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Neutrosophic Nano Regular Generalized Star Star B – Continuous and Irresolute Mappings in Neutrosophic Nano Topological Spaces

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Abstract: Neutrosophic sets, proposed by Smarandache, extend fuzzy sets and intuitionistic fuzzy sets to better manage imprecise, incomplete, and inconsistent information frequently encountered in real-world problems. A neutrosophic set is characterized by three independent components: truth, indeterminacy, and falsity. Based on this framework, Salama introduced the notion of neutrosophic topological spaces. The concept of a Neutrosophic Nano Topological Space further integrates neutrosophic set theory with nano topology to effectively model uncertainty, indeterminacy, and vagueness in highly granular or fine-scale systems and structures. In 2020, N. Rajeshwari, G. Vasanthakannan, and J. Subashini introduced the concepts of neutrosophic nano regular generalized star star b-closed ($N_{cut}N_{ano}rg^{**}b$ -closed) sets and studied some basic properties in neutrosophic nano topological space ($N_{cut}N_{ano}$ -Top-Spaces). In this research paper, we introduce the concepts of $N_{cut}N_{ano}rg^{**}b$ -continuous mappings, $N_{cut}N_{ano}rg^{**}b$ -irresolute mappings, $N_{cut}N_{ano}rg^{**}b$ -closed mappings, $N_{cut}N_{ano}rg^{**}b$ -open mappings, strongly $N_{cut}N_{ano}rg^{**}b$ -continuous mappings, perfectly $N_{cut}N_{ano}rg^{**}b$ -continuous mappings, $N_{cut}N_{ano}contra$ - $rg^{**}b$ -continuous mappings, $N_{cut}N_{ano}contra$ - $rg^{**}b$ -irresolute mappings and totally $N_{cut}N_{ano}rg^{**}b$ -continuous mappings in neutrosophic topological spaces ($N_{cut}N_{ano}$ -Top-Spaces). We investigate and obtain several properties and characterizations concerning these mappings in $N_{cut}N_{ano}$ -Top-Spaces.

Keywords: $N_{cut}N_{ano}rg^{**}b$ -continuous map, $N_{cut}N_{ano}rg^{**}b$ -irresolute map, $N_{cut}N_{ano}rg^{**}b$ -open map, $N_{cut}N_{ano}rg^{**}b$ -closed map, $N_{cut}N_{ano}contra$ - $rg^{**}b$ -irresolute map

Introduction

Many real-world problems inherently involve uncertainty, imprecision, and indeterminacy, particularly in disciplines such as computer science, engineering, artificial intelligence, medical diagnosis, and decision-making. Conventional set theory and topology, which are founded on precise and deterministic information, often fall short in handling such complexities. To address these shortcomings, various mathematical models have been developed, including fuzzy sets proposed by Zadeh (1965), intuitionistic fuzzy sets introduced by Atanassov (1986), rough sets formulated by Pawlak (1982), soft sets by Molodtsov (1999), and neutrosophic sets by Smarandache (1999). These approaches have made substantial contributions to the modeling and analysis of incomplete, inconsistent, and uncertain data. The investigation of $N_{cut}N_{ano}$ -Top-Space is significant from both theoretical and applied perspectives. Theoretically, it expands the framework of topological structures by combining the adaptability of neutrosophic sets with the rigor of nano topology. In practice, $N_{cut}N_{ano}$ -Top-Space demonstrates strong potential for applications such as knowledge representation, data mining, image processing, pattern recognition, and decision-making, where the effective handling of indeterminate and inconsistent information is essential. As a result, $N_{cut}N_{ano}$ -Top-Space provides a robust hybrid framework that fosters new research directions and innovations in environments characterized by uncertainty. In this research paper, we introduce the concepts of $N_{cut}N_{ano}rg^{**}b$ -continuous mappings,

$N_{cut}N_{ano}rg^{**b}$ -irresolute mappings, $N_{cut}N_{ano}rg^{**b}$ -closed mappings, $N_{cut}N_{ano}rg^{**b}$ -open mappings, strongly $N_{cut}N_{ano}rg^{**b}$ -continuous mappings, perfectly $N_{cut}N_{ano}rg^{**b}$ -continuous mappings, $N_{cut}N_{ano}contra-\alpha\mathcal{G}\#\psi$ -continuous mappings, $N_{cut}N_{ano}contra-rg^{**b}$ -irresolute mappings and totally $N_{cut}N_{ano}rg^{**b}$ -continuous mappings in $N_{cut}N_{ano}$ -Top-Space. We investigate and obtain several properties and characterizations concerning these mappings in $N_{cut}N_{ano}$ -Top-Spaces.

Preliminaries

Definition 2.1. Let X be a non-empty fixed set. A neutrosophic set (briefly N_{eu} -set) A is an object having a form $A = \{\langle x, T_A(x), Q_A(x), F_A(x) \rangle : x \in X\}$, where $T_A : X \rightarrow [0,1]$, $Q_A : X \rightarrow [0,1]$ and $F_A : X \rightarrow [0,1]$ are the functions such that $T_A(x)$ -represents the truth-membership degree of membership, $Q_A(x)$ - represents the degree of indeterminacy, and $F_A(x)$ - represents the degree of non-membership.

Definition 2.2. Let $A = \{\langle x, T_A(x), Q_A(x), F_A(x) \rangle : x \in X\}$, and $B = \{\langle x, T_B(x), Q_B(x), F_B(x) \rangle : x \in X\}$

1. $0_{NN} = \{\langle x, 0, 0, 1 \rangle : x \in X\}$, $1_{NN} = \{\langle x, 1, 1, 0 \rangle : x \in X\}$
2. Subset : $A \subseteq B \Leftrightarrow T_A(x) \leq T_B(x), Q_A(x) \leq Q_B(x), F_A(x) \geq F_B(x), " x \in X$
3. Equal : $A = B \Leftrightarrow A \subseteq B \text{ and } B \subseteq A$.
4. Complement (Supplement) : $A^C = \{\langle x, F_A(x), 1 - Q_A(x), T_A(x) \rangle : x \in X\}$,
5. Union : $A \cup B = \{\langle x, \max(T_A(x), T_B(x)), \min(Q_A(x), Q_B(x)), \min(F_A(x), F_B(x)) \rangle : x \in X\}$
6. Intersection : $A \cap B = \{\langle x, \min(T_A(x), T_B(x)), \max(Q_A(x), Q_B(x)), \max(F_A(x), F_B(x)) \rangle : x \in X\}$.

Definition 2.3. A neutrosophic topology on a non-empty X is a family of T_N of neutrosophic sets in X satisfying the following axioms:

- (01) $0_N, 1_N \in T_N$
- (I) " $G, H \in T_N \Rightarrow G \cap H \in T_N$
- (U) " $\{G_\lambda : \lambda \in \Lambda\} \subseteq T_N \Rightarrow \bigcup_{\lambda \in \Lambda} G_\lambda \in T_N$

Definition 2.4. Let U be a non-empty finite set of objects called the universe, and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Let X be a subset of U . Then the lower approximation of X with respect to R is denoted by $\underline{R} = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by $x \in U$.

Definition 2.5. The upper approximation of X with respect to R is the set of all objects that can be possibly classified as X with respect to R and it is denoted by $\overline{R} = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \emptyset\}$.

Definition 2.6. The boundary region of X with respect to R is the set of all objects that can be possibly classified neither as X nor as not X with respect to R and it is denoted by $B_R = \overline{R} - \underline{R}$.

Definition 2.7. Let U be a universe and R be an equivalence relation on U and let S be a neutrosophic subset of U . Then the neutrosophic nano topology

$$\begin{aligned} \Upsilon_{\mathbb{N}\mathbb{N}}(S) &= \{0_{\mathbb{N}\mathbb{N}}, 1_{\mathbb{N}\mathbb{N}}, \overline{\mathbb{N}}(S), \underline{\mathbb{N}}(S), B_N(S)\}, \text{ where } \overline{\mathbb{N}}(S) = \left\{ \langle x, M_{\overline{\mathbb{R}}(x)}, I_{\overline{\mathbb{R}}(x)}, N_{\overline{\mathbb{R}}(x)}(x) \rangle : z \in [x]_{\mathbb{R}}, x \in U \right\}. \\ \underline{\mathbb{N}}(S) &= \left\{ \langle x, M_{\underline{\mathbb{R}}(x)}, I_{\underline{\mathbb{R}}(x)}, N_{\underline{\mathbb{R}}(x)}(x) \rangle : z \in [x]_{\mathbb{R}}, x \in U \right\}. \\ B_N(S) &= \overline{\mathbb{N}}(S) - \underline{\mathbb{N}}(S). \text{ where } M_{\underline{\mathbb{R}}(x)} = \inf \{M_S(z) : z \in [x]_{\mathbb{R}}\}, \\ I_{\underline{\mathbb{R}}(x)} &= \inf \{I_S(z) : z \in [x]_{\mathbb{R}}\}, N_{\underline{\mathbb{R}}(x)} = \sup \{N_S(z) : z \in [x]_{\mathbb{R}}\}, M_{\overline{\mathbb{R}}(x)} = \sup \{M_S(z) : z \in [x]_{\mathbb{R}}\}, \\ I_{\overline{\mathbb{R}}(x)} &= \sup \{I_S(z) : z \in [x]_{\mathbb{R}}\}, N_{\overline{\mathbb{R}}(x)} = \inf \{N_S(z) : z \in [x]_{\mathbb{R}}\}. \end{aligned}$$

Definition 2.8. Let U be a universe, $\mathbb{R}$ be an equivalence relation on U and S be a neutrosophic set in U and if the collection $\Upsilon_{\mathbb{N}\mathbb{N}}(S) = \{0_{\mathbb{N}\mathbb{N}}, 1_{\mathbb{N}\mathbb{N}}, \underline{\mathbb{N}}(S), \overline{\mathbb{N}}(S), B_N(S)\}$ containing neutrosophic nano subsets of U satisfies the axioms of a topology, namely (i) $0_{\mathbb{N}\mathbb{N}}, 1_{\mathbb{N}\mathbb{N}} \in \Upsilon_{\mathbb{N}\mathbb{N}}(S)$ (ii) $G \cap H \in \Upsilon_{\mathbb{N}\mathbb{N}}(S)$ for every $G, H \in \Upsilon_{\mathbb{N}\mathbb{N}}(S)$ (iii) $\bigcup_{j \in J} G_j \in \Upsilon_{\mathbb{N}\mathbb{N}}(S)$ for every $\{G_j : j \in J\} \subseteq \Upsilon_{\mathbb{N}\mathbb{N}}(S)$. Then the pair $(U, \Upsilon_{\mathbb{N}\mathbb{N}}(S))$ is called a neutrosophic nano topological space (briefly $N_{\text{eut}}N_{\text{ano}}\text{-Top-Space}$). The elements of $\Upsilon_{\mathbb{N}\mathbb{N}}(S)$ are called neutrosophic nano rg^{**}b -open (briefly $N_{\text{eut}}N_{\text{ano}}\text{rg}^{**}\text{b}$ -open) sets in U . A $N_{\text{eut}}N_{\text{ano}}$ -set A in U is called a neutrosophic nano rg^{**}b -closed (briefly $N_{\text{eut}}N_{\text{ano}}\text{rg}^{**}\text{b}$ -closed) set if and only if its complement A^C is a $N_{\text{eut}}N_{\text{ano}}\alpha\text{g}\#\psi$ -open set.

Definition 2.9. Let $(U, \Upsilon_{\mathbb{N}\mathbb{N}}(S))$ be a $N_{\text{eut}}N_{\text{ano}}\text{-Top-Space}$ and A be a $N_{\text{eut}}N_{\text{ano}}$ -set. Then

- (i) The neutrosophic nano interior of A , denoted by $N_{\text{eut}}N_{\text{ano}}\text{-Int}(A)$ is the union of all $N_{\text{eut}}N_{\text{ano}}$ -open subsets of A . $N_{\text{eut}}N_{\text{ano}}\text{-Int}(A) = \bigcup \{G : G \text{ is a } N_{\text{eut}}N_{\text{ano}}\text{-open set in } U \text{ and } G \subseteq A\}$.
- (ii) The neutrosophic nano closure of A denoted by $N_{\text{eut}}N_{\text{ano}}\text{-Cl}(A)$ is the intersection of all $N_{\text{eut}}N_{\text{ano}}$ -closed sets containing A . $N_{\text{eut}}N_{\text{ano}}\text{-Cl}(A) = \bigcap \{K : K \text{ is a } N_{\text{eut}}N_{\text{ano}}\text{-closed set in } U \text{ and } A \subseteq K\}$.

Definition 2.10. Let A be a $N_{\text{eut}}N_{\text{ano}}$ -subset of a $N_{\text{eut}}N_{\text{ano}}\text{-Top-Space}$ $(U, \Upsilon_{\mathbb{N}\mathbb{N}}(S))$. Then A is called

- (i) $N_{\text{eut}}N_{\text{ano}}$ -semi-open if $A \subseteq N_{\text{eut}}N_{\text{ano}}\text{-Cl}(N_{\text{eut}}N_{\text{ano}}\text{-Int}(A))$.
- (ii) $N_{\text{eut}}N_{\text{ano}}\text{-}\alpha$ -open set if $A \subseteq N_{\text{eut}}N_{\text{ano}}\text{-Int}(N_{\text{eut}}N_{\text{ano}}\text{-Cl}(N_{\text{eut}}N_{\text{ano}}\text{-Int}(A)))$.
- (iii) $N_{\text{eut}}N_{\text{ano}}$ -pre-open set if $A \subseteq N_{\text{eut}}N_{\text{ano}}\text{-Int}(N_{\text{eut}}N_{\text{ano}}\text{-Cl}(A))$.
- (iv) $N_{\text{eut}}N_{\text{ano}}$ -sregular-open set if $A = N_{\text{eut}}N_{\text{ano}}\text{-Int}(N_{\text{eut}}N_{\text{ano}}\text{-Cl}(A))$.
- (v) $N_{\text{eut}}N_{\text{ano}}\text{-b}$ -open set if $A \subseteq [N_{\text{eut}}N_{\text{ano}}\text{-Cl}(N_{\text{eut}}N_{\text{ano}}\text{-Int}(A))] \cup [N_{\text{eut}}N_{\text{ano}}\text{-Int}(N_{\text{eut}}N_{\text{ano}}\text{-Cl}(A))]$
- (vi) $N_{\text{eut}}N_{\text{ano}}$ -semi pre open if $A \subseteq N_{\text{eut}}N_{\text{ano}}\text{-Cl}(N_{\text{eut}}N_{\text{ano}}\text{-Int}(N_{\text{eut}}N_{\text{ano}}\text{-Cl}(A)))$.

The complement of the above-mentioned sets are called their respective $N_{\text{eut}}N_{\text{ano}}$ -closed sets.

Definition 2.11. Let A be a $N_{\text{eut}}N_{\text{ano}}$ -subset of a $N_{\text{eut}}N_{\text{ano}}\text{-Top-Space}$ $(U, \Upsilon_{\mathbb{N}\mathbb{N}}(S))$. Then A is called a

- (i) $N_{\text{eut}}N_{\text{ano}}$ -generalized-closed (briefly $N_{\text{eut}}N_{\text{ano}}\text{g-closed}$) set if $N_{\text{eut}}N_{\text{ano}}\text{-Cl}(A) \subseteq V$ whenever $A \subseteq V$ and V is $N_{\text{eut}}N_{\text{ano}}$ -open in $(U, \Upsilon_{\mathbb{N}\mathbb{N}}(S))$.
- (ii) $N_{\text{eut}}N_{\text{ano}}$ -generalized-star-closed (briefly $N_{\text{eut}}N_{\text{ano}}\text{g}^*\text{-closed}$) set if $N_{\text{eut}}N_{\text{ano}}\text{-Cl}(A) \subseteq V$ whenever $A \subseteq V$ and V is $N_{\text{eut}}N_{\text{ano}}\text{g}$ -open set in $(U, \Upsilon_{\mathbb{N}\mathbb{N}}(S))$.

- (iii) $N_{eut}N_{ano}$ generalized α -closed (briefly $N_{eut}N_{ano}g\alpha$ -closed) set if $N_{eut}N_{ano}\alpha-Cl(A) \subseteq V$ whenever $A \subseteq V$ and V is $N_{eut}N_{ano}\alpha$ -open in $(U, \Upsilon_{NN}(S))$.
- (iv) $N_{eut}N_{ano}$ generalized p -pre-closed (briefly $N_{eut}N_{ano}gp$ -closed) set if $N_{eut}N_{ano}p-Cl(A) \subseteq V$ whenever $A \subseteq V$ and V is $N_{eut}N_{ano}$ -open in $(U, \Upsilon_{NN}(S))$.
- (v) $N_{eut}N_{ano}$ generalized p -regular closed (briefly $N_{eut}N_{ano}gpr$ -closed) set if $N_{eut}N_{ano}p-Cl(A) \subseteq V$ whenever $A \subseteq V$ and V is $N_{eut}N_{ano}$ -regular open in $(U, \Upsilon_{NN}(S))$.
- (vi) $N_{eut}N_{ano}$ regular generalized $\star$ -closed (briefly $N_{eut}N_{ano}rg\star$ -closed) set if $N_{eut}N_{ano}\star-Cl(A) \subseteq V$ whenever $A \subseteq V$ and V is $N_{eut}N_{ano}$ -regular open in $(U, \Upsilon_{NN}(S))$.
- (vii) $N_{eut}N_{ano}$ regular generalized $\star$ -b closed (briefly $N_{eut}N_{ano}rg\star b$ -closed) set if $N_{eut}N_{ano}b-Cl(A) \subseteq V$ whenever $A \subseteq V$ and V is $N_{eut}N_{ano}rg\star$ -open in $(U, \Upsilon_{NN}(S))$.

Definition 2.12. Let $(U, \Upsilon_{NN}(S))$ be a $N_{eut}N_{ano}$ -Top-Space and A be a $N_{eut}N_{ano}$ -set of U . Then A is said to be a Neutrosophic Nano regular generalized star star b - closed (briefly $N_{eut}N_{ano}rg\star\star b$ -closed) set if $N_{eut}N_{ano}rg\star\star b-Cl(A) \subseteq V$ whenever $A \subseteq V$ and V is $N_{eut}N_{ano}$ -open in $(U, \Upsilon_{NN}(S))$.

Definition 2.13. Let $(U, \Upsilon_{NN}(S))$ be a $N_{eut}N_{ano}$ -Top-Space and A be a $N_{eut}N_{ano}$ -set of U . Then A is said to be a Neutrosophic Nano regular generalized star star b - open (briefly $N_{eut}N_{ano}rg\star\star b$ -open) set if its complement A^c is $N_{eut}N_{ano}rg\star\star b$ -closed in $(U, \Upsilon_{NN}(S))$.

The collection of all $N_{eut}N_{ano}rg\star\star b$ -open (resp. $N_{eut}N_{ano}rg\star\star b$ -closed) sets in a $N_{eut}N_{ano}$ -Top-Space $(U, \Upsilon_{NN}(S))$ is denoted by $N_{eut}N_{ano}rg\star\star b-O(U, \Upsilon_{NN}(S))$ [*resp.* $N_{eut}N_{ano}rg\star\star b-C(U, \Upsilon_{NN}(S))$].

Theorem 2.14. In a $N_{eut}N_{ano}$ -Top-Space $(U, \Upsilon_{NN}(S))$, we have the following conditions.

- (i) 0_{NN} and 1_{NN} are $N_{eut}N_{ano}rg\star\star b$ -open and $N_{eut}N_{ano}rg\star\star b$ -closed sets in $(U, \Upsilon_{NN}(S))$.
- (ii) The Union of two $N_{eut}N_{ano}rg\star\star b$ -closed sets in $(U, \Upsilon_{NN}(S))$ is $N_{eut}N_{ano}rg\star\star b$ -closed set.
- (iii) The Intersection of two $N_{eut}N_{ano}rg\star\star b$ -open sets in $(U, \Upsilon_{NN}(S))$ is $N_{eut}N_{ano}rg\star\star b$ -open

Definition 2.15. A $N_{eut}N_{ano}$ -set A in a $N_{eut}N_{ano}$ -Top-Space $(U, \Upsilon_{NN}(S))$ is called a $N_{eut}N_{ano}rg\star\star b$ -interior of A (briefly $N_{eut}N_{ano}rg\star\star b-Int(A)$) and $N_{eut}N_{ano}rg\star\star b$ -closure of A (briefly $N_{eut}N_{ano}rg\star\star b-Cl(A)$) are defined as follows:

- (i) $N_{eut}N_{ano}rg\star\star b-Int(A) = \bigcup \{G : G \text{ is a } N_{eut}N_{ano}rg\star\star b\text{-open set in } U \text{ and } G \subseteq A\}$.
- (ii) $N_{eut}N_{ano}rg\star\star b-Cl(A) = \bigcap \{K : K \text{ is a } N_{eut}N_{ano}rg\star\star b\text{-closed set in } U \text{ and } A \subseteq K\}$.

Theorem 2.16. Every $N_{eut}N_{ano}$ -closed set is $N_{eut}N_{ano}rg\star\star b$ -closed set.

Theorem 2.17. Every $N_{eut}N_{ano}$ -Regular-closed set is $N_{eut}N_{ano}rg\star\star b$ -closed set.

Theorem 2.18. Every $N_{eut}N_{ano}\alpha$ -closed set is $N_{eut}N_{ano}rg\star\star b$ -closed set.

Theorem 2.19. Every $N_{eut}N_{ano}g$ -closed set is $N_{eut}N_{ano}rg\star\star b$ -closed set.

Theorem 2.20. Every $N_{eut}N_{ano}g\star$ -closed set is $N_{eut}N_{ano}rg\star\star b$ -closed set.

Theorem 2.21. Every $N_{eut}N_{ano}g\alpha$ -closed set is $N_{eut}N_{ano}rg^{**}b$ -closed set.

Theorem 2.22. Let $(U, \Upsilon_{NN}(S))$ be a $N_{eut}N_{ano}$ -Top-Space. Then for any $N_{eut}N_{ano}$ -subsets A and B of U , the following statements are true.

- (i) $N_{eut}N_{ano}rg^{**}b-Int(A) \subseteq A \subseteq N_{eut}N_{ano}rg^{**}b-Cl(A)$.
- (ii) A is $N_{eut}N_{ano}rg^{**}b$ -open set in U if and only if $N_{eut}N_{ano}rg^{**}b-Int(A) = A$.
- (iii) A is $N_{eut}N_{ano}rg^{**}b$ -closed set in U if and only if $N_{eut}N_{ano}rg^{**}b-Cl(A) = A$.
- (iv) $N_{eut}N_{ano}rg^{**}b-Int[N_{eut}N_{ano}rg^{**}b-Int(A)] = N_{eut}N_{ano}rg^{**}b-Int(A)$.
- (v) $N_{eut}N_{ano}rg^{**}b-Cl[N_{eut}N_{ano}rg^{**}b-Cl(A)] = N_{eut}N_{ano}rg^{**}b-Cl(A)$.
- (vi) If $A \subseteq B$, then $N_{eut}N_{ano}rg^{**}b-Int(A) \subseteq N_{eut}N_{ano}rg^{**}b-Int(B)$
- (vii) If $A \subseteq B$, then $N_{eut}N_{ano}rg^{**}b-Cl(A) \subseteq N_{eut}N_{ano}rg^{**}b-Cl(B)$
- (viii) $(N_{eut}N_{ano}rg^{**}b-Cl(A))^C = N_{eut}N_{ano}rg^{**}b-Int(A^C)$
- (iX) $(N_{eut}N_{ano}rg^{**}b-Int(A))^C = N_{eut}N_{ano}rg^{**}b-Cl(A^C)$
- (X) $N_{eut}N_{ano}rg^{**}b-Int(0_{NN}) = 0_{NN}$, $N_{eut}N_{ano}rg^{**}b-Int(1_{NN}) = 1_{NN}$.
- (Xi) $N_{eut}N_{ano}rg^{**}b-Cl(0_{NN}) = 0_{NN}$, $N_{eut}N_{ano}rg^{**}b-Cl(1_{NN}) = 1_{NN}$
- (Xiii) $N_{eut}N_{ano}rg^{**}b-Int(A \cap B) \subseteq [N_{eut}N_{ano}rg^{**}b-Int(A)] \cap [N_{eut}N_{ano}rg^{**}b-Int(B)]$
- (Xiv) $N_{eut}N_{ano}rg^{**}b-Cl(A) \cup N_{eut}N_{ano}rg^{**}b-Cl(B) \subseteq N_{eut}N_{ano}rg^{**}b-Cl(A \cup B)$
- (Xv) $[N_{eut}N_{ano}rg^{**}b-Int(A)] \cup [N_{eut}N_{ano}rg^{**}b-Int(B)] \subseteq N_{eut}N_{ano}rg^{**}b-Int(A \cup B)$
- (Xvi) $N_{eut}N_{ano}rg^{**}b-Cl(A \cap B) \subseteq [N_{eut}N_{ano}rg^{**}b-Cl(A)] \cap [N_{eut}N_{ano}rg^{**}b-Cl(B)]$

Definition 2.23. A map $\xi: (U, \Upsilon_{NN}(S)) \rightarrow (W, \Gamma_{NN}(T))$ is called a $N_{eut}N_{ano}$ -continuous mapping if $\xi^{-1}(B)$ is a $N_{eut}N_{ano}$ -open (resp. $N_{eut}N_{ano}$ -closed) set in U , for every $N_{eut}N_{ano}$ -open (resp. $N_{eut}N_{ano}$ -closed) set B in W .

3. Neutrosophic Nano Regular Generalized Star Star b-Continuous Mappings

In this section, we introduce the concepts of neutrosophic nano regular generalized star star b-continuous ($N_{eut}N_{ano}rg^{**}b$ -continuous) mappings in $N_{eut}N_{ano}$ -Top-Spaces. Also, we study some of the main results depending on $N_{eut}N_{ano}rg^{**}b$ -open sets.

Definition 3.1. A map $\xi: (U, \Upsilon_{NN}(S)) \rightarrow (W, \Gamma_{NN}(T))$ is called a $N_{eut}N_{ano}rg^{**}b$ -continuous function if $\xi^{-1}(B)$ is a $N_{eut}N_{ano}rg^{**}b$ -open (resp. $N_{eut}N_{ano}rg^{**}b$ -closed) set in U , for every $N_{eut}N_{ano}$ -open (resp. $N_{eut}N_{ano}$ -closed) set B in W .

Theorem 3.2. Every $N_{eut}N_{ano}$ -continuous mapping is $N_{eut}N_{ano}rg^{**}b$ -continuous mapping.

Proof. Let $\xi: (U, \Upsilon_{NN}(S)) \rightarrow (W, \Gamma_{NN}(T))$ be a $N_{eut}N_{ano}$ -continuous mapping. Let H be a $N_{eut}N_{ano}$ -open set in $(W, \Gamma_{NN}(T))$. Then $\xi^{-1}(H)$ is $N_{eut}N_{ano}$ -open set in $(U, \Upsilon_{NN}(S))$. Since every

$N_{\text{cut}}N_{\text{ano}}$ -open set is $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -open, $\xi^{-1}(H)$ is $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -open set in $(U, \Upsilon_{\text{NN}}(S))$. Hence ξ is $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -continuous mapping.

Theorem 3.3. Let $(U, \Upsilon_{\text{NN}}(S))$, $(W, \Gamma_{\text{NN}}(T))$ and $(Z, \Psi_{\text{NN}}(T^*))$ be $N_{\text{cut}}N_{\text{ano}}$ -Top-Spaces. If $\xi: (U, \Upsilon_{\text{NN}}(S)) \rightarrow (W, \Gamma_{\text{NN}}(T))$ is a $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -continuous mapping and $\eta: (W, \Gamma_{\text{NN}}(T)) \rightarrow (Z, \Psi_{\text{NN}}(T^*))$ is $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -continuous, then $\eta \circ \xi: (U, \Upsilon_{\text{NN}}(S)) \rightarrow (Z, \Psi_{\text{NN}}(T^*))$ is a $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -continuous mapping.

Proof. Let G be a $N_{\text{cut}}N_{\text{ano}}$ -open set in Z . Since $\eta: (W, \Gamma_{\text{NN}}(T)) \rightarrow (Z, \Psi_{\text{NN}}(T^*))$ is $N_{\text{cut}}N_{\text{ano}}$ -continuous, $\eta^{-1}(G)$ is $N_{\text{cut}}N_{\text{ano}}$ -open in W . Since ξ is a $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -continuous mapping, $\xi^{-1}[\eta^{-1}(G)]$ is $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -open in U . But $\xi^{-1}[\eta^{-1}(G)] = (\eta \circ \xi)^{-1}(G)$. Then $(\eta \circ \xi)^{-1}(G)$ is $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -open set in U . Hence, $\eta \circ \xi$ is a $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -continuous mapping.

Theorem 3.4. Let $(U, \Upsilon_{\text{NN}}(S))$ and $(W, \Gamma_{\text{NN}}(T))$ be two $N_{\text{cut}}N_{\text{ano}}$ -Top-Spaces. Then prove that $\xi: (U, \Upsilon_{\text{NN}}(S)) \rightarrow (W, \Gamma_{\text{NN}}(T))$ is $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -continuous if and only if $\xi^{-1}(B)$ is $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -closed set in U for every $N_{\text{cut}}N_{\text{ano}}$ -closed set B in W .

Proof. Let B be a $N_{\text{cut}}N_{\text{ano}}$ -closed set in W . Then B^C is $N_{\text{cut}}N_{\text{ano}}$ -open set in W . Since ξ is $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -continuous. Therefore $\xi^{-1}(B^C)$ is a $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -open set in U . Since $\xi^{-1}(B^C) = [\xi^{-1}(B)]^C$, $\xi^{-1}(B)$ is $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -closed set in U .

Conversely, Let B be a $N_{\text{cut}}N_{\text{ano}}$ -open set in W . Then B^C is $N_{\text{cut}}N_{\text{ano}}$ -closed set in W . By assumption $\xi^{-1}(B^C)$ is $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -closed set in U . Since $\xi^{-1}(B^C) = [\xi^{-1}(B)]^C$, $\xi^{-1}(B)$ is $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -open set in U . Hence ξ is $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -continuous.

Theorem 3.5. Let $(U, \mathfrak{I}_{\text{NN}}(S))$ and $(W, \Omega_{\text{NN}}(T))$ be two $N_{\text{cut}}N_{\text{ano}}$ -Top-Spaces and $\xi: U \rightarrow W$ be a mapping. Then ξ is a $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -continuous mapping if and only if $\xi(N_{\text{cut}}N_{\text{ano}}rg^{**}b\text{-Cl}(P)) \subseteq N_{\text{trs}}N_{\text{n}}\text{-Cl}(\xi(P))$ for every $N_{\text{cut}}N_{\text{ano}}$ -set P in U .

Proof. Let P be a $N_{\text{cut}}N_{\text{ano}}$ -set in U and ξ be a $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -continuous mapping. We note that evidently $\xi(P) \subseteq N_{\text{cut}}N_{\text{ano}}rg^{**}b\text{-Cl}[\xi(P)]$. Then we obtain $P \subseteq \xi^{-1}[\xi(P)] \subseteq \xi^{-1}[N_{\text{cut}}N_{\text{ano}}rg^{**}b\text{-Cl}(\xi(P))]$ and $N_{\text{cut}}N_{\text{ano}}rg^{**}b\text{-Cl}(P) \subseteq N_{\text{cut}}N_{\text{ano}}rg^{**}b\text{-Cl}[\xi^{-1}(N_{\text{cut}}N_{\text{ano}}rg^{**}b\text{-Cl}(\xi(P)))]$. Since ξ is a $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -continuous mapping and $N_{\text{cut}}N_{\text{ano}}rg^{**}b\text{-Cl}[\xi(P)]$ is a $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -closed set. Thus we have $N_{\text{cut}}N_{\text{ano}}rg^{**}b\text{-Cl}[\xi^{-1}(N_{\text{cut}}N_{\text{ano}}rg^{**}b\text{-Cl}(\xi(P)))] = \xi^{-1}[N_{\text{cut}}N_{\text{ano}}rg^{**}b\text{-Cl}(\xi(P))]$. Hence, we have $\xi[N_{\text{cut}}N_{\text{ano}}rg^{**}b\text{-Cl}(P)] \subseteq N_{\text{cut}}N_{\text{ano}}\text{-Cl}[\xi(P)]$.

Conversely, let $\xi[N_{\text{cut}}N_{\text{ano}}rg^{**}b\text{-Cl}(P)] \subseteq N_{\text{cut}}N_{\text{ano}}\text{-Cl}[\xi(P)]$, for each $N_{\text{trs}}N_{\text{n}}$ -set P in U . Let Q be a $N_{\text{trs}}N_{\text{n}}$ -closed set in W . Then $N_{\text{cut}}N_{\text{ano}}rg^{**}b\text{-Cl}[\xi^{-1}(Q)] \subseteq N_{\text{cut}}N_{\text{ano}}rg^{**}b\text{-Cl}(Q) = Q$. By assumption, $\xi[N_{\text{cut}}N_{\text{ano}}rg^{**}b\text{-Cl}(\xi^{-1}(Q))] \subseteq N_{\text{cut}}N_{\text{ano}}\text{-Cl}[\xi(\xi^{-1}(Q))] \subseteq Q$ and so

$N_{\text{eut}}N_{\text{ano}}rg^{**}b - Cl[\xi^{-1}(Q)] \subseteq \xi^{-1}(Q)$. Since $\xi^{-1}(Q) \subseteq N_{\text{eut}}N_{\text{ano}}rg^{**}b - Cl[\xi^{-1}(Q)]$. Thus we have $N_{\text{eut}}N_{\text{ano}}rg^{**}b - Cl[\xi^{-1}(Q)] = \xi^{-1}(Q)$. This implies that $\xi^{-1}(Q)$ is a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -closed set in U . Therefore by Theorem 3.4, ξ is a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -continuous mapping.

Theorem 3.6. Let $\xi : (U, \mathfrak{T}_{NN}(S)) \rightarrow (W, \Omega_{NN}(S^*))$ be a mapping. Then ξ is a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -continuous mapping if and only if $N_{\text{eut}}N_{\text{ano}}rg^{**}b - Cl[\xi^{-1}(Q)] \subseteq \xi^{-1}[N_{\text{eut}}N_{\text{ano}} - Cl(Q)]$ for each $N_{\text{eut}}N_{\text{ano}}$ -set Q in W .

Proof. Suppose that $\xi : (U, \mathfrak{T}_{NN}(S)) \rightarrow (W, \Omega_{NN}(S^*))$ is a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -continuous mapping. Take Q be any $N_{\text{eut}}N_{\text{ano}}$ -set in W . Clearly $Q \subseteq N_{\text{eut}}N_{\text{ano}} - Cl(Q)$ implies $\xi^{-1}(Q) \subseteq \xi^{-1}[N_{\text{eut}}N_{\text{ano}}rg^{**}b - Cl(Q)]$. Then $N_{\text{eut}}N_{\text{ano}}rg^{**}b - Cl[\xi^{-1}(Q)] \subseteq N_{\text{eut}}N_{\text{ano}}rg^{**}b - Cl[\xi^{-1}(N_{\text{eut}}N_{\text{ano}} - Cl(Q))]$. Since $N_{\text{eut}}N_{\text{ano}} - Cl(Q)$ is $N_{\text{eut}}N_{\text{ano}}$ -closed set in W . So by Theorem 3.4, $\xi^{-1}[N_{\text{eut}}N_{\text{ano}} - Cl(Q)]$ is a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -closed set in U . Therefore, $N_{\text{eut}}N_{\text{ano}}rg^{**}b - Cl[\xi^{-1}(N_{\text{eut}}N_{\text{ano}} - Cl(Q))]$ is $\xi^{-1}[N_{\text{eut}}N_{\text{ano}} - Cl(Q)]$. Hence, we have $N_{\text{eut}}N_{\text{ano}}rg^{**}b - Cl[\xi^{-1}(Q)] \subseteq \xi^{-1}[N_{\text{eut}}N_{\text{ano}} - Cl(Q)]$.

Conversely, let $N_{\text{eut}}N_{\text{ano}}rg^{**}b - Cl[\xi^{-1}(Q)] \subseteq \xi^{-1}[N_{\text{eut}}N_{\text{ano}} - Cl(Q)]$ for all $N_{\text{eut}}N_{\text{ano}}$ -sets Q in W . Let F be a $N_{\text{eut}}N_{\text{ano}}$ -closed set in W . Since every $N_{\text{eut}}N_{\text{ano}}$ -closed set is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -closed set, $N_{\text{eut}}N_{\text{ano}}rg^{**}b - Cl[\xi^{-1}(F)] \subseteq \xi^{-1}[N_{\text{eut}}N_{\text{ano}} - Cl(F)] = \xi^{-1}(F)$. This implies that $\xi^{-1}(F)$ is a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -closed set in U . Thus by Theorem 3.4, ξ is a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -continuous mapping.

Theorem 3.7. Let $(U, \Upsilon_{NN}(S))$ and $(W, \Gamma_{NN}(T))$ be two $N_{\text{eut}}N_{\text{ano}}$ -Top-Spaces and $\xi : U \rightarrow W$ be a mapping.. Then ξ is a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -continuous mapping if and only if $\xi^{-1}[N_{\text{eut}}N_{\text{ano}} - Int(B)] \subseteq N_{\text{eut}}N_{\text{ano}}rg^{**}b - Int[\xi^{-1}(B)]$ for every $N_{\text{eut}}N_{\text{ano}}$ -set B in W .

Proof. Let B be any $N_{\text{eut}}N_{\text{ano}}$ -set in W and ξ be a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -continuous mapping. Then $N_{\text{eut}}N_{\text{ano}} - Int(B) \subseteq B$ implies $\xi^{-1}[N_{\text{eut}}N_{\text{ano}} - Int(B)] \subseteq \xi^{-1}(B)$. Therefore we obtain $N_{\text{eut}}N_{\text{ano}}rg^{**}b - Int[\xi^{-1}(N_{\text{eut}}N_{\text{ano}} - Int(B))]$ $\subseteq N_{\text{eut}}N_{\text{ano}}rg^{**}b - Int[\xi^{-1}(B)]$. Since $N_{\text{eut}}N_{\text{ano}} - Int(B)$ is $N_{\text{eut}}N_{\text{ano}}$ -open set in W and ξ is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -continuous, $\xi^{-1}[N_{\text{eut}}N_{\text{ano}} - Int(B)]$ is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -open set in U . So $N_{\text{eut}}N_{\text{ano}}rg^{**}b - Int[\xi^{-1}(N_{\text{eut}}N_{\text{ano}} - Int(B))]$ $= \xi^{-1}[N_{\text{eut}}N_{\text{ano}} - Int(B)] \subseteq N_{\text{eut}}N_{\text{ano}}rg^{**}b - Int[\xi^{-1}(B)]$.

Conversely, let $\xi^{-1}[N_{\text{eut}}N_{\text{ano}}rg^{**}b - Int(B)] \subseteq N_{\text{eut}}N_{\text{ano}}rg^{**}b - Int[\xi^{-1}(B)]$ for every $N_{\text{eut}}N_{\text{ano}}$ -set B in W . Let G be any $N_{\text{eut}}N_{\text{ano}}$ -open set in W and hence $N_{\text{eut}}N_{\text{ano}} - Int(G) = G$. By hypothesis, we have $\xi^{-1}[N_{\text{eut}}N_{\text{ano}} - Int(G)] \subseteq N_{\text{eut}}N_{\text{ano}}rg^{**}b - Int[\xi^{-1}(G)]$. Therefore, $\xi^{-1}(G) \subseteq N_{\text{eut}}N_{\text{ano}}rg^{**}b - Int[\xi^{-1}(G)]$. Also $N_{\text{eut}}N_{\text{ano}}rg^{**}b - Int[\xi^{-1}(G)] \subseteq [\xi^{-1}(G)]$. So $N_{\text{eut}}N_{\text{ano}}rg^{**}b - Int[\xi^{-1}(G)] = [\xi^{-1}(G)]$. This implies that $\xi^{-1}(G)$ is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -open set in U . Hence ξ is a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -continuous mapping.

Definition 3.8. Let $x_{(r,t,s)}$ be a $N_{\text{cut}}N_{\text{ano}}$ -point of a $N_{\text{cut}}N_{\text{ano}}$ -Top-Space $(U, \Upsilon_{\text{NN}}(S))$. A $N_{\text{cut}}N_{\text{ano}}$ -set A of U is called $N_{\text{cut}}N_{\text{ano}}$ -neighbourhood of $x_{(r,t,s)}$ if there exists a $N_{\text{cut}}N_{\text{ano}}$ -open set B such that $x_{(r,t,s)} \in B \subseteq A$.

Theorem 3.9. Let ξ be a mapping from a $N_{\text{cut}}N_{\text{ano}}$ -Top-Space $(U, \Upsilon_{\text{NN}}(S))$ to a $N_{\text{cut}}N_{\text{ano}}$ -Top-Space $(W, \Gamma_{\text{NN}}(T))$. Then the following assertions are equivalent.

- (i) ξ is $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -continuous.
- (ii) For each $N_{\text{cut}}N_{\text{ano}}$ -point $x_{(r,t,s)} \in U$ and every $N_{\text{cut}}N_{\text{ano}}$ -neighbourhood A of $\xi(x_{(r,t,s)})$, there exists a $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -open set B such that $x_{(r,t,s)} \in B \subseteq \xi^{-1}(A)$.
- (iii) For each $N_{\text{cut}}N_{\text{ano}}$ -point $x_{(r,t,s)} \in U$ and every $N_{\text{cut}}N_{\text{ano}}$ -neighbourhood A of $\xi(x_{(r,t,s)})$, there exists a $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -open set B in U such that $x_{(r,t,s)} \in B$ and $\xi(B) \subseteq A$.

Proof. (i) $\Rightarrow$ (ii): Let $x_{(r,t,s)} \in U$ be a $N_{\text{cut}}N_{\text{ano}}$ -point in U and let A be a $N_{\text{cut}}N_{\text{ano}}$ -neighbourhood of $N_{\text{cut}}N_{\text{ano}}$ -open set B in W such that $\xi(x_{(r,t,s)}) \in B \subseteq A$. Since ξ is $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -continuous, we know that $\xi^{-1}(B)$ is a $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -open set in U and $x_{(r,t,s)} \in \xi^{-1}(\xi(x_{(r,t,s)})) \subseteq \xi^{-1}(B) \subseteq \xi^{-1}(A)$. This implies (ii) is true.

(ii) $\Rightarrow$ (iii): Let $x_{(r,t,s)}$ be a $N_{\text{cut}}N_{\text{ano}}$ -point in U and let A be a $N_{\text{cut}}N_{\text{ano}}$ -neighbourhood of $\xi(x_{(r,t,s)})$. The condition (ii) implies that there exists a $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -open set B in U such that $x_{(r,t,s)} \in B \subseteq \xi^{-1}(A)$. Thus $x_{(r,t,s)} \in B$ and $\xi(B) \subseteq \xi[\xi^{-1}(A)] \subseteq A$. Hence (iii) is true.

(iii) $\Rightarrow$ (i): Let B be a $N_{\text{cut}}N_{\text{ano}}$ -open set in W and let $x_{(r,t,s)} \in \xi^{-1}(B)$. Since B is $N_{\text{cut}}N_{\text{ano}}$ -open set, $\xi(x_{(r,t,s)}) \in B$, and so B is $N_{\text{cut}}N_{\text{ano}}$ -neighbourhood of $\xi(x_{(r,t,s)})$. It follows from (iii) that there exists a $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -open set A in U such that $x_{(r,t,s)} \in A$ and $\xi(A) \subseteq B$ so that $x_{(r,t,s)} \in A \subseteq \xi^{-1}[\xi(A)] \subseteq \xi^{-1}(B)$. This implies by definition that $\xi^{-1}(B)$ is a $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -open set in U . Therefore, ξ is a $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -continuous mapping.

Definition 3.10. Let $x_{(r,t,s)}$ be a $N_{\text{cut}}N_{\text{ano}}$ -point of a $N_{\text{cut}}N_{\text{ano}}$ -Top-Space $(U, \Upsilon_{\text{NN}}(S))$. A $N_{\text{cut}}N_{\text{ano}}$ -set A of U is called $N_{\text{cut}}N_{\text{ano}}$ -neighbourhood of $x_{(r,t,s)}$ if there exists a $N_{\text{cut}}N_{\text{ano}}$ -open set B such that $x_{(r,t,s)} \in B \subseteq A$.

Theorem 3.11. Let ξ be a mapping from a $N_{\text{cut}}N_{\text{ano}}$ -Top-Space $(U, \Upsilon_{\text{NN}}(S))$ to a $N_{\text{cut}}N_{\text{ano}}$ -Top-Space $(W, \Gamma_{\text{NN}}(T))$. Then the following assertions are equivalent.

- (i) ξ is $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -continuous.
- (ii) For each $N_{\text{cut}}N_{\text{ano}}$ -point $x_{(r,t,s)} \in U$ and every $N_{\text{cut}}N_{\text{ano}}$ -neighbourhood A of $\xi(x_{(r,t,s)})$, there exists a $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -open set B such that $x_{(r,t,s)} \in B \subseteq \xi^{-1}(A)$.

(iii) For each $N_{\text{cut}}N_{\text{ano}}$ -point $x_{(r,t,s)} \in U$ and every $N_{\text{cut}}N_{\text{ano}}$ -neighbourhood A of $\xi(x_{(r,t,s)})$, there exists a $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -open set B in U such that $x_{(r,t,s)} \in B$ and $\xi(B) \subseteq A$.

Proof. (i) $\Rightarrow$ (ii): Let $x_{(r,t,s)} \in U$ be a $N_{\text{cut}}N_{\text{ano}}$ -point in U and let A be a $N_{\text{cut}}N_{\text{ano}}$ -neighbourhood of $\xi(x_{(r,t,s)})$. Then there exists a $N_{\text{cut}}N_{\text{ano}}$ -open set B in W such that $\xi(x_{(r,t,s)}) \in B \subseteq A$. Since ξ is $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -continuous, we know that $\xi^{-1}(B)$ is a $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -open set in U and $x_{(r,t,s)} \in \xi^{-1}(B) \subseteq \xi^{-1}(A)$. This implies (ii) is true.

(ii) $\Rightarrow$ (iii): Let $x_{(r,t,s)}$ be a $N_{\text{cut}}N_{\text{ano}}$ -point in U and let A be a $N_{\text{cut}}N_{\text{ano}}$ -neighbourhood of $\xi(x_{(r,t,s)})$. The condition (ii) implies that there exists a $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -open set B in U such that $x_{(r,t,s)} \in B \subseteq \xi^{-1}(A)$. Thus $x_{(r,t,s)} \in B$ and $\xi(B) \subseteq \xi[\xi^{-1}(A)] \subseteq A$. Hence (iii) is true.

(iii) $\Rightarrow$ (i): Let B be a $N_{\text{cut}}N_{\text{ano}}$ -open set in W and let $x_{(r,t,s)} \in \xi^{-1}(B)$. Since B is $N_{\text{cut}}N_{\text{ano}}$ -open set, $\xi(x_{(r,t,s)}) \in B$, and so B is $N_{\text{cut}}N_{\text{ano}}$ -neighbourhood of $\xi(x_{(r,t,s)})$. It follows from (iii) that there exists a $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -open set A in U such that $x_{(r,t,s)} \in A$ and $\xi(A) \subseteq B$ so that $x_{(r,t,s)} \in A \subseteq \xi^{-1}[\xi(A)] \subseteq \xi^{-1}(B)$. This implies by definition that $\xi^{-1}(B)$ is a $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -open set in U .

Therefore ξ is $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -continuous.

4. Neutrosophic Nan Regular Generalized Star Star B-Irresolute Maps

In this section, we introduce the concept of neutrosophic nano $\alpha\mathcal{G}\#\psi$ -irresolute ($N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -irresolute) mappings in $N_{\text{cut}}N_{\text{ano}}$ -Top-Spaces. Also, we discuss the relationship with $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -continuous mappings.

Definition 4.1. A map $\xi: (U, \Upsilon_{\text{NN}}(S)) \rightarrow (W, \Gamma_{\text{NN}}(T))$ is called a $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -irresolute function if $\xi^{-1}(B)$ is a $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -open set in U , for every $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -open set B in W .

Theorem 4.2. Let $(U, \Upsilon_{\text{NN}}(S))$ and $(W, \Gamma_{\text{NN}}(T))$ be two $N_{\text{cut}}N_{\text{ano}}$ -Top-Spaces. Prove that a mapping $\xi: U \rightarrow W$ is $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -irresolute if the inverse image of every $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -closed set in W is $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -closed set in U .

Proof. Let A be any $N_{\text{cut}}N_{\text{ano}}\alpha\mathcal{G}\#\psi$ -closed set in W . Then A^c is $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -open set in W . Since ξ is $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -irresolute, $\xi^{-1}(A^c)$ is $N_{\text{cut}}N_{\text{ano}}\alpha\mathcal{G}\#\psi$ -open set in U and $\xi^{-1}(A^c) = [\xi^{-1}(A)]^c$ which implies that $\xi^{-1}(A)$ is $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -closed set in U .

Conversely, Let B be any $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -open set in W . Then B^c is $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -closed set in W . Thus $\xi^{-1}(B^c)$ is $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -closed set in U and $\xi^{-1}(B^c) = [\xi^{-1}(B)]^c$ which implies that $\xi^{-1}(B)$ is $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -open set in U . Hence $\xi: U \rightarrow W$ is $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -irresolute.

Theorem 4.3. Every $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -irresolute mapping is $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -continuous.

Proof. Let $(U, \Upsilon_{\text{NN}}(S))$ and $(W, \Gamma_{\text{NN}}(T))$ be two $N_{\text{eut}}N_{\text{ano}}$ -Top-Spaces and $\xi: U \rightarrow W$ be a mapping. Let V be a $N_{\text{eut}}N_{\text{ano}}$ -open set in W . Since every $N_{\text{eut}}N_{\text{ano}}$ -open set is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -open, V is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -open. Since ξ is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -irresolute, $\xi^{-1}(V)$ is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -open in U . Therefore ξ is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -continuous.

Theorem 4.4. Let $(U, \Upsilon_{\text{NN}}(S))$ and $(W, \Gamma_{\text{NN}}(T))$ be two $N_{\text{eut}}N_{\text{ano}}$ -Top-Spaces and $\xi: U \rightarrow W$ be a mapping. Then ξ is a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -irresolute mapping if and only if $\xi(N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl(A)) \subseteq N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl(\xi(A))$ for every $N_{\text{eut}}N_{\text{ano}}$ -set A in U .

Proof. Let A be a $N_{\text{eut}}N_{\text{ano}}$ -set in U and ξ be a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -irresolute mapping. Then evidently $\xi(A) \subseteq N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl[\xi(A)]$. Then we have $A \subseteq \xi^{-1}[\xi(A)] \subseteq \xi^{-1}[N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl(\xi(A))]$ and hence $N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl(A) \subseteq N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl[\xi^{-1}(N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl(\xi(A)))]$. Since ξ is a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -irresolute mapping and $N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl[\xi(A)]$ is a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -closed set. Therefore $N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl[\xi^{-1}(N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl(\xi(A)))] = \xi^{-1}[N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl(\xi(A))]$. Thus $\xi[N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl(A)] \subseteq N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl[\xi(A)]$. Conversely, let $\xi[N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl(A)] \subseteq N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl[\xi(A)]$, for each $N_{\text{eut}}N_{\text{ano}}$ -set A in U . Let F be a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -closed set in W . Then $N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl(F) = F$. By assumption, $\xi[N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl(\xi^{-1}(F))] \subseteq N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl[\xi(\xi^{-1}(F))] \subseteq N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl(F) = F$ and then this implies that $N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl[\xi^{-1}(F)] \subseteq \xi^{-1}(F)$. Since $\xi^{-1}(F) \subseteq N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl[\xi^{-1}(F)]$. Thus we have $N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl[\xi^{-1}(F)] = \xi^{-1}(F)$. This shows that $\xi^{-1}(F)$ is a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -closed set in U . Thus, by Theorem 4.3, ξ is a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -irresolute mapping.

Theorem 4.5. Let $(U, \Upsilon_{\text{NN}}(S))$ and $(W, \Gamma_{\text{NN}}(T))$ be two $N_{\text{eut}}N_{\text{ano}}$ -Top-Spaces and $\xi: U \rightarrow W$ be a mapping. Then ξ is a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -irresolute mapping if and only if $N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl[\xi^{-1}(B)] \subseteq \xi^{-1}[N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl(B)]$ for each $N_{\text{eut}}N_{\text{ano}}$ -set B in W .

Proof. Let $\xi: U \rightarrow W$ be a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -irresolute mapping. Let B be any $N_{\text{eut}}N_{\text{ano}}$ -set set in W . Since $B \subseteq N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl(B)$ and therefore we get $\xi^{-1}(B) \subseteq \xi^{-1}[N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl(B)]$. Then we obtain $N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl[\xi^{-1}(B)] \subseteq N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl[\xi^{-1}(N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl(B))]$. Since $N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl(B)$ is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -closed set in W and ξ is a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -irresolute mapping. So by Theorem 4.3, $\xi^{-1}[N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl(B)]$ is a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -closed set in U . Therefore, $N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl[\xi^{-1}(N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl(B))]$ is $\xi^{-1}[N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl(B)]$. Hence, we have $N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl[\xi^{-1}(B)] \subseteq \xi^{-1}[N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl(B)]$. Conversely, let $N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl[\xi^{-1}(B)] \subseteq \xi^{-1}[N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl(B)]$ for all $N_{\text{eut}}N_{\text{ano}}$ -sets B in W . Let F be a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -closed set in W . By assumption, $N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl[\xi^{-1}(F)] \subseteq \xi^{-1}[N_{\text{eut}}N_{\text{ano}}rg^{**}b-Cl(F)] = \xi^{-1}(F)$. This implies that $\xi^{-1}(F)$ is a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -closed set in U . Thus by Theorem 4.3, ξ is a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -irresolute mapping.

Theorem 4.6. Let $(U, \Upsilon_{\text{NN}}(S))$ and $(W, \Gamma_{\text{NN}}(T))$ be two $N_{\text{eut}}N_{\text{ano}}$ -Top-Spaces and $\xi: U \rightarrow W$ be a bijective mapping. Then ξ is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -irresolute if and only if $N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int[\xi(A)] \subseteq \xi[N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int(A)]$ for every $N_{\text{eut}}N_{\text{ano}}$ -set A in U .

Proof. Let A be any $N_{\text{eut}}N_{\text{ano}}$ -set in U and ξ be a bijective and $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -irresolute mapping. Let $\xi(A) = B$. Since $N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int(B) \subseteq B$ and hence we obtain $\xi^{-1}[N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int(B)] \subseteq \xi^{-1}(B)$. Since ξ is an injective mapping, so $\xi^{-1}(B) = A$. Thus $\xi^{-1}[N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int(B)] \subseteq A$. Then this implies that $N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int[\xi^{-1}(N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int(B))]$ $\subseteq N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int(A)$. Since ξ is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -irresolute, $\xi^{-1}[N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int(B)]$ is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -open set in U and therefore $\xi^{-1}[N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int(B)] \subseteq N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int(A)$. Hence $\xi[\xi^{-1}(N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int(B))]$ $\subseteq \xi[N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int(A)]$. Thus $N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int[\xi(A)] \subseteq \xi[N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int(A)]$.
Conversely, let $N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int[\xi(A)] \subseteq \xi[N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int(A)]$ for every $N_{\text{eut}}N_{\text{ano}}$ -set A in U . Let V be a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -open set in W . Since ξ is surjective and therefore, by hypothesis, $V = N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int(V) = N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int[\xi(\xi^{-1}(V))] \subseteq \xi[N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int(\xi^{-1}(V))]$. It follows that $\xi^{-1}(V) \subseteq N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int[\xi^{-1}(V)]$. Therefore $\xi^{-1}(V)$ is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -open set in U . Hence ξ is a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -irresolute mapping.

Theorem 4.7. Let $(U, \Upsilon_{\text{NN}}(S))$ and $(W, \Gamma_{\text{NN}}(T))$ be two $N_{\text{eut}}N_{\text{ano}}$ -Top-Spaces and $\xi: U \rightarrow W$ be a mapping.. Then ξ is a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -irresolute mapping if and only if $\xi^{-1}[N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int(B)] \subseteq N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int[\xi^{-1}(B)]$ for every $N_{\text{eut}}N_{\text{ano}}$ -set B in W .

Proof. Let B be any $N_{\text{eut}}N_{\text{ano}}$ -set in W and ξ be a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -irresolute mapping. Then $N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int(B) \subseteq B$ implies $\xi^{-1}[N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int(B)] \subseteq \xi^{-1}(B)$. Hence, $N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int[\xi^{-1}(N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int(B))]$ $\subseteq N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int[\xi^{-1}(B)]$. Since $N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int(B)$ is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -open set in W and ξ is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -irresolute, $\xi^{-1}[N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int(B)]$ is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -open set in U . Thus $N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int[\xi^{-1}(N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int(B))]$ $= \xi^{-1}[N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int(B)]$. Then we obtain $\xi^{-1}[N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int(B)] \subseteq N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int[\xi^{-1}(B)]$.
Conversely, let $\xi^{-1}[N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int(B)] \subseteq N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int[\xi^{-1}(B)]$ for every $N_{\text{eut}}N_{\text{ano}}$ -set B in W . Let G be any $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -open set in W . Then by assumption, $\xi^{-1}(G) = \xi^{-1}[N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int(G)] \subseteq N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int[\xi^{-1}(G)]$ and therefore $\xi^{-1}(G) \subseteq N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int[\xi^{-1}(G)]$. This implies that $\xi^{-1}(G)$ is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -open set in U . Hence ξ is a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -irresolute mapping.

Theorem 4.8. Let $(U, \Upsilon_{\text{NN}}(S))$ and $(W, \Gamma_{\text{NN}}(T))$ be two $N_{\text{eut}}N_{\text{ano}}$ -Top-Spaces and $\xi: U \rightarrow W$ be a bijective mapping. Then ξ is a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -irresolute mapping. Then prove that $\xi[N_{\text{eut}}N_{\text{ano}}rg^{**}b-Fr(A)] \subseteq N_{\text{eut}}N_{\text{ano}}rg^{**}b-Fr[\xi(A)]$ for every $N_{\text{eut}}N_{\text{ano}}$ -set A in U .

Proof. Let ξ be a bijective and $N_{eut}N_{ano}rg^{**}b$ -irresolute mapping. Let A be a $N_{eut}N_{ano}$ -set in U . By definition, $N_{eut}N_{ano}rg^{**}b-Fr(A) = N_{eut}N_{ano}rg^{**}b-Cl(A) \cap N_{eut}N_{ano}rg^{**}b-Cl(A^c)$. By Theorem 4.6, $N_{eut}N_{ano}rg^{**}b-Int[\xi(A)] \subseteq \xi[N_{eut}N_{ano}rg^{**}b-Int(A)]$ and from Theorem 4.4, $\xi[N_{eut}N_{ano}rg^{**}b-Cl(A)] \subseteq N_{eut}N_{ano}rg^{**}b-Cl[\xi(A)]$. So $\xi[N_{eut}N_{ano}rg^{**}b-Fr(A)] = \xi[N_{eut}N_{ano}rg^{**}b-Cl(A)] \cap \xi[N_{eut}N_{ano}rg^{**}b-Cl(A^c)] \subseteq N_{eut}N_{ano}rg^{**}b-Cl[\xi(A)] \cap N_{eut}N_{ano}rg^{**}b-Cl[\xi(A)^c] = N_{eut}N_{ano}rg^{**}b-Fr[\xi(A)]$.

Theorem 4.9. Let $(U, \Upsilon_{NN}(S))$ and $(W, \Gamma_{NN}(T))$ be two $N_{eut}N_{ano}$ -Top-Spaces and $\xi: U \rightarrow W$ be a bijective $N_{eut}N_{ano}rg^{**}b$ -irresolute mapping. Then we have $N_{eut}N_{ano}rg^{**}b-Fr[\xi^{-1}(B)] \subseteq \xi^{-1}[N_{eut}N_{ano}rg^{**}b-Fr(B)]$ for every $N_{eut}N_{ano}$ -set B in W .

Proof. Let ξ be a bijective and $N_{eut}N_{ano}rg^{**}b$ -irresolute mapping. Let B be a $N_{eut}N_{ano}$ -set in W . By Theorem 4.5, we have $N_{eut}N_{ano}rg^{**}b-Cl[\xi^{-1}(B)] \subseteq \xi^{-1}[N_{eut}N_{ano}rg^{**}b-Cl(B)]$. Now $\xi^{-1}[N_{eut}N_{ano}rg^{**}b-Fr(B)] = \xi^{-1}[(N_{eut}N_{ano}rg^{**}b-Cl(B)) \cap N_{eut}N_{ano}rg^{**}b-Cl(B^c)] = \xi^{-1}[N_{eut}N_{ano}rg^{**}b-Cl(B)] \cap \xi^{-1}[N_{eut}N_{ano}rg^{**}b-Cl(B^c)] \supseteq N_{eut}N_{ano}rg^{**}b-Cl[\xi^{-1}(B)] \cap N_{eut}N_{ano}rg^{**}b-Cl[\xi^{-1}(B^c)] = N_{eut}N_{ano}rg^{**}b-Cl[\xi^{-1}(B)] \cap N_{eut}N_{ano}rg^{**}b-Cl[(\xi^{-1}(B))^c] = N_{eut}N_{ano}rg^{**}b-Fr[\xi^{-1}(B)]$. Therefore we obtain $N_{eut}N_{ano}rg^{**}b-Fr[\xi^{-1}(B)] \subseteq \xi^{-1}[N_{eut}N_{ano}rg^{**}b-Fr(B)]$.

Theorem 4.10. If $\xi: (U, \Upsilon_{NN}(S)) \rightarrow (W, \Gamma_{NN}(S^*))$ and $\eta: (W, \Gamma_{NN}(S^*)) \rightarrow (Z, \Psi_{NN}(S^{**}))$ are $N_{eut}N_{ano}rg^{**}b$ -irresolute, then their composition function $\eta \circ \xi: (U, \Upsilon_{NN}(S)) \rightarrow (Z, \Psi_{NN}(S^{**}))$ is also $N_{eut}N_{ano}rg^{**}b$ -irresolute.

Proof. Let V be a $N_{eut}N_{ano}rg^{**}b$ -open set in Z . Since η is a $N_{eut}N_{ano}rg^{**}b$ -irresolute mapping, $\eta^{-1}(V)$ is $N_{eut}N_{ano}rg^{**}b$ -open in W . Since ξ is a $N_{eut}N_{ano}rg^{**}b$ -irresolute mapping, $\xi^{-1}[\eta^{-1}(V)] = (\eta \circ \xi)^{-1}(V)$ is $N_{eut}N_{ano}rg^{**}b$ -open in U . Therefore $\eta \circ \xi$ is $N_{eut}N_{ano}rg^{**}b$ -irresolute.

Theorem 4.11. If $\xi: (U, \Upsilon_{NN}(S)) \rightarrow (W, \Gamma_{NN}(S^*))$ is $N_{eut}N_{ano}rg^{**}b$ -irresolute and $\eta: (W, \Gamma_{NN}(S^*)) \rightarrow (Z, \Psi_{NN}(S^{**}))$ is $N_{eut}N_{ano}rg^{**}b$ -continuous, then their composition function $\eta \circ \xi: (U, \Upsilon_{NN}(S)) \rightarrow (Z, \Psi_{NN}(S^{**}))$ is also a $N_{eut}N_{ano}rg^{**}b$ -continuous function.

Proof. Let V be a $N_{eut}N_{ano}$ -open set in Z . Since η is a $N_{eut}N_{ano}rg^{**}b$ -continuous mapping, $\eta^{-1}(V)$ is $N_{eut}N_{ano}rg^{**}b$ -open set in W . Since ξ is a $N_{eut}N_{ano}rg^{**}b$ -irresolute mapping, $\xi^{-1}[\eta^{-1}(V)] = (\eta \circ \xi)^{-1}(V)$ is $N_{eut}N_{ano}rg^{**}b$ -open in U . Therefore $\eta \circ \xi$ is $N_{eut}N_{ano}rg^{**}b$ -continuous.

5. Neutrosophic Nano Regular Generalized Star Star B-Closed Maps and Neutrosophic Nano Regular Generalized Star Star B-Open Maps

In this section, we introduce neutrosophic nano $rg^{**}b$ -closed ($N_{eut}N_{ano}rg^{**}b$ -closed) mappings and neutrosophic nano $rg^{**}b$ -open ($N_{eut}N_{ano}rg^{**}b$ -open) mappings in $N_{eut}N_{ano}$ -Top-Spaces and obtain certain characterizations of these classes of mappings.

Definition 5.1. Let $(U, \Upsilon_{NN}(S))$ and $(W, \Gamma_{NN}(S^*))$ be two $N_{eut}N_{ano}$ -Top-Spaces. A function $\xi: (U, \Upsilon_{NN}(S)) \rightarrow (W, \Gamma_{NN}(S^*))$ is said to be $N_{eut}N_{ano}rg^{**}b$ -closed if the image of each $N_{eut}N_{ano}$ -closed set in U is $N_{eut}N_{ano}rg^{**}b$ -closed in W .

Definition 5.2. Let $(U, \Upsilon_{NN}(S))$ and $(W, \Gamma_{NN}(S^*))$ be two $N_{eut}N_{ano}$ -Top-Spaces. A function $\xi: (U, \Upsilon_{NN}(S)) \rightarrow (W, \Gamma_{NN}(S^*))$ is said to be $N_{eut}N_{ano}rg^{**}b$ -open if the image of each $N_{eut}N_{ano}$ -open set in U is $N_{eut}N_{ano}rg^{**}b$ -open in W .

Theorem 5.3. Prove that a function $\xi: (U, \Upsilon_{NN}(S)) \rightarrow (W, \Gamma_{NN}(S^*))$ is $N_{eut}N_{ano}rg^{**}b$ -closed if and only if $N_{eut}N_{ano}rg^{**}b-Cl[\xi(A)] \subseteq \xi[N_{eut}N_{ano}-Cl(A)]$ for every $N_{eut}N_{ano}$ -set A of U .

Proof. Suppose $\xi: (U, \Upsilon_{NN}(S)) \rightarrow (W, \Gamma_{NN}(S^*))$ is a $N_{eut}N_{ano}rg^{**}b$ -closed function and A is any $N_{eut}N_{ano}$ -set in U . Then $N_{eut}N_{ano}-Cl(A)$ is a $N_{eut}N_{ano}$ -closed set in U . Since ξ is $N_{eut}N_{ano}rg^{**}b$ -closed, $\xi[N_{eut}N_{ano}-Cl(A)]$ is a $N_{eut}N_{ano}rg^{**}b$ -closed set in W . Thus $N_{eut}N_{ano}rg^{**}b-Cl[\xi(N_{eut}N_{ano}-Cl(A))] = \xi[N_{eut}N_{ano}-Cl(A)]$. Therefore we obtain $N_{eut}N_{ano}rg^{**}b-Cl[\xi(A)] \subseteq N_{eut}N_{ano}rg^{**}b-Cl[\xi(N_{eut}N_{ano}-Cl(A))] = \xi(N_{eut}N_{ano}-Cl(A))$. Hence $N_{eut}N_{ano}rg^{**}b-Cl[\xi(A)] \subseteq \xi(N_{eut}N_{ano}-Cl(A))$.

Conversely, let A be a $N_{eut}N_{ano}$ -closed set in U . Then $N_{eut}N_{ano}-Cl(A) = A$ and so $\xi(A) = \xi[N_{eut}N_{ano}-Cl(A)]$. By our assumption $N_{eut}N_{ano}rg^{**}b-Cl[\xi(A)] \subseteq \xi(A)$. But $\xi(A) \subseteq N_{eut}N_{ano}rg^{**}b-Cl[\xi(A)]$. Hence $N_{eut}N_{ano}rg^{**}b-Cl[\xi(A)] = \xi(A)$ and therefore $\xi(A)$ is $N_{eut}N_{ano}rg^{**}b$ -closed set in W . Thus ξ is a $N_{eut}N_{ano}rg^{**}b$ -closed mapping.

Theorem 5.4. A mapping $\xi: (U, \Upsilon_{NN}(S)) \rightarrow (W, \Gamma_{NN}(S^*))$ is $N_{eut}N_{ano}rg^{**}b$ -closed if and only if for each $N_{eut}N_{ano}$ -set H of W and for each $N_{eut}N_{ano}$ -open set G of U containing $\xi^{-1}(H)$ there exists a $N_{eut}N_{ano}rg^{**}b$ -open set V of W such that $H \subseteq V$ and $\xi^{-1}(V) \subseteq G$.

Proof. Suppose ξ is a $N_{eut}N_{ano}rg^{**}b$ -closed mapping. Let H be any $N_{eut}N_{ano}$ -set in W and G be a $N_{eut}N_{ano}$ -open set of U such that $\xi^{-1}(H) \subseteq G$. Then $V = [\xi(G^c)]^c$ is $N_{eut}N_{ano}rg^{**}b$ -open in W such that $\xi^{-1}(V) \subseteq G$.

Conversely, let H be a $N_{eut}N_{ano}$ -closed of W . Then $\xi^{-1}[(\xi(H))^c] \subseteq H^c$ and H^c is $N_{eut}N_{ano}$ -open in U . By assumption, there exists a $N_{eut}N_{ano}rg^{**}b$ -open set V of W such that $[\xi(H)]^c \subseteq V$ and $\xi^{-1}(V) \subseteq H^c$ and so $H \subseteq [\xi^{-1}(V)]^c$. Hence $V^c \subseteq \xi(H) \subseteq \xi([\xi^{-1}(V)]^c) \subseteq V^c$, which implies $\xi(H) = V^c$.

Since V^c is $N_{eut}N_{ano}rg^{**}b$ -closed, $\xi(H)$ is $N_{eut}N_{ano}rg^{**}b$ -closed and ξ is $N_{eut}N_{ano}rg^{**}b$ -closed mapping.

Theorem 5.5. Let $\xi : (U, \Upsilon_{\text{NN}}(S)) \rightarrow (W, \Gamma_{\text{NN}}(S^*))$ be a $N_{\text{eut}}N_{\text{ano}}$ -closed mapping and $\eta : (W, \Gamma_{\text{NN}}(S^*)) \rightarrow (Z, \Psi_{\text{NN}}(S^{**}))$ be a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -closed mapping. Then their composition $\eta \circ \xi : (U, \Upsilon_{\text{NN}}(S)) \rightarrow (Z, \Psi_{\text{NN}}(S^{**}))$ is a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -closed mapping.

Proof. Let F be a $N_{\text{eut}}N_{\text{ano}}$ -closed set in U . Since ξ is $N_{\text{eut}}N_{\text{ano}}$ -closed, $\xi(F)$ is $N_{\text{eut}}N_{\text{ano}}$ -closed in W . Since η is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -closed, $\eta[\xi(F)] = (\eta \circ \xi)(F)$ is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -closed in Z . Hence $\eta \circ \xi$ is a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -closed mapping.

Theorem 5.6. Let $\xi : (U, \Upsilon_{\text{NN}}(S)) \rightarrow (W, \Gamma_{\text{NN}}(S^*))$ and $\eta : (W, \Gamma_{\text{NN}}(S^*)) \rightarrow (Z, \Psi_{\text{NN}}(S^{**}))$ be two mappings such that their composition $\eta \circ \xi : (U, \Upsilon_{\text{NN}}(S)) \rightarrow (Z, \Psi_{\text{NN}}(S^{**}))$ is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -closed. Then the following statements are true.

- (i) If ξ is $N_{\text{eut}}N_{\text{ano}}$ -continuous and surjective, then η is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -closed.
- (ii) If η is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -irresolute and injective, then ξ is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -closed.

Proof. (i) Let A be a $N_{\text{eut}}N_{\text{ano}}$ -closed set of W . Since ξ is $N_{\text{eut}}N_{\text{ano}}$ -continuous, $\xi^{-1}(A)$ is $N_{\text{eut}}N_{\text{ano}}$ -closed in U . Since $\eta \circ \xi$ is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -closed, $(\eta \circ \xi)(\xi^{-1}(A))$ is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -closed in Z . Since ξ is surjective, $(\eta \circ \xi)(\xi^{-1}(A)) = \eta(A)$ is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -closed in Z . Hence η is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -closed.

(ii) Let B be any $N_{\text{eut}}N_{\text{ano}}$ -closed set of U . Since $\eta \circ \xi$ is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -closed, $(\eta \circ \xi)(B)$ is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -closed in Z . Since η is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -irresolute, $\eta^{-1}(\eta \circ \xi(B))$ is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -closed in W . Since η is injective, $\eta^{-1}(\eta \circ \xi(B)) = \eta^{-1}(\eta(\xi(B))) = \xi(B)$ is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -closed in W . Hence ξ is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -closed.

Theorem 5.7. A function $\xi : (U, \Upsilon_{\text{NN}}(S)) \rightarrow (W, \Gamma_{\text{NN}}(T))$ is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -open if and only if $\xi[N_{\text{eut}}Int(A)] \subseteq N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int[\xi(A)]$, for every $N_{\text{eut}}N_{\text{ano}}$ -set A of U .

Proof. Suppose $\xi : (U, \Upsilon_{\text{NN}}(S)) \rightarrow (W, \Gamma_{\text{NN}}(T))$ is a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -open function and A is any $N_{\text{eut}}N_{\text{ano}}$ -set in U . Then $N_{\text{eut}}N_{\text{ano}}-Int(A)$ is a $N_{\text{eut}}N_{\text{ano}}$ -open set in U . Since ξ is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -open, $\xi[N_{\text{eut}}N_{\text{ano}}-Int(A)]$ is a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -open set. Since $N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int[\xi(N_{\text{eut}}N_{\text{ano}}-Int(A))] \subseteq N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int[\xi(A)]$, $\xi[N_{\text{eut}}N_{\text{ano}}-Int(A)] \subseteq N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int[\xi(A)]$.

Conversely, $\xi[N_{\text{eut}}Int(A)] \subseteq N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int[\xi(A)]$ for every $N_{\text{eut}}N_{\text{ano}}$ -set A in U . Let U be a $N_{\text{eut}}N_{\text{ano}}$ -open set in U . Then $N_{\text{eut}}N_{\text{ano}}-Int(U) = U$ and by hypothesis, $\xi(U) \subseteq N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int[\xi(U)]$. But $N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int[\xi(U)] \subseteq \xi(U)$. Therefore, $\xi(U) = N_{\text{eut}}N_{\text{ano}}rg^{**}b-Int[\xi(U)]$. Then $\xi(U)$ is $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -open. Hence ξ is a $N_{\text{eut}}N_{\text{ano}}rg^{**}b$ -open mapping.

Theorem 5.8. Let $\xi: (U, \Upsilon_{\text{NN}}(S)) \rightarrow (W, \Gamma_{\text{NN}}(S^*))$ be a $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -open function. If for each $x_{(r,s,t)} \in U$ and for each $N_{\text{cut}}N_{\text{ano}}$ -neighbourhood G of $x_{(r,s,t)}$ in U , there exists a $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -neighborhood H of $\xi(x_{(r,s,t)})$ in W such that $H \subseteq \xi(G)$.

Proof. Let $\xi: (U, \Upsilon_{\text{NN}}(S)) \rightarrow (W, \Gamma_{\text{NN}}(S^*))$ be a $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -open function. Let $x_{(r,s,t)} \in U$ and G be any arbitrary $N_{\text{cut}}N_{\text{ano}}$ -neighbourhood of $x_{(r,s,t)}$ in U . Then there exists a $N_{\text{cut}}N_{\text{ano}}$ -open set P such that $x_{(r,s,t)} \in P \subseteq G$. By Theorem 5.7, $\xi(P) = \xi[N_{\text{cut}}N_{\text{ano}}\text{-Int}(P)] \subseteq N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}\text{-Int}[\xi(P)]$. But, $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}\text{-Int}[\xi(P)] \subseteq \xi(P)$. Therefore, $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}\text{-Int}[\xi(P)] = \xi(P)$ and hence $\xi(P)$ is $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -open in W . Since $x_{(r,s,t)} \in P \subseteq G$, $\xi(x_{(r,s,t)}) \in \xi(P) \subseteq \xi(G)$ and so the result follows by taking $H = \xi(P)$.

Theorem 5.9. For any bijective mapping $\xi: (U, \Upsilon_{\text{NN}}(S)) \rightarrow (W, \Gamma_{\text{NN}}(S^*))$ the following statements are equivalent:

- (i) $\xi^{-1}: (W, \Gamma_{\text{NN}}(S^*)) \rightarrow (U, \Upsilon_{\text{NN}}(S))$ is $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -continuous.
- (ii) ξ is $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -open.
- (iii) ξ is $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -closed.

Proof. (i) $\Rightarrow$ (ii): Let G be a $N_{\text{cut}}N_{\text{ano}}$ -set set in U . By assumption, $(\xi^{-1})^{-1}(G) = \xi(G)$ is $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -open in W and so ξ is $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -open.

(ii) $\Rightarrow$ (iii): Let F be a $N_{\text{cut}}N_{\text{ano}}$ -closed set of U . Then F^c is a $N_{\text{cut}}N_{\text{ano}}$ -open set in U . By assumption $\xi(F^c)$ is $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -open in W . But $\xi(F^c) = [\xi(F)]^c$. Therefore $\xi(F)$ is $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -closed set in W . Hence, ξ is $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -closed.

(iii) $\Rightarrow$ (i): Let F be a $N_{\text{cut}}N_{\text{ano}}$ -closed set of U . By assumption, $\xi(F)$ is $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -closed set in W . But $\xi(F) = (\xi^{-1})^{-1}(F)$ and therefore by Theorem 3.4, $\xi^{-1}: W \rightarrow U$ is $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -continuous.

6. Strongly Neutrosophic Nano Regular Generalized Star Star B-Continuous Maps and Perfectly Neutrosophic Nano Regular Generalized Star Star B-Continuous Maps

In this section, we introduce and study the concepts of strongly $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -continuous and perfectly $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -continuous mappings in $N_{\text{cut}}N_{\text{ano}}$ -Top-Spaces.

Definition 6.1. A function $\xi: (U, \Upsilon_{\text{NN}}(S)) \rightarrow (W, \Gamma_{\text{NN}}(S^*))$ is called strongly $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -continuous if the inverse image of every $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -open set in W is $N_{\text{cut}}N_{\text{ano}}$ -open in U .

Definition 6.2. A function $\xi: (U, \Upsilon_{\text{NN}}(S)) \rightarrow (W, \Gamma_{\text{NN}}(S^*))$ is called perfectly $N_{\text{cut}}N_{\text{ano}}$ -continuous if the inverse image of every $N_{\text{cut}}N_{\text{ano}}$ -open set in W is $N_{\text{cut}}N_{\text{ano}}$ -clopen in U .

Definition 6.3. A function $\xi: (U, \Upsilon_{\text{NN}}(S)) \rightarrow (W, \Gamma_{\text{NN}}(S^*))$ is called perfectly $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -continuous if the inverse image of every $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -open set in W is $N_{\text{cut}}N_{\text{ano}}$ -clopen in U .

Theorem 6.4. Let $\xi: (U, \Upsilon_{\text{NN}}(S)) \rightarrow (W, \Gamma_{\text{NN}}(S^*))$ be a mapping. Then the following statements are true:

- (i) If ξ is perfectly $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -continuous, then ξ is perfectly $N_{\text{cut}}N_{\text{ano}}$ -continuous.
- (ii) If ξ is strongly $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -continuous, then ξ is $N_{\text{cut}}N_{\text{ano}}$ -continuous.

Proof. (i) Let $\xi: (U, \Upsilon_{\text{NN}}(S)) \rightarrow (W, \Gamma_{\text{NN}}(S^*))$ be perfectly $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -continuous. Let V be a $N_{\text{cut}}N_{\text{ano}}$ -open set in W . Then V is $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -open set in W . Since ξ is perfectly $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -continuous, $\xi^{-1}(V)$ is $N_{\text{cut}}N_{\text{ano}}$ -clopen in U . Hence ξ is perfectly $N_{\text{cut}}N_{\text{ano}}$ -continuous.

(ii) Let $\xi: U \rightarrow W$ be strongly $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -continuous. Let G be $N_{\text{cut}}N_{\text{ano}}$ -open set in W . Then G is $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -open set in W . Since ξ is strongly $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -continuous, $\xi^{-1}(G)$ is $N_{\text{cut}}N_{\text{ano}}$ -open in U . Therefore ξ is $N_{\text{cut}}N_{\text{ano}}$ -continuous.

Theorem 6.5. Let $\xi: (U, \Upsilon_{\text{NN}}(S)) \rightarrow (W, \Gamma_{\text{NN}}(S^*))$ be strongly $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -continuous and A be a $N_{\text{cut}}N_{\text{ano}}$ -open set in W . Then the restriction map, $\xi_A: A \rightarrow W$ is strongly $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -continuous.

Proof. Let V be any $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -open set in W . Since ξ is strongly $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -continuous, $\xi^{-1}(V)$ is $N_{\text{cut}}N_{\text{ano}}$ -open in U . But $\xi_A^{-1}(V) = A \cap \xi^{-1}(V)$. Since A and $\xi^{-1}(V)$ are $N_{\text{cut}}N_{\text{ano}}$ -open, $\xi_A^{-1}(V)$ is $N_{\text{cut}}N_{\text{ano}}$ -open in A . Hence ξ_A is strongly $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -continuous.

Theorem 6.5. Every perfectly $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -continuous mapping $\xi: (U, \Upsilon_{\text{NN}}(S)) \rightarrow (W, \Gamma_{\text{NN}}(S^*))$ is strongly $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -continuous.

Proof. Let $\xi: U \rightarrow W$ be perfectly $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -continuous and V be $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -open set in W . Since ξ is perfectly $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -continuous, $\xi^{-1}(V)$ is $N_{\text{cut}}N_{\text{ano}}$ -clopen in U . Thus $\xi^{-1}(V)$ is $N_{\text{cut}}N_{\text{ano}}$ -open in U . Hence ξ is strongly $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -continuous.

Theorem 6.6. If $\xi: (U, \Upsilon_{\text{NN}}(S)) \rightarrow (W, \Gamma_{\text{NN}}(S^*))$ and $\eta: (W, \Gamma_{\text{NN}}(S^*)) \rightarrow (Z, \Psi_{\text{NN}}(S^{**}))$ are strongly $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -continuous, then $\eta \circ \xi: (U, \Upsilon_{\text{NN}}(S)) \rightarrow (Z, \Psi_{\text{NN}}(S^{**}))$ is also strongly $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -continuous.

Proof. Let V be a $N_{\text{cut}}N_{\text{ano}}$ -open set in Z . Since η is a strongly $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -continuous mapping, $\eta^{-1}(V)$ is $N_{\text{cut}}N_{\text{ano}}$ -open in W . Since ξ is a strongly $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -continuous mapping, $\xi^{-1}[\eta^{-1}(V)] = (\eta \circ \xi)^{-1}(V)$ is $N_{\text{cut}}N_{\text{ano}}$ -open in U . Therefore, $\eta \circ \xi$ is strongly $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -continuous.

Theorem 6.7. If $\xi: (U, \Upsilon_{\text{NN}}(S)) \rightarrow (W, \Gamma_{\text{NN}}(S^*))$ and $\eta: (W, \Gamma_{\text{NN}}(S^*)) \rightarrow (Z, \Psi_{\text{NN}}(S^{**}))$ are perfectly $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -continuous mappings, then their composition $\eta \circ \xi: (U, \Upsilon_{\text{NN}}(S)) \rightarrow (Z, \Psi_{\text{NN}}(S^{**}))$ is also perfectly $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -continuous mapping.

Proof. Let V be a $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -open set in Z . Since η is a perfectly $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -continuous mapping, $\eta^{-1}(V)$ is $N_{\text{cut}}N_{\text{ano}}$ -clopen in W . That is $\eta^{-1}(V)$ both $N_{\text{cut}}N_{\text{ano}}$ -open and $N_{\text{cut}}N_{\text{ano}}$ -closed in

U . Then $\eta^{-1}(V)$ is $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -open set in U . Since ξ is a perfectly $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -continuous mapping, $\xi^{-1}[\eta^{-1}(V)] = (gof)^{-1}(V)$ is $N_{\text{cut}}N_{\text{ano}}$ -clopen in U . Therefore $\eta o \xi$ is perfectly $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -continuous.

Theorem 6.8. Let $\xi: (U, \Upsilon_{\text{NN}}(S)) \rightarrow (W, \Gamma_{\text{NN}}(S^*))$ and $\eta: (W, \Gamma_{\text{NN}}(S^*)) \rightarrow (Z, \Psi_{\text{NN}}(S^{**}))$ be mappings. Then the following statements are true.

- (i) If η is strongly $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -continuous and ξ is $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -continuous, then $\eta o \xi$ is $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -irresolute.
- (ii) If η is perfectly $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -continuous and ξ is $N_{\text{cut}}N_{\text{ano}}$ -continuous, then $\eta o \xi$ is strongly $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -continuous.
- (iii) If η is strongly $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -continuous and ξ is perfectly $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -continuous, then $\eta o \xi$ is perfectly $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -continuous.
- (iv) If η is $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -continuous and ξ is strongly $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -continuous, then $\eta o \xi$ is $N_{\text{cut}}N_{\text{ano}}$ -continuous,

Proof. (i) Let V be a $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -open set in Z . Since η is a strongly $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -continuous mapping, $\eta^{-1}(V)$ is $N_{\text{cut}}N_{\text{ano}}$ -open set in W . Since ξ is a $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -continuous mapping, $\xi^{-1}[\eta^{-1}(V)] = (\eta o \xi)^{-1}(V)$ is $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -open set in U . Hence $\eta o \xi$ is $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -irresolute.

(ii) Let V be a $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -open set in Z . Since η is a perfectly $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -continuous mapping, $\eta^{-1}(V)$ is $N_{\text{cut}}N_{\text{ano}}$ -clopen set in W . That is, $\eta^{-1}(V)$ is both $N_{\text{cut}}N_{\text{ano}}$ -open and $N_{\text{cut}}N_{\text{ano}}$ -closed. Since ξ is a $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -continuous mapping, $\xi^{-1}[\eta^{-1}(V)] = (\eta o \xi)^{-1}(V)$ is $N_{\text{cut}}N_{\text{ano}}$ -open in U . Therefore $\eta o \xi$ is strongly $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -continuous.

(iii) Let V be a $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -open set in Z . Since η is a strongly $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -continuous mapping, $\eta^{-1}(V)$ is $N_{\text{cut}}N_{\text{ano}}$ -open set in W . Since every $N_{\text{cut}}N_{\text{ano}}$ -open set is $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -open set. So $\eta^{-1}(V)$ is $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -open set in U . Since ξ is a perfectly $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -continuous mapping, $\xi^{-1}[\eta^{-1}(V)] = (\eta o \xi)^{-1}(V)$ is $N_{\text{cut}}N_{\text{ano}}$ -clopen in U . Hence $\eta o \xi$ is perfectly $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -continuous.

(iv) Let V be a $N_{\text{cut}}N_{\text{ano}}$ -open set in Z . Since η is a $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -continuous mapping, $\eta^{-1}(V)$ is $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -open set in W . Since ξ is a strongly $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -continuous map, $\xi^{-1}[\eta^{-1}(V)] = (\eta o \xi)^{-1}(V)$ is $N_{\text{cut}}N_{\text{ano}}$ -open in U . So $\eta o \xi$ is $N_{\text{cut}}N_{\text{ano}}$ -continuous.

7. Neutrosophic Nano Contra Regular Generalized Star Star B-Continuous Maps and Neutrosophic Nano Contra Regular Generalized Star Star B-Irresolute Maps

In this section, we introduce the concepts of neutrosophic nano contra $N_{\text{cut}}N_{\text{ano}}rg^{**}b$ -continuous ($N_{\text{cut}}N_{\text{ano}}\text{Contra-}rg^{**}b$ -continuous) mappings and neutrosophic nano contra $rg^{**}b$ -irresolute ($N_{\text{cut}}N_{\text{ano}}\text{Contra-}rg^{**}b$ -irresolute) mappings and investigate their fundamental properties and characterizations.

Definition 7.1. A function $\xi: (U, \Upsilon_{\text{NN}}(S)) \rightarrow (W, \Gamma_{\text{NN}}(S^*))$ is said to be $N_{\text{cut}}N_{\text{ano}}\text{Contra}$ -continuous if the inverse image of every $N_{\text{cut}}N_{\text{ano}}$ -open set in W is $N_{\text{cut}}N_{\text{ano}}$ -closed set in U .

Definition 7.2. A function $\xi: (U, \Upsilon_{\text{NN}}(S)) \rightarrow (W, \Gamma_{\text{NN}}(S^*))$ is said to be $N_{\text{cut}}N_{\text{ano}}\text{Contra-rg**b}$ -continuous if the inverse image of every $N_{\text{cut}}N_{\text{ano}}$ -open set in W is $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -closed in U .

Theorem 7.3. Let $\xi: (U, \Upsilon_{\text{NN}}(S)) \rightarrow (W, \Gamma_{\text{NN}}(S^*))$ be a $N_{\text{cut}}N_{\text{ano}}\text{Contra}$ -continuous mapping. Then ξ is $N_{\text{cut}}N_{\text{ano}}\text{Contra-rg**b}$ -continuous.

Proof. Let V be any $N_{\text{cut}}N_{\text{ano}}$ -open set in W . Since ξ is $N_{\text{cut}}N_{\text{ano}}\text{Contra}$ -continuous, $\xi^{-1}(V)$ is $N_{\text{cut}}N_{\text{ano}}$ -closed set in U . As every $N_{\text{cut}}N_{\text{ano}}$ -closed set is $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -closed, we have $\xi^{-1}(V)$ is $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -closed set in U . Therefore ξ is $N_{\text{cut}}N_{\text{ano}}\text{Contra-rg**b}$ -continuous.

Theorem 7.4. Prove that a function $\xi: (U, \Upsilon_{\text{NN}}(S)) \rightarrow (W, \Gamma_{\text{NN}}(S^*))$ is $N_{\text{cut}}N_{\text{ano}}\text{Contra-rg**b}$ -continuous if and only if the inverse image of every $N_{\text{cut}}N_{\text{ano}}$ -closed set in W is $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -open set in U .

Proof. Let V be a $N_{\text{cut}}N_{\text{ano}}$ -closed set in W . Then V^C is $N_{\text{cut}}N_{\text{ano}}$ -open set in W . Since ξ is $N_{\text{cut}}N_{\text{ano}}\text{Contra-rg**b}$ -continuous, $\xi^{-1}(V^C)$ is $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -closed set in U . But $\xi^{-1}(V^C) = (\xi^{-1}(V))^C$ and so $\xi^{-1}(V)$ is $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -open set in U .

Conversely, assume that the inverse image of every $N_{\text{cut}}N_{\text{ano}}$ -closed set in W is $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -open in U . Let G be a $N_{\text{cut}}N_{\text{ano}}$ -open set in W . Then G^C is $N_{\text{cut}}N_{\text{ano}}$ -closed in W . By hypothesis $\xi^{-1}(G^C) = (\xi^{-1}(G))^C$ is $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -open in U , and so $\xi^{-1}(G)$ is $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -closed set in U . Thus ξ is $N_{\text{cut}}N_{\text{ano}}\text{Contra-rg**b}$ -continuous.

Theorem 7.5. If a map $\xi: (U, \Upsilon_{\text{NN}}(S)) \rightarrow (W, \Gamma_{\text{NN}}(S^*))$ is $N_{\text{cut}}N_{\text{ano}}\text{Contra-rg**b}$ -continuous and $\eta: (W, \Gamma_{\text{NN}}(S^*)) \rightarrow (Z, \Psi_{\text{NN}}(S^{**}))$ is $N_{\text{cut}}N_{\text{ano}}$ -continuous, then their composition $\eta \circ \xi: (U, \Upsilon_{\text{NN}}(S)) \rightarrow (Z, \Psi_{\text{NN}}(S^{**}))$ is $N_{\text{cut}}N_{\text{ano}}\text{Contra-rg**b}$ -continuous.

Proof. Let G be a $N_{\text{cut}}N_{\text{ano}}$ -open set in Z . Since η is $N_{\text{cut}}N_{\text{ano}}$ -continuous, $\eta^{-1}(G)$ is $N_{\text{cut}}N_{\text{ano}}$ -open set in W . Since ξ is $N_{\text{cut}}N_{\text{ano}}\text{Contra-rg**b}$ -continuous, $\xi^{-1}[\eta^{-1}(G)]$ is $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -closed set in U . But $(\eta \circ \xi)^{-1}(G) = \xi^{-1}[\eta^{-1}(G)]$. Thus $\eta \circ \xi$ is $N_{\text{cut}}N_{\text{ano}}\text{Contra-rg**b}$ -continuous.

Definition 7.6. A mapping $\xi: (U, \Upsilon_{\text{NN}}(S)) \rightarrow (W, \Gamma_{\text{NN}}(S^*))$ is called $N_{\text{cut}}N_{\text{ano}}\text{Contra-rg**b}$ -irresolute if the inverse image of every $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -open set in W is $N_{\text{cut}}N_{\text{ano}}\text{rg**b}$ -closed in U .

Theorem 7.7. If a mapping $\xi: (U, \Upsilon_{\text{NN}}(S)) \rightarrow (W, \Gamma_{\text{NN}}(S^*))$ is $N_{\text{cut}}N_{\text{ano}}\text{Contra-rg**b}$ -irresolute, then it is $N_{\text{cut}}N_{\text{ano}}\text{Contra-rg**b}$ -continuous.

Proof. Let V be a $N_{cut}N_{ano}$ -open set in W . Since every $N_{cut}N_{ano}$ -open set is $N_{cut}N_{ano}rg^{**}b$ -open, V is $N_{cut}N_{ano}rg^{**}b$ -open set in W . Since ξ is $N_{cut}N_{ano}Contra-rg^{**}b$ -irresolute, $\xi^{-1}(V)$ is $N_{cut}N_{ano}rg^{**}b$ -closed set in U . Thus ξ is $N_{cut}N_{ano}Contra-rg^{**}b$ -continuous.

Theorem 7.8. Let $(U, \Upsilon_{NN}(S))$, $(W, \Gamma_{NN}(S^*))$ and $(Z, \Psi_{NN}(S^{**}))$ be $N_{cut}N_{ano}$ -Top-Spaces. If a function $\xi: (U, \Upsilon_{NN}(S)) \rightarrow (W, \Gamma_{NN}(S^*))$ is $N_{cut}N_{ano}Contra-rg^{**}b$ -irresolute and a function $\eta: (W, \Gamma_{NN}(S^*)) \rightarrow (Z, \Psi_{NN}(S^{**}))$ is $N_{cut}N_{ano}rg^{**}b$ -continuous, then $\eta \circ \xi: (U, \Upsilon_{NN}(S)) \rightarrow (Z, \Psi_{NN}(S^{**}))$ is $N_{cut}N_{ano}Contra-rg^{**}b$ -continuous.

Proof. Let V be any $N_{cut}N_{ano}$ -open set in Z . Since η is $N_{cut}N_{ano}rg^{**}b$ -continuous, $\eta^{-1}(V)$ is $N_{cut}N_{ano}rg^{**}b$ -open set in W . Since ξ is $N_{cut}N_{ano}Contra-rg^{**}b$ -irresolute, $\xi^{-1}[\eta^{-1}(V)]$ is $N_{cut}N_{ano}rg^{**}b$ -closed set in U . But $(\eta \circ \xi)^{-1}(V) = \xi^{-1}[\eta^{-1}(V)]$. Thus $\eta \circ \xi$ is $N_{cut}N_{ano}Contra-rg^{**}b$ -continuous.

Theorem 7.9. If $\xi: (U, \Upsilon_{NN}(S)) \rightarrow (W, \Gamma_{NN}(S^*))$ is $N_{cut}N_{ano}rg^{**}b$ -irresolute and $\eta: (W, \Gamma_{NN}(S^*)) \rightarrow (Z, \Psi_{NN}(S^{**}))$ is $N_{cut}N_{ano}Contra-rg^{**}b$ -irresolute, then their composition $\eta \circ \xi: (U, \Upsilon_{NN}(S)) \rightarrow (Z, \Psi_{NN}(S^{**}))$ is $N_{cut}N_{ano}Contra-rg^{**}b$ -irresolute mapping.

Proof. Let H be any $N_{cut}N_{ano}rg^{**}b$ -open set in Z . Since η is $N_{cut}N_{ano}Contra-rg^{**}b$ -irresolute, $\eta^{-1}(H)$ is $N_{cut}N_{ano}rg^{**}b$ -closed set in W . Since ξ is $N_{cut}N_{ano}rg^{**}b$ -irresolute, $\xi^{-1}[\eta^{-1}(H)]$ is $N_{cut}N_{ano}rg^{**}b$ -closed set in U . But $(\eta \circ \xi)^{-1}(H) = \xi^{-1}[\eta^{-1}(H)]$. Thus $\eta \circ \xi$ is $N_{cut}N_{ano}Contra-rg^{**}b$ -irresolute.

8. Totally Neutrosophic Nano Regular Generalized Star Star B-Continuous Maps

In this section, we introduce totally $N_{cut}N_{ano}rg^{**}b$ -continuous continuous functions and discuss some of their interesting properties.

Definition 8.1. A function $\xi: (U, \Upsilon_{NN}(S)) \rightarrow (W, \Gamma_{NN}(S^*))$ is said to be totally $N_{cut}N_{ano}rg^{**}b$ -continuous if the inverse image of every $N_{cut}N_{ano}$ -closed set in $(W, \Gamma_{NN}(S^*))$ is both $N_{cut}N_{ano}rg^{**}b$ -closed set and $N_{cut}N_{ano}rg^{**}b$ -open set ($\exists, N_{cut}N_{ano}rg^{**}b$ -clopen set) in $(U, \Upsilon_{NN}(S))$.

Theorem 8.2. Every perfectly $N_{cut}N_{ano}rg^{**}b$ -continuous function is totally $N_{cut}N_{ano}rg^{**}b$ -continuous function.

Proof. Let $\xi: (U, \Upsilon_{NN}(S)) \rightarrow (W, \Gamma_{NN}(S^*))$ be a perfectly $N_{cut}N_{ano}rg^{**}b$ -continuous function. Let H be a $N_{cut}N_{ano}$ -closed set in $(W, \Gamma_{NN}(S^*))$. Then H is $N_{cut}N_{ano}rg^{**}b$ -closed in $(W, \Gamma_{NN}(S^*))$. Since ξ is a perfectly $N_{cut}N_{ano}rg^{**}b$ -continuous function, $\xi^{-1}(H)$ is both $N_{cut}N_{ano}$ -closed set and $N_{cut}N_{ano}$ -open set in $(U, \Upsilon_{NN}(S))$. Which implies $\xi^{-1}(H)$ is both $N_{cut}N_{ano}rg^{**}b$ -closed set and $N_{cut}N_{ano}rg^{**}b$ -open set in $(U, \Upsilon_{NN}(S))$. Hence, ξ is totally $N_{cut}N_{ano}rg^{**}b$ -continuous function.

Theorem 8.3. Prove that every totally $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**b}$ -continuous function is $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**b}$ -continuous function.

Proof. Let $\xi: (U, \Upsilon_{\text{NN}}(S)) \rightarrow (W, \Gamma_{\text{NN}}(S^*))$ be a totally $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**b}$ -continuous function. Let H be a $N_{\text{cut}}N_{\text{ano}}$ -closed set in $(W, \Gamma_{\text{NN}}(S^*))$. Since ξ is a totally $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**b}$ -continuous function, $\xi^{-1}(H)$ is both $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**b}$ -closed set and $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**b}$ -open set in $(U, \Upsilon_{\text{NN}}(S))$. Which implies $\xi^{-1}(H)$ is $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**b}$ -closed in $(U, \Upsilon_{\text{NN}}(S))$. Therefore, ξ is $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**b}$ -continuous function.

Definition 8.4. A $N_{\text{cut}}N_{\text{ano}}$ -Top-Space $(U, \Upsilon_{\text{NN}}(S))$ is called a $T_{\text{NN}}\text{rg}^{**b}$ -Space if every $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**b}$ -closed set is $N_{\text{cut}}N_{\text{ano}}$ -closed in $(U, \Upsilon_{\text{NN}}(S))$.

Theorem 8.5. Let $\xi: (U, \Upsilon_{\text{NN}}(S)) \rightarrow (W, \Gamma_{\text{NN}}(S^*))$ be totally $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**b}$ -continuous function and $(W, \Gamma_{\text{NN}}(S^*))$ be $T_{\text{NN}}\text{rg}^{**b}$ -Space. Then ξ is $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**b}$ -irresolute.

Proof. Let H be any $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**b}$ -closed set in $(W, \Gamma_{\text{NN}}(S^*))$. Since $(W, \Gamma_{\text{NN}}(S^*))$ is $T_{\text{NN}}\text{rg}^{**b}$ -Space, H is $N_{\text{cut}}N_{\text{ano}}$ -closed set in $(W, \Gamma_{\text{NN}}(S^*))$. Since ξ is totally $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**b}$ -continuous, $\xi^{-1}(H)$ is both $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**b}$ -closed set and $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**b}$ -open set in $(U, \Upsilon_{\text{NN}}(S))$. Hence $\xi^{-1}(H)$ is $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**b}$ -closed set in $(U, \Upsilon_{\text{NN}}(S))$. Therefore, ξ is $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**b}$ -irresolute.

Theorem 8.6. Let $\xi: (U, \Upsilon_{\text{NN}}(S)) \rightarrow (W, \Gamma_{\text{NN}}(S^*))$ be a $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**b}$ -irresolute function. Let $\eta: (W, \Gamma_{\text{NN}}(S^*)) \rightarrow (Z, \Psi_{\text{NN}}(S^{**}))$ be a totally $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**b}$ -continuous function. Then $(\eta \circ \xi): (U, \Upsilon_{\text{NN}}(S)) \rightarrow (Z, \Psi_{\text{NN}}(S^{**}))$ is totally $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**b}$ -continuous function.

Proof. Let H be a $N_{\text{cut}}N_{\text{ano}}$ -closed set in $(Z, \Psi_{\text{NN}}(S^{**}))$. Since η is totally $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**b}$ -continuous, $\eta^{-1}(H)$ is $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**b}$ -closed set and $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**b}$ -open set in $(W, \Gamma_{\text{NN}}(S^*))$. Since ξ is $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**b}$ -irresolute function, $\xi^{-1}(\eta^{-1}(H)) = (\eta \circ \xi)^{-1}(H)$ is $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**b}$ -closed set and $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**b}$ -open set in $(U, \Upsilon_{\text{NN}}(S))$. Hence $\eta \circ \xi$ is totally $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**b}$ -continuous function.

Theorem 8.7. Let $\xi: (U, \Upsilon_{\text{NN}}(S)) \rightarrow (W, \Gamma_{\text{NN}}(S^*))$ be a totally $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**b}$ -continuous function. Let $\eta: (W, \Gamma_{\text{NN}}(S^*)) \rightarrow (Z, \Psi_{\text{NN}}(S^{**}))$ be a $N_{\text{cut}}N_{\text{ano}}$ -continuous function. Then $(\eta \circ \xi): (U, \Upsilon_{\text{NN}}(S)) \rightarrow (Z, \Psi_{\text{NN}}(S^{**}))$ is totally $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**b}$ -continuous function.

Proof. Let H be a $N_{\text{cut}}N_{\text{ano}}$ -closed set in $(Z, \Psi_{\text{NN}}(S^{**}))$. By hypothesis, $\eta^{-1}(H)$ is a $N_{\text{cut}}N_{\text{ano}}$ -closed set in $(W, \Gamma_{\text{NN}}(S^*))$. Since ξ is totally $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**b}$ -continuous, $\xi^{-1}(\eta^{-1}(H)) = (\eta \circ \xi)^{-1}(H)$ is $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**b}$ -closed set and $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**b}$ -open set in $(U, \Upsilon_{\text{NN}}(S))$. Hence $\eta \circ \xi$ is totally $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**b}$ -continuous function.

Theorem 8.8. Let $\xi: (U, \Upsilon_{\text{NN}}(S)) \rightarrow (W, \Gamma_{\text{NN}}(S^*))$ and $\eta: (W, \Gamma_{\text{NN}}(S^*)) \rightarrow (Z, \Psi_{\text{NN}}(S^{**}))$ be totally $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -continuous functions and $(W, \Gamma_{\text{NN}}(S^*))$ be $T_{\text{NN}}\text{rg}^{**}\mathbf{b}$ -Space. Then $(\eta \circ \xi): (U, \Upsilon_{\text{NN}}(S)) \rightarrow (Z, \Psi_{\text{NN}}(S^{**}))$ is totally $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -continuous function.

Proof. Let H be any $N_{\text{cut}}N_{\text{ano}}$ -closed set in $(Z, \Psi_{\text{NN}}(S^{**}))$. Since η is totally $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -continuous, $\eta^{-1}(H)$ is both $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -closed set and $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -open set in $(W, \Gamma_{\text{NN}}(S^*))$. Since $(W, \Gamma_{\text{NN}}(S^*))$ is $T_{\text{NN}}\text{rg}^{**}\mathbf{b}$ -Space, $\eta^{-1}(H)$ is $N_{\text{cut}}N_{\text{ano}}$ -closed set in $(W, \Gamma_{\text{NN}}(S^*))$. Since ξ is totally $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -continuous, $\xi^{-1}(\eta^{-1}(H)) = (\eta \circ \xi)^{-1}(H)$ is both $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -closed set and $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -open set in $(U, \Upsilon_{\text{NN}}(S))$. Therefore, $\eta \circ \xi$ is totally $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -continuous function.

Conclusion

Neutrosophic set theory has emerged as a powerful tool for handling a wide range of complex problems in engineering, environmental science, economics, and several advanced areas of mathematics. In this research article, we have introduced and studied the properties of $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -continuous functions, $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -irresolute functions, $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -closed functions, $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -open functions, strongly $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -continuous functions, perfectly $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -continuous functions, $N_{\text{cut}}N_{\text{ano}}\text{Contra-rg}^{**}\mathbf{b}$ -continuous functions, $N_{\text{cut}}N_{\text{ano}}\text{Contra-rg}^{**}\mathbf{b}$ -irresolute functions and totally $N_{\text{cut}}N_{\text{ano}}\text{rg}^{**}\mathbf{b}$ -continuous functions in $N_{\text{cut}}N_{\text{ano}}$ -Top-Spaces and established the relations between them. We have obtained fundamental characterizations of these mappings and investigated preservation properties. We expect the results in this paper will be the basis for further applications of mappings in $N_{\text{cut}}N_{\text{ano}}$ -Top-Spaces. The results obtained in this study are expected to serve as a foundation for future investigations and applications of mappings in. Moreover, the findings presented here may contribute to the advancement of information systems and exert a positive influence on related disciplines such as engineering, physics, and computer science. Finally, we hope that this work will inspire further research in neutrosophic topology, paving the way toward the development of a robust and comprehensive framework for practical applications in these fields.

Scientific Ethics Declaration

* The authors declare that the scientific ethical and legal responsibility of this article published in EPSTEM journal belongs to the authors.

Conflict of Interest

* The authors declare that they have no conflicts of interest

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