 1\right) \cdot \left(\frac{1}{M(x_0,x_1,t)} - 1\right) \leq 2\left(\frac{1}{M(x_0,x_1,t)} - 1\right)$$

Which proves that the sequence $\{P_n\}$ is not bounded.

Case 5. Studying $\tilde{N}(x_{0,},x_{2r_n+2}) = \beta\left(\frac{1}{M(x_{2r_n+2},x_{2r_n+1},t)} - 1\right)(\frac{1}{M(x_{2r_n+2},x_{2r_n+1},t)} - 1)$

we take

$P_n \le \frac{1}{M(x_0,x_1,t)} - 1 + \beta\left(\frac{1}{M(x_{2r_n+2},x_{2r_n+1},t)} - 1\right)(\frac{1}{M(x_{2r_n+2},x_{2r_n+1},t)} - 1)$

$$P_n < \frac{1}{M(x_0,x_1,t)} - 1 + \beta\left(\frac{1}{M(x_{2r_n+2},x_{2r_n+1},t)} - 1\right) \cdot P_n$$

Calculating, we have $1 - \frac{\left(\frac{1}{M(x_0,x_1,t)} - 1\right)}{P_n} \le \beta\left(\frac{1}{M(x_{2r_n+2},x_{2r_n+1},t)} - 1\right) < 1$

By taking limits in the above inequality, it yields $\lim_{n\to\infty} \beta\left(\frac{1}{M(x_{2r_n+2},x_{2r_n+1},t)} - 1\right) = 1$ and $\lim_{n\to\infty}(\frac{1}{M(x_{2r_n+2},x_{2r_n+1},t)} - 1) = 0.$

Consequently $\lim_{n\to\infty} P_n = \frac{1}{M(x_0,x_1,t)} - 1$ which is a contradiction.

Since we achieve a contradiction in all cases, we have that the sequence $\{P_n\}_{n\in N}$ is bounded.

The sequence $\{P_n\}_{n\in N}$ is increasing and bounded from above, as a result, it converges.

Constructing the sequence $q_n = sup\left\{\frac{1}{M(x_k,x_s,t)} - 1,\ k,s \ge n, k,s \in N\right\}$

We have that $0 \le q_n \le q_{n-1} \le \cdots \le q_0 \le \lim_{n\to\infty} P_n \le q$

Since the sequence $\{q_n\}_{n\in N}$is decreasing and bounded from below, it converges to $q \ge 0$.

Let us prove that $q = 0$.

Suppose that $q > 0$.

We see the following inequality

$$\begin{aligned}
\frac{1}{M(x_{2k+1},x_{2s+2},t)} - 1 \\
&\le max\Big\{\beta\left(\frac{1}{M(x_{2k},x_{2s+1},t)} - 1\right)(\frac{1}{M(x_{2k},x_{2s+1},t)} - 1), \\
&\beta\left(\frac{1}{M(x_{2k+1},x_{2s+1},t)} - 1\right)(\frac{1}{M(x_{2k+1},x_{2s+1},t)} \\
&- 1), \beta\left(\frac{1}{M(x_{2k},x_{2s+2},t)} - 1\right)\ (\frac{1}{M(x_{2k},x_{2s+2},t)} - 1), \\
&\beta\left(\frac{1}{M(x_{2k},x_{2s+1},t)} - 1\right)(\frac{1}{M(x_{2k},x_{2s+1},t)} - 1), \\
&\beta\left(\frac{1}{M(x_{2s+1},x_{2s+1},t)} - 1\right)(\frac{1}{M(x_{2s+1},x_{2s+1},t)} - 1)\Big\}
\end{aligned}$$

Analysing the cases we have:

Case 1.

$$\frac{1}{\mathrm{M}(x_{2k+1}, x_{2s+2}, t)} - 1 \leq \beta\left(\frac{1}{\mathrm{M}(x_{2k}, x_{2s+1}, t)} - 1\right)\left(\frac{1}{\mathrm{M}(x_{2k}, x_{2s+1}, t)} - 1\right),$$

Taking the limit when $k, s \to \infty, t > 0$, of the supremum of both sides, we have

$$q \leq \lim_{\mathrm{k,s} \to +\infty,} Sup\ \beta\left(\frac{1}{\mathrm{M}(x_{2k}, x_{2s+1}, t)} - 1\right) \cdot q$$

As a result, $\lim_{\mathrm{k,s} \to +\infty} Sup\ \beta\left(\frac{1}{\mathrm{M}(x_{2k}, x_{2s+1}, t)} - 1\right) = 1$ and consequently $\lim_{\mathrm{k,s} \to +\infty} \frac{1}{\mathrm{M}(x_{2k}, x_{2s+1}, t)} - 1 = r = 0$

Using the same method, we can prove that $q = 0$ in all cases.

Since $\lim_{\mathrm{k,s} \to +\infty} \frac{1}{\mathrm{M}(x_{2k+1}, x_{2s+2}, t)} - 1 = 0$, the sequence $\{x_{2n}\}_{n\in N}$ is Cauchy.

From the completeness of the fuzzy space $(X, M, *)$ we have that the sequence $\{x_{2n}\}_{n\in N}$ converges to a point $x \in M$.

Next, we have to prove that x is a common fixed point of f and g.

$$\frac{1}{\mathrm{M}(x_{2n+1}, fx^*, t)} - 1 = \frac{1}{\mathrm{M}(gx_{2n}, fx^*, t)} - 1 \leq max\left\{\beta\left(\frac{1}{\mathrm{M}(x_{2n}, x^*, t)} - 1\right)\left(\frac{1}{\mathrm{M}(x_{2n}, x^*, t)} - 1\right), \beta\left(\frac{1}{\mathrm{M}(gx_{2n}, x_{2n}, t)} - 1\right)\left(\frac{1}{\mathrm{M}(gx_{2n}, x_{2n}, t)} - 1\right),\ \beta\left(\frac{1}{\mathrm{M}(fx^*, x^*, t)} - 1\right)\left(\frac{1}{\mathrm{M}(fx^*, x^*, t)} - 1\right),\ \beta\left(\frac{1}{\mathrm{M}(x_{2n+1}, x^*, t)} - 1\right)\left(\frac{1}{\mathrm{M}(x_{2n+1}, x^*, t)} - 1\right), \beta\left(\frac{1}{\mathrm{M}(fx^*, x_{2n}, t)} - 1\right)\left(\frac{1}{\mathrm{M}(fx^*, x_{2n}, t)} - 1\right)\right\}$$

Let us discuss the following cases:

Case 1. Taking

$$\frac{1}{\mathrm{M}(x_{2n+1}, fx^*, t)} - 1 \leq \beta\left(\frac{1}{\mathrm{M}(x_{2n}, x^*, t)} - 1\right) \cdot \left(\frac{1}{\mathrm{M}(x_{2n}, x^*, t)} - 1\right)$$

we see that

$$\frac{1}{\mathrm{M}(x^*, fx^*, t)} - 1 \leq \frac{1}{\mathrm{M}(x^*, x_{2n+1}, t)} - 1 + \frac{1}{\mathrm{M}(x_{2n+1}, fx^*, t)} - 1 \leq \frac{1}{\mathrm{M}(x^*, x_{2n+1}, t)} - 1 + \beta\left(\frac{1}{\mathrm{M}(x_{2n}, x^*, t)} - 1\right) \cdot \left(\frac{1}{\mathrm{M}(x_{2n}, x^*, t)} - 1\right)$$

Considering the limit of the inequality, we get

$$0 \leq \frac{1}{\mathrm{M}(x^*, fx^*, t)} - 1 \leq \lim_{n\to\infty} \frac{1}{\mathrm{M}(x_{2n+1}, fx^*, t)} - 1 \leq 0$$

So $\frac{1}{\mathrm{M}(x^*, fx^*, t)} - 1 = 0$ and $\mathrm{M}(x^*, fx^*, t) = 1\ and\ fx^* = x^*$

Using the same method, we prove that $gx^* = x^*$

So $fx^* = gx^* = x^*\ and\ \ x^*$is a common fixed point for f and g.

Let us prove that x^* is unique.

Suppose that there exists another point y^*such that

$fy^* = gy^* = y^*$

$\frac{1}{M(x^*,y^*,t)} - 1 = \frac{1}{M(fx^*,gy^*,t)} - 1 \leq \beta\left(\frac{1}{M(x^*,y^*,t)} - 1\right) max\left\{\beta\left(\frac{1}{M(x^*,y^*,t)} - 1\right)\left(\frac{1}{M(x^*,y^*,t)} - 1\right), \beta\left(\frac{1}{M(fx^*,y^*,t)} - 1\right)\left(\frac{1}{M(fx^*,y^*,t)} - 1\right), \beta\left(\frac{1}{M(x^*,gy^*,t)} - 1\right)\left(\frac{1}{M(x^*,gy^*,t)} - 1\right), \beta\left(\frac{1}{M(x^*,fx^*,t)} - 1\right)\left(\frac{1}{M(x^*,fx^*,t)} - 1\right), \beta\left(\frac{1}{M(y^*,gy^*,t)} - 1\right)\left(\frac{1}{M(y^*,gy^*,t)} - 1\right)\right\}$

$= \beta\left(\frac{1}{M(x^*,y^*,t)} - 1\right) \cdot \left(\frac{1}{M(x^*,y^*,t)} - 1\right)$

$$\left(\frac{1}{M(x^*,y^*,t)} - 1\right) \leq \beta\left(\frac{1}{M(x^*,y^*,t)} - 1\right) \cdot \left(\frac{1}{M(x^*,y^*,t)} - 1\right)$$

which holds only for $\frac{1}{M(x^*,y^*,t)} - 1 = 0$ since $\beta\left(\frac{1}{M(x^*,y^*,t)} - 1\right) < 1$

Example 2

Let $X = R^+, t > 0, *$ is continuous

$$M: X \times X \times (0, +\infty) \to [0,1] \quad M(x,y,t) = \frac{t}{t + |x - y|}$$

For every $x, y \in X, t > 0$, a fuzzy metric space. The triple $(X, M, *)$ is a fuzzy metric space, and it is complete and triangular.

Let $f: X \to X$, $f(x) = \begin{cases} \frac{4}{5}, & x \neq 1 \\ 1, & x = 1 \end{cases}$ $g: X \to X$ $g(y) = \begin{cases} \frac{1}{5}, & x \neq 1 \\ 1, & x = 1 \end{cases}$

be two functions, and

$$\beta(s) = \begin{cases} \frac{1}{s}, & 1 < s < \frac{5}{4} \\ \frac{9}{10}, & otherwise \end{cases}$$

We see that

$\frac{1}{M(fx,gy,t)} - 1 \leq max\left\{\beta\left(\frac{1}{M(x,y,t)} - 1\right)\left(\frac{1}{M(x,y,t)} - 1\right), \beta\left(\frac{1}{M(fx,y,t)} - 1\right)\left(\frac{1}{M(fx,y,t)} - 1\right), \beta\left(\frac{1}{M(x,gy,t)} - 1\right)\left(\frac{1}{M(x,gy,t)} - 1\right), \beta\left(\frac{1}{M(x,fx,t)} - 1\right)\left(\frac{1}{M(x,fx,t)} - 1\right), \beta\left(\frac{1}{M(y,g\,y,t)} - 1\right)\left(\frac{1}{M(y,g\,y,t)} - 1\right)\right\}$

Consequently, the functions f and g have a common fixed point

$x = 1, f1 = g1 = 1.$

Theorem 3. Let $(X, M, *)$ be a complete, triangular fuzzy metric space and $f: X \to X$ a Geraghty-Ciric fuzzy contraction function. Then f has a unique fixed point in X.

Proof. If we take $f = g$ in Theorem 3.1, our result is true.

Corollary 4. Let $(X, M, *)$ be a complete, triangular fuzzy metric space and $f: X \to X$ a Ciric fuzzy contraction function.

$\frac{1}{M(fx,gy,t)} - 1 \leq k \; max\left\{\frac{1}{M(x,y,t)} - 1, \frac{1}{M(fx,y,t)} - 1, \frac{1}{M(x,fy,t)} - 1, \frac{1}{M(fx,x,t)} - 1, \frac{1}{M(y,y,t)} - 1\right\}$

For $0 < k < 1, x, y \in X, t > 0$.

Then f has a unique fixed point.

Conclusion

We investigated common fixed-point results for a pair of weakly compatible mappings satisfying a Geraghty-Cirić-type contraction condition in the setting of complete fuzzy metric spaces in this article. Using the properties of Cauchy and Convergent sequences, we proved the existence and uniqueness of common fixed point of the mappings under consideration. The results we get extend and generalise the existing theorems in the literature (especially that of Shu-Fang Li et al) with a view about weak compatibility and contractive condition in fuzzy metric spaces. This shows that how generalized contraction conditions are combined with the structure of fuzzy metric space is promising for further developments in fixed point theory. A relevant example has been given to validate the main theorem. A corollary and a special case of the main theorem are derived thus showing the strength of the said approach.

Recommendations

Based on the results obtained in this study, several directions for future research can be suggested. The proposed approach can be extended to more general classes of mappings, such as multivalued mappings, cyclic mappings, or mappings satisfying weaker compatibility conditions. Furthermore, it would be of interest to investigate similar fixed-point results in more generalized spaces, including intuitionistic fuzzy metric spaces, probabilistic metric spaces, or other hybrid structures that capture uncertainty in a more refined way.In addition, the methods developed in this work could be applied to problems in applied mathematics, such as integral equations, differential equations, and models involving uncertainty, where fuzzy metric frameworks are particularly well-suited.

Scientific Ethics Declaration

* The authors declare that the scientific ethical and legal responsibility of this article published in EPSTEM journal belongs to the authors.

Conflict of Interest

* The authors declare that they have no conflicts of interest

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