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PNPS α -Open Sets and PNPS α -Continuous, Irresolute, Open, Closed and Homeomorphic Mappings in Pentapartitioned Neutrosophic Pythagorean Topological Spaces

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Abstract: Pentapartitioned Neutrosophic Pythagorean Topological Spaces (PNP-Top-Spaces) provide an effective mathematical framework for handling uncertainty, indeterminacy, inconsistency, contradiction, and ignorance in complex decision-making and information systems. Motivated by the development of generalized open sets in neutrosophic topology, this paper introduces and studies the concepts of Pentapartitioned Neutrosophic Pythagorean Semi- α -open (PNPS α -open) and PNPS α -closed sets. Fundamental definitions are established, and several basic properties and characterizations of these sets are obtained. The relationships between PNPS α -open sets and existing classes of PNP open, α -open, semi-open, and closed sets are investigated, and inclusion relations among these classes are discussed. Using these newly defined sets, the notions of PNPS α -continuous, PNPS α -irresolute, PNPS α -open, PNPS α -closed, and PNPS α -homeomorphic mappings are introduced. Various properties and characterizations of these mappings are derived by using PNPS α -open and PNPS α -closed sets. Several fundamental properties concerning preservation and composition of mappings are established. In addition, relationships among ordinary continuous mappings, PNPS α -continuous mappings, and PNPS α -irresolute mappings are examined. The behavior of PNPS α -open and PNPS α -closed mappings under composition and homeomorphic equivalence is also investigated.

Keywords: Pentapartitioned Neutrosophic Pythagorean Topological Space, PNPS α -open set, PNPS α -closed set, PNPS α -continuous mapping, PNPS α -irresolute mapping, PNPS α -open mapping, PNPS α -closed mapping, PNPS α -homeomorphism

Introduction

The theory of neutrosophic sets, introduced by Smarandache, has become an important framework for modeling uncertainty, indeterminacy, and inconsistency encountered in real-world problems. By incorporating the independent components of truth-membership, indeterminacy-membership, and falsity-membership, neutrosophic sets provide a more comprehensive representation of uncertain information than classical fuzzy and intuitionistic fuzzy models. Consequently, neutrosophic topology has emerged as an active area of research, leading to the introduction of numerous generalized topological concepts and mappings that extend classical topological theory.

Among the recent developments, Pythagorean fuzzy sets have attracted considerable attention due to their greater flexibility in handling uncertainty by allowing the squared sum of membership and non-membership degrees to be bounded by one. The integration of neutrosophic and Pythagorean concepts has given rise to Pentapartitioned Neutrosophic Pythagorean Sets (PNP-sets), which provide a richer mathematical structure for representing complex information. These sets partition uncertainty into five independent components, enabling a more refined description of positive, neutral, contradictory, incomplete, and negative information. Such an extension has opened new directions for the study of topological structures under highly generalized uncertainty environments.

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The investigation of generalized open sets occupies a central position in topology because they provide deeper insights into continuity, separation axioms, compactness, connectedness, and decomposition of mappings. Since the introduction of α -open sets by Njåstad, many generalized open sets, including semi-open, pre-open, β -open, regular open, generalized α -open, and their corresponding closed sets, have been studied extensively in classical, fuzzy, intuitionistic fuzzy, and neutrosophic topological spaces. These generalized classes have significantly enriched the theory of continuous and irresolute mappings and have found applications in decision analysis, information systems, artificial intelligence, and uncertainty modelling.

Motivated by these developments, this paper introduces and investigates a new class of generalized open sets, called Pentapartitioned Neutrosophic Pythagorean α -open sets (PNPS α -open sets), in Pentapartitioned Neutrosophic Pythagorean Topological Spaces (PNPS-Top-Spaces). The proposed class extends several existing notions of generalized openness and provides a broader framework for studying topological properties in pentapartitioned neutrosophic environments. Fundamental properties of PNPS α -open and PNPS α -closed sets are established, together with their relationships with other generalized open and closed sets already available in the literature.

Furthermore, based on the newly introduced PNPS α -open sets, several important classes of mappings are defined and studied, namely PNPS α -continuous mappings, PNPS α -irresolute mappings, PNPS α -open mappings, PNPS α -closed mappings, and PNPS α -homeomorphisms. Characterizations of these mappings are obtained through inverse images and direct images of PNPS α -open and PNPS α -closed sets. Various preservation results, equivalent conditions, composition properties, and interrelationships among these mappings are also established, thereby extending several well-known results from classical topology and neutrosophic topology to the framework of Pentapartitioned Neutrosophic Pythagorean Topological Spaces.

The results presented in this paper contribute to the theoretical development of generalized neutrosophic topology by providing a unified framework for studying generalized openness and continuity in PNPSTs. It is expected that the concepts introduced here will serve as a foundation for further investigations into separation axioms, compactness, connectedness, quotient spaces, decomposition of mappings, and applications of Pentapartitioned Neutrosophic Pythagorean structures in intelligent decision-making, information sciences, data analysis, and computational uncertainty.

2. Preliminaries

Definition 2.1. Let X be a universe. A Pentapartitioned Neutrosophic Pythagorean (briefly PNP-set) set M with T , F , C , and U as dependent neutrosophic components and I as an independent component for M on X is an object of the form $M = \{ \langle x, T_M, C_M, I_M, U_M, F_M \rangle : x \in X \}$ where $T_M + F_M \leq 1$, $C_M + U_M \leq 1$ and $(T_M)^2 + (C_M)^2 + (I_M)^2 + (U_M)^2 + (F_M)^2 \leq 3$. We note that $T_M(x)$ is the truth membership, $C_M(x)$ is contradiction membership, $U_M(x)$ is ignorance membership, $F_M(x)$ is the false membership, and $I_M(x)$ is an unknown membership.

Definition 2.2. Let X be a non-empty set. A PNP-set M over X characterizes each element x in X : a truth membership function T_M , a contradiction membership function C_M , an ignorance membership function G_M , unknown membership function U_M and a false membership function F_M , such that for each x in X . $T_M + C_M + G_M + U_M + F_M \leq 5$.

Definition 2.3. The complement of a PNP-set M on X denoted by M^c and is defined as $M^c = \{ \langle x, F_M(x), U_M(x), 1 - G_M(x), C_M(x), T_M(x) \rangle : x \in X \}$.

Definition 2.4. Let $M = \{ \langle x, T_M(x), C_M(x), G_M(x), U_M(x), F_M(x) \rangle : x \in X \}$ and $Q = \{ \langle x, T_Q(x), C_Q(x), G_Q(x), U_Q(x), F_Q(x) \rangle : x \in X \}$ are PNP-sets Then

$$M \cup Q = \left\{ \left\langle x, \max(T_M(x), T_Q(x)), \max(C_M(x), C_Q(x)), \min(G_M(x), G_Q(x)), \min(U_M(x), U_Q(x)), \min(F_M(x), F_Q(x)) \right\rangle : x \in X \right\}$$

$$M \cap Q = \left\{ \left\langle x, \min(T_M(x), T_Q(x)), \min(C_M(x), C_Q(x)), \max(G_M(x), G_Q(x)), \max(U_M(x), U_Q(x)), \max(F_M(x), F_Q(x)) \right\rangle : x \in X \right\}$$

Definition 2.5. A pentapartitioned neutrosophic Pythagorean topology on a nonempty set X is a family T_{PNP} of PNP-sets in X satisfying the following axioms

- (1) $0, 1 \in T_{\text{PNP}}$
- (2) $M_1 \cap M_2 \in T_{\text{PNP}}$ for any $M_1, M_2 \in T_{\text{PNP}}$
- (3) $\bigcup_{i \in I} M_i \in T_{\text{PNP}}$ for any $\{M_i : i \in I\} \subseteq T_{\text{PNP}}$

The pair (X, T_{PNP}) is called a PNP-Top-Space.

Definition 2.6. Let (X, T_{PNP}) be a PNP-Top-Space and $M \in T_{\text{PNP}}$. Then M is called a PNP-open set in X . The complement of a PNP-open set M is called a PNP-closed set in X and it is denoted by M^c .

Definition 2.7. Let (X, T_{PNP}) be a PNP-Top-Space and

$M = \left\{ \left\langle x, T_M(x), C_M(x), G_M(x), U_M(x), F_M(x) \right\rangle : x \in X \right\}$ be a PNP-set in X . Then the PNP-interior and the PNP-closure of M are denoted by $\text{PNP-Int}(M)$ and $\text{PNP-CI}(M)$ respectively and are defined as follows.

$\text{PNP-Int}(M) = \bigcap \{Q : Q \text{ is a PNP-open set and } M \subseteq Q\}$ and

$\text{PNP-CI}(M) = \bigcup \{K : K \text{ is a PNP-closed set and } K \subseteq M\}$

Theorem 2.8. Let (X, T_{PNP}) be a PNP-Top-Space and let M and Q be PNP-set subset of X . Then the following statements hold.

- (i) $\text{PNP-Int}(M)$ is a PNP-open set in X .
- (ii) $\text{PNP-Int}(\phi) = \phi$ and $\text{PNP-Int}(X) = X$.
- (iii) $\text{PNP-CI}(M)$ is a PNP-closed set in X .
- (iv) $\text{PNP-CI}(\phi) = \phi$ and $\text{PNP-CI}(X) = X$.
- (v) M is PNP-open set if and only if $M = \text{PNP-Int}(M)$
- (vi) M is PNP-closed set if and only if $M = \text{PNP-CI}(M)$
- (vii) $\text{PNP-Int}[\text{PNP-Int}(P)] = \text{PNP-Int}(M)$
- (viii) $\text{PNP-CI}[\text{PNP-CI}(P)] = \text{PNP-CI}(M)$
- (ix) $M \subseteq Q$ implies that $\text{PNP-Int}(M) \subseteq \text{PNP-Int}(Q)$
- (x) $M \subseteq Q$ implies that $\text{PNP-CI}(M) \subseteq \text{PNP-CI}(Q)$
- (xi) $\text{PNP-Int}(M \cap Q) = [\text{PNP-Int}(M)] \cap [\text{PNP-Int}(Q)]$
- (xii) $\text{PNP-CI}(M \cap Q) \subseteq [\text{PNP-CI}(M)] \cap [\text{PNP-CI}(Q)]$
- (xiii) $[\text{PNP-Int}(M)] \cup [\text{PNP-Int}(Q)] \subseteq \text{PNP-Int}(M \cup Q)$

Definition 2.9. Let (X, T_{PNP}) be a PNP-Top-Space and M be a PNP-set. M is said to be a Pentapartitioned Neutrosophic Pythagorean Pre-open (briefly PNPP-open) set if $M \subseteq \text{PNP-Int}(\text{PNP-CI}(M))$.

The family of all PNPP-open sets in (X, T_{PNP}) is denoted by $\text{PNPP-O}(X, T_{\text{PNP}})$. The complement of a PNPP-open set is called a $\text{Pentapartitioned Neutrosophic Pythagorean Pre-closed}$ (briefly PNPP-closed) set in X .

Definition 2.10. Let (X, T_{PNP}) be a PNP-Top-Space and M be a PNP-set . M is said to be a $\text{Pentapartitioned Neutrosophic Pythagorean Semi-open}$ (briefly PNPS-open) set if $M \subseteq \text{PNPP-Cl}(\text{PNP-Int}(M))$. The family of all PNPS-open sets in (X, T_{PNP}) is denoted by $\text{PNPS-O}(X, T_{\text{PNP}})$. The complement of a PNPS-open is called a $\text{Pentapartitioned Neutrosophic Pythagorean Semi-closed}$ (briefly PNPS-closed) set in X .

Definition 2.11. Let (X, T_{PNP}) be a PNP-Top-Space and M be a PNP-set . M is said to be a $\text{Pentapartitioned Neutrosophic Pythagorean } \alpha\text{-open}$ (briefly $\text{PNP}\alpha\text{-open}$) set if $M \subseteq \text{PNP-Int}(\text{PNPP-Cl}(\text{PNP-Int}(M)))$. The family of all $\text{PNP}\alpha\text{-open}$ sets in (X, T_{PNP}) is denoted by $\text{PNP}\alpha\text{-O}(X, T_{\text{PNP}})$. The complement of a $\text{PNP}\alpha\text{-open}$ set is called a $\text{Pentapartitioned Neutrosophic Pythagorean } \alpha\text{-closed}$ (briefly $\text{PNP}\alpha\text{-closed}$) set in X .

Proposition 2.12. A PNP-open subset M of a PNP-Top-Space (X, T_{PNP}) is a $\text{PNP}\alpha\text{-open}$ set in X if and only if M is a PNPS-open set and PNPP-open set.

Definition 2.13. Let (X, T_{PNP}) be a PNP-Top-Space and $M = \{ \langle x, T_M(x), C_M(x), G_M(x), U_M(x), F_M(x) \rangle : x \in X \}$ be a PNP-set in X . Then the $\text{PNP}\alpha\text{-interior}$ and the $\text{PNP}\alpha\text{-closure}$ of M are denoted by $\text{PNP}\alpha\text{-Int}(M)$ and $\text{PNP}\alpha\text{-Cl}(M)$, and are defined as follows.

$$\begin{aligned} \text{PNP}\alpha\text{-Int}(M) &= \bigcup \{ Q : Q \text{ is a } \text{PNP}\alpha\text{-open set and } Q \subseteq M \} \text{ and} \\ \text{PNP}\alpha\text{-Cl}(M) &= \bigcap \{ K : K \text{ is a } \text{PNP}\alpha\text{-closed set and } M \subseteq K \} \end{aligned}$$

Theorem 2.14. Let (X, T_{PNP}) be a PNP-Top-Space and M be a PNP-set in X . Then M is a $\text{PNP}\alpha\text{-open}$ set in X if and only if there exists a PNP-open set Q in X such that $Q \subseteq M \subseteq \text{PNP-Int}(\text{PNP-Cl}(Q))$.

Proof. Let M be a $\text{PNP}\alpha\text{-open}$ set in X . Then $M \subseteq \text{PNP-Int}[\text{PNP-Cl}(\text{PNP-Int}(M))]$. Then this implies that $\text{PNP-Int}(M) \subseteq M \subseteq \text{PNP-Int}[\text{PNP-Cl}(\text{PNP-Int}(M))]$. Let $Q = \text{PNP-Int}(M)$. Then there exists a PNP-open set Q such that $Q \subseteq M \subseteq \text{PNP-Int}[\text{PNP-Cl}(Q)]$.

Conversely, suppose that there is a PNP-open set Q such that $Q \subseteq M \subseteq \text{PNP-Int}[\text{PNP-Cl}(Q)]$. Then $Q = \text{PNP-Int}(Q) \subseteq \text{PNP-Int}(M)$. Hence $\text{PNP-Cl}(Q) \subseteq \text{PNP-Cl}(\text{PNP-Int}(M))$, then we obtain $\text{PNP-Int}(\text{PNP-Cl}(Q)) \subseteq \text{PNP-Int}[\text{PNP-Cl}(\text{PNP-Int}(M))]$. But by hypothesis, we have $Q \subseteq M \subseteq \text{PNP-Int}[\text{PNP-Cl}(Q)]$. Then $M \subseteq \text{PNP-Int}[\text{PNP-Cl}(\text{PNP-Int}(M))]$. Therefore, M is a $\text{PNP}\alpha\text{-open}$ set in X .

Theorem 2.15. The union of any family of $\text{PNP}\alpha\text{-open}$ sets in a PNP-Top-Space (X, T_{PNP}) is a $\text{PNP}\alpha\text{-open}$ set in X .

Proof. Let $\{M_i : i \in I\}$ be a family of $\text{PNP}\alpha$ -open sets in X . Then $M_i \subseteq \text{PNP-Int}(\text{PNP-Cl}(\text{PNP-Int}(M_i)))$, for each $i \in I$. Since $\bigcup_{i \in I} \text{PNP-Int}(M_i) \subseteq \text{PNP-Int}(\bigcup_{i \in I} M_i)$ and $\bigcup_{i \in I} \text{PNP-Cl}(M_i) \subseteq \text{PNP-Cl}(\bigcup_{i \in I} M_i)$. We have $\bigcup_{i \in I} M_i \subseteq \bigcup_{i \in I} \text{PNP-Int}(\text{PNP-Cl}(\text{PNP-Int}(M_i))) \subseteq \text{PNP-Int}(\bigcup_{i \in I} \text{PNP-Cl}(\text{PNP-Int}(M_i)))$
 $\subseteq \text{PNP-Int}(\text{PNP-Cl}(\bigcup_{i \in I} \text{PNP-Int}(M_i))) \subseteq \text{PNP-Int}(\text{PNP-Cl}(\text{PNP-Int}(\bigcup_{i \in I} M_i)))$. Hence, we conclude that $\bigcup_{i \in I} M_i$ is a $\text{PNP}\alpha$ -open set in X .

Theorem 2.16. Let (X, T_{PNP}) be a PNP-Top-Space . Then the following statements hold.

- (i) Every PNP-open (resp. PNP-closed) set is a $\text{PNP}\alpha$ -open (resp. $\text{PNP}\alpha$ -closed) set.
- (ii) Every $\text{PNP}\alpha$ -open (resp. $\text{PNP}\alpha$ -closed) set is a PNPS-open (resp. PNPS-closed) set.
- (iii) Every $\text{PNP}\alpha$ -open (resp. $\text{PNP}\alpha$ -closed) set is a PNPP-open (resp. PNPP-closed) set.

Lemma 2.17. Let (X, T_{PNP}) be a PNP-Top-Space and let M be a PNP-set in X .

- (i) If M is a PNP-open set, then $\text{PNPS-Cl}(M) = \text{PNP-Int}(\text{PNP-Cl}(M))$.
- (ii) $\text{PNPS-Int}(\text{PNP-Cl}(M)) = \text{PNP-Cl}(\text{PNP-Int}(\text{PNP-Cl}(M)))$.

3. PNPS α -Open Sets in Pentapartitioned Neutrosophic Pythagorean Topological Spaces

Definition 3.1. Let $(U, \mathfrak{T}_{\text{PNP}})$ be a PNP-Top-Space and M be a PNP-set . M is said to be a Pentapartitioned Neutrosophic Pythagorean semi α -open (briefly $\text{PNPS}\alpha$ -open) set if there exists a $\text{PNP}\alpha$ -open set H in U such that $H \subseteq M \subseteq \text{PNP-Cl}(H)$ or equivalently if $M \subseteq \text{PNP-Cl}(\text{PNP}\alpha\text{-Int}(M))$. The family of all $\text{PNPS}\alpha$ -open sets in $(U, \mathfrak{T}_{\text{PNP}})$ is denoted by $\text{PNPS}\alpha\text{-O}(U, \mathfrak{T}_{\text{PNP}})$. The complement of a $\text{PNPS}\alpha$ -open set is called a Pentapartitioned Neutrosophic Pythagorean semi α -closed (briefly $\text{PNPS}\alpha$ -closed) set in U .

Theorem 3.2. Let $(U, \mathfrak{T}_{\text{PNP}})$ be a PNP-Top-Space .

- (i) Every PNP-open (resp. PNP-closed) set is a $\text{PNPS}\alpha$ -open (resp. $\text{PNPS}\alpha$ -closed) set.
- (ii) Every $\text{PNP}\alpha$ -open (resp. $\text{PNP}\alpha$ -closed) set is a $\text{PNPS}\alpha$ -open (resp. $\text{PNPS}\alpha$ -closed) set.

Theorem 3.3. Let $(U, \mathfrak{T}_{\text{PNP}})$ be a PNP-Top-Space and M be a PNP-set . The following properties are equivalent:

- (i) $M \in \text{PNPS}\alpha\text{-O}(U, \mathfrak{T}_{\text{PNP}})$.
- (ii) There exists a PNP-open set H such that $H \subseteq M \subseteq \text{PNP-Cl}(\text{PNP-Int}(\text{PNP-Cl}(H)))$.
- (iii) $M \subseteq \text{PNP-Cl}(\text{PNP-Int}(\text{PNP-Cl}(\text{PNP-Int}(M))))$.

Proof. (i) $\Rightarrow$ (ii). Let M be a $\text{PNPS}\alpha$ -open set in $(U, \mathfrak{T}_{\text{PNP}})$. Then there exists a $\text{PNP}\alpha$ -open set Q such that $Q \subseteq M \subseteq \text{PNP-Cl}(Q)$. Hence there exists a PNP-open G such that $G \subseteq Q \subseteq \text{PNP-Int}(\text{PNP-Cl}(G))$ (by Theorem 2.14). Therefore, $\text{PNP-Cl}(G) \subseteq \text{PNP-Cl}(Q) \subseteq \text{PNP-Cl}(\text{PNP-Int}(\text{PNP-Cl}(G)))$. This implies

that $\text{PNP-Cl}(Q) \subseteq \text{PNP-Cl}(\text{PNP-Int}(\text{PNP-Cl}(G)))$. Then we obtain $G \subseteq Q \subseteq M \subseteq \text{PNP-Cl}(Q) \subseteq \text{PNP-Cl}(\text{PNP-Int}(\text{PNP-Cl}(G)))$. Therefore, $G \subseteq M \subseteq \text{PNP-Cl}(\text{PNP-Int}(\text{PNP-Cl}(G)))$, for some PNP-open set G .

(ii) $\Rightarrow$ (iii). Suppose that there exists PNP-open set G such that $G \subseteq M \subseteq \text{PNP-Cl}(\text{PNP-Int}(\text{PNP-Cl}(G)))$. We know that $\text{PNP-Int}(M) \subseteq M$. On the other hand, $G \subseteq \text{PNP-Int}(M)$ since $\text{PNP-Int}(M)$ is the largest PNP-open set contained in M . Hence $\text{PNP-Cl}(G) \subseteq \text{PNP-Cl}(\text{PNP-Int}(M))$. Then $\text{PNP-Int}(\text{PNP-Cl}(G)) \subseteq \text{PNP-Int}(\text{PNP-Cl}(\text{PNP-Int}(M)))$. Therefore, we obtain $\text{PNP-Cl}(\text{PNP-Int}(\text{PNP-Cl}(G))) \subseteq \text{PNP-Cl}(\text{PNP-Int}(\text{PNP-Cl}(\text{PNP-Int}(M))))$. But by hypothesis, $M \subseteq \text{PNP-Cl}(\text{PNP-Int}(\text{PNP-Cl}(M)))$. Hence $M \subseteq \text{PNP-Cl}(\text{PNP-Int}(\text{PNP-Cl}(G))) \subseteq \text{PNP-Cl}(\text{PNP-Int}(\text{PNP-Cl}(\text{PNP-Int}(M))))$. Then $M \subseteq \text{PNP-Cl}(\text{PNP-Int}(\text{PNP-Cl}(\text{PNP-Int}(M))))$.

(iii) $\Rightarrow$ (i). Let $M \subseteq \text{PNP-Cl}(\text{PNP-Int}(\text{PNP-Cl}(\text{PNP-Int}(M))))$. To prove M is $\text{PNPS}\alpha\text{-open}$ set. Let $Q = \text{PNP-Int}(M)$; we know that $\text{PNP-Int}(M) \subseteq M$. We need to prove $M \subseteq \text{PNP-Cl}(\text{PNP-Int}(M))$. Since $\text{PNP-Int}(\text{PNP-Cl}(\text{PNP-Int}(M))) \subseteq \text{PNP-Cl}(\text{PNP-Int}(M))$. So $\text{PNP-Cl}(\text{PNP-Int}(\text{PNP-Cl}(\text{PNP-Int}(M)))) \subseteq \text{PNP-Cl}(\text{PNP-Cl}(\text{PNP-Int}(M))) = \text{PNP-Cl}(\text{PNP-Int}(M))$. But by hypothesis $M \subseteq \text{PNP-Cl}(\text{PNP-Int}(\text{PNP-Cl}(\text{PNP-Int}(M))))$. Hence, $M \subseteq \text{PNP-Cl}(\text{PNP-Int}(\text{PNP-Cl}(\text{PNP-Int}(M)))) \subseteq \text{PNP-Cl}(\text{PNP-Int}(M))$. This implies $M \subseteq \text{PNP-Cl}(\text{PNP-Int}(M))$. Thus, there exists a PNP-open set, say Q , such that $Q \subseteq M \subseteq \text{PNP-Cl}(M)$. On the other hand, Q is a $\text{PNP}\alpha\text{-open}$ set (since Q is a PNP-open set). Hence M is a $\text{PNP}\alpha\text{-open}$ set in U .

Corollary 3.4. Let $(U, \mathfrak{T}_{\text{PNP}})$ be a PNP-Top-Space and M be a PNP-set . Then the following statements are equivalent:

- (i) M be a $\text{PNPS}\alpha\text{-closed}$ set.
- (ii) There exists a PNP-closed set H such that $\text{PNP-Int}(\text{PNP-Cl}(\text{PNP-Int}(H))) \subseteq M \subseteq H$.
- (iii) $\text{PNP-Int}(\text{PNP-Cl}(\text{PNP-Int}(\text{PNP-Cl}(M)))) \subseteq M$.

Proof. (i) $\Rightarrow$ (ii). Let M be a $\text{PNPS}\alpha\text{-closed}$ set in $(U, \mathfrak{T}_{\text{PNP}})$. Then M^c is a $\text{PNPS}\alpha\text{-open}$ set in U . Hence, by Theorem 3.3, there is a PNP-open set Q such that $Q \subseteq M^c \subseteq \text{PNP-Cl}(\text{PNP-Int}(\text{PNP-Cl}(Q)))$. Hence $(\text{PNP-Cl}(\text{PNP-Int}(\text{PNP-Cl}(Q))))^c \subseteq (M^c)^c \subseteq Q^c \Rightarrow \text{PNP-Int}(\text{PNP-Cl}(\text{PNP-Int}(Q^c))) \subseteq M \subseteq Q^c$. Put $H = Q^c$. Then H is a PNP-closed set in U . This implies that $\text{PNP-Int}(\text{PNP-Cl}(\text{PNP-Int}(H))) \subseteq M \subseteq H$, for some PNP-closed set H in U .

(ii) $\Rightarrow$ (iii). Assume there exists a PNP-closed set Q such that $\text{PNP-Int}(\text{PNP-Cl}(\text{PNP-Int}(Q))) \subseteq M \subseteq Q$, but $\text{PNP-Cl}(M)$ is the smallest PNP-closed set containing M . Then $\text{PNP-Cl}(M) \subseteq Q$, and therefore, we get $\text{PNP-Int}(\text{PNP-Cl}(M)) \subseteq \text{PNP-Int}(Q)$. Thus $\text{PNP-Cl}(\text{PNP-Int}(\text{PNP-Cl}(M))) \subseteq \text{PNP-Cl}(\text{PNP-Int}(Q))$. Then

this implies $\text{PNP-Int}\left(\text{PNP-Cl}\left(\text{PNP-Int}\left(\text{PNP-Cl}(M)\right)\right)\right) \subseteq \text{PNP-Int}\left(\text{PNP-Cl}\left(\text{PNP-Int}(Q)\right)\right) \subseteq M$. Therefore we obtain $\text{PNP-Int}\left(\text{PNP-Cl}\left(\text{PNP-Int}\left(\text{PNP-Cl}(M)\right)\right)\right) \subseteq M$.

(iii) $\Rightarrow$ (i). Assume that $\text{PNP-Int}\left(\text{PNP-Cl}\left(\text{PNP-Int}\left(\text{PNP-Cl}(M)\right)\right)\right) \subseteq M$. Then it follows that $M^c \subseteq \left(\text{PNP-Int}\left(\text{PNP-Cl}\left(\text{PNP-Int}\left(\text{PNP-Cl}(M)\right)\right)\right)\right)^c = \text{PNP-Cl}\left(\text{PNP-Int}\left(\text{PNP-Cl}\left(\text{PNP-Int}(M^c)\right)\right)\right)$. Therefore, we obtain $M^c \subseteq \text{PNP-Cl}\left(\text{PNP-Int}\left(\text{PNP-Cl}\left(\text{PNP-Int}(M^c)\right)\right)\right)$. Hence M^c is a $\text{PNPS}\alpha$ -open set in U . Thus M is a $\text{PNPS}\alpha$ -closed set in U .

Theorem 3.5. The union of any family of $\text{PNPS}\alpha$ -open sets in a $\text{PNP-Top-Space } (U, \mathfrak{T}_{\text{PNP}})$ is a $\text{PNPS}\alpha$ -open set in U .

Proof. Let $\Omega = \{M_\lambda : \lambda \in \Lambda\}$ be a family of $\text{PNPS}\alpha$ -open sets in $(U, \mathfrak{T}_{\text{PNP}})$. Since for each $\lambda \in \Lambda$, M_λ is $\text{PNPS}\alpha$ -open set, there exists a $\text{PNP}\alpha$ -open set Q_λ in U such that $Q_\lambda \subseteq M_\lambda \subseteq \text{PNP-Cl}(Q_\lambda)$. This implies that $\bigcup_{\lambda \in \Lambda} Q_\lambda \subseteq \bigcup_{\lambda \in \Lambda} M_\lambda \subseteq \bigcup_{\lambda \in \Lambda} \text{PNP-Cl}(Q_\lambda) \subseteq \text{PNP-Cl}\left(\bigcup_{\lambda \in \Lambda} Q_\lambda\right)$. By Theorem 2.13, $\bigcup_{\lambda \in \Lambda} Q_\lambda$ is $\text{PNP}\alpha$ -open set in U . Therefore, $\bigcup_{\lambda \in \Lambda} M_\lambda$ is a $\text{PNPS}\alpha$ -open set in U .

Proposition 3.6. The intersection of any family of $\text{PNPS}\alpha$ -closed sets in a $\text{PNP-Top-Space } (U, \mathfrak{T}_{\text{PNP}})$ is a $\text{PNPS}\alpha$ -closed set in $(U, \mathfrak{T}_{\text{PNP}})$.

Proof. This follows directly from Theorem 3.5.

Definition 3.7. Let $M = \{\langle x, T_M(x), C_M(x), G_M(x), U_M(x), F_M(x) \rangle : x \in X\}$ be a PNP-set in a $\text{PNP-Top-Space } (U, \mathfrak{T}_{\text{PNP}})$. Then the $\text{PNPS}\alpha$ -interior and the $\text{PNPS}\alpha$ -closure of M are denoted by $\text{PNPS}\alpha\text{-Int}(M)$ and $\text{PNPS}\alpha\text{-Cl}(M)$, and are defined as follows.

$$\text{PNPS}\alpha\text{-Int}(M) = \bigcup \{Q : Q \text{ is a } \text{PNPS}\alpha\text{-open set and } Q \subseteq M\} \text{ and}$$

$$\text{PNPS}\alpha\text{-Cl}(M) = \bigcap \{K : K \text{ is a } \text{PNPS}\alpha\text{-closed set and } M \subseteq K\}$$

Theorem 3.8. Let $(U, \mathfrak{T}_{\text{PNP}})$ be a PNP-Top-Space and M be PNP-set in U . Then the following statements are true:

- (i) $\text{PNPS}\alpha\text{-Int}(M) = M$ if and only if M is $\text{PNPS}\alpha$ -open set.
- (ii) $\text{PNPS}\alpha\text{-Cl}(M) = M$ if and only if M is $\text{PNPS}\alpha$ -closed set.
- (iii) $\text{PNPS}\alpha\text{-Int}(M)$ is the largest $\text{PNPS}\alpha$ -open set contained in M .
- (iv) $\text{PNPS}\alpha\text{-Cl}(M)$ is the smallest $\text{PNPS}\alpha$ -closed set containing M .

Proof. (i), (ii), (iii) and (iv) are obvious.

Theorem 3.9. Let $(U, \mathfrak{T}_{\text{PNP}})$ be a PNP-Top-Space and M be a PNP-set in U . Then the following statements are true

$$(i) \text{PNPS}\alpha\text{-Int}(1_{\text{PNP}} - M) = 1_{\text{PNP}} - (\text{PNPS}\alpha\text{-Cl}(M)).$$

$$(ii) \text{PNPS}\alpha\text{-Cl}(1_{\text{PNP}} - M) = 1_{\text{PNP}} - (\text{PNPS}\alpha\text{-Int}(M)).$$

Proof. (i) By definition, $\text{PNPS}\alpha\text{-Cl}(M) = \bigcap \{K : K \text{ is a } \text{PNPS}\alpha\text{-closed set and } M \subseteq K\}$

$$\begin{aligned} 1_{\text{PNP}} - (\text{PNPS}\alpha\text{-Cl}(M)) &= 1_{\text{PNP}} - \bigcap \{K : K \text{ is a } \text{PNPS}\alpha\text{-closed set and } M \subseteq K\} \\ &= \bigcup \{1_{\text{PNP}} - K : K \text{ is a } \text{PNPS}\alpha\text{-closed set and } M \subseteq K\} = \bigcup \{Q : Q \subseteq 1_{\text{PNP}} - M, Q \text{ is a } \text{PNPS}\alpha\text{-open set}\} \\ &= \text{PNPS}\alpha\text{-Int}(1_{\text{PNP}} - M). \end{aligned}$$

(ii) The proof is similar to (i).

Theorem 3.10. Let $(U, \mathfrak{T}_{\text{PNP}})$ be a PNP -Top-Space and M and Q be PNP -sets in X . Then the following statements are true:

- (i) $\text{PNPS}\alpha\text{-Int}(0_{\text{PNP}}) = 0_{\text{PNP}}$, $\text{PNPS}\alpha\text{-Int}(1_{\text{PNP}}) = 1_{\text{PNP}}$.
- (ii) $\text{PNPS}\alpha\text{-Int}(M) \subseteq M$.
- (iii) $M \subseteq Q \Rightarrow \text{PNPS}\alpha\text{-Int}(M) \subseteq \text{PNPS}\alpha\text{-Int}(Q)$.
- (iv) $\text{PNPS}\alpha\text{-Int}(M \cap Q) \subseteq (\text{PNPS}\alpha\text{-Int}(M)) \cap (\text{PNPS}\alpha\text{-Int}(Q))$.
- (v) $(\text{PNPS}\alpha\text{-Int}(M)) \cup (\text{PNPS}\alpha\text{-Int}(Q)) \subseteq \text{PNPS}\alpha\text{-Int}(M \cup Q)$.
- (vi) $\text{PNPS}\alpha\text{-Int}(\text{PNPS}\alpha\text{-Int}(M)) = \text{PNPS}\alpha\text{-Int}(M)$.

Proof. (i), (ii), (iii), (iv), (v) and (vi) are obvious.

Theorem 3.11. Let $(U, \mathfrak{T}_{\text{PNP}})$ be a PNP -Top-Space and M and Q be PNP -sets in U . Then the following statements are true:

- (i) $\text{PNPS}\alpha\text{-Cl}(0_{\text{PNP}}) = 0_{\text{PNP}}$, $\text{PNPS}\alpha\text{-Cl}(1_{\text{PNP}}) = 1_{\text{PNP}}$.
- (ii) $M \subseteq \text{PNPS}\alpha\text{-Cl}(M)$.
- (iii) $M \subseteq Q \Rightarrow \text{PNPS}\alpha\text{-Cl}(M) \subseteq \text{PNPS}\alpha\text{-Cl}(Q)$.
- (iv) $\text{PNPS}\alpha\text{-Cl}(M \cap Q) \subseteq (\text{PNPS}\alpha\text{-Cl}(M)) \cap (\text{PNPS}\alpha\text{-Cl}(Q))$.
- (v) $(\text{PNPS}\alpha\text{-Cl}(M)) \cup (\text{PNPS}\alpha\text{-Cl}(Q)) \subseteq \text{PNPS}\alpha\text{-Cl}(M \cup Q)$.
- (vi) $\text{PNPS}\alpha\text{-Cl}(\text{PNPS}\alpha\text{-Cl}(M)) = \text{PNPS}\alpha\text{-Cl}(M)$.

Proof. (i) and (ii) are evident.

(iii) By part (ii), $Q \subseteq \text{PNPS}\alpha\text{-Cl}(Q)$. Since $M \subseteq Q$, we have $M \subseteq \text{PNPS}\alpha\text{-Cl}(Q)$. But $\text{PNPS}\alpha\text{-Cl}(Q)$ is a $\text{PNPS}\alpha\text{-closed}$ set. Thus $\text{PNPS}\alpha\text{-Cl}(Q)$ is a $\text{PNPS}\alpha\text{-closed}$ set containing M . Since $\text{PNPS}\alpha\text{-Cl}(M)$ is the smallest $\text{PNPS}\alpha\text{-closed}$ set containing M , we have $\text{PNPS}\alpha\text{-Cl}(M) \subseteq \text{PNPS}\alpha\text{-Cl}(Q)$. Hence $M \subseteq Q$ implies that $\text{PNPS}\alpha\text{-Cl}(M) \subseteq \text{PNPS}\alpha\text{-Cl}(Q)$.

(iv) We know that $M \cap Q \subseteq M$ and $M \cap Q \subseteq Q$. Therefore, by part (iii), $\text{PNPS}\alpha\text{-Cl}(M \cap Q) \subseteq \text{PNPS}\alpha\text{-Cl}(M)$ and $\text{PNPS}\alpha\text{-Cl}(M \cap Q) \subseteq \text{PNPS}\alpha\text{-Cl}(Q)$. Hence, we obtain $\text{PNPS}\alpha\text{-Cl}(M \cap Q) \subseteq \text{PNPS}\alpha\text{-Cl}(M) \cap \text{PNPS}\alpha\text{-Cl}(Q)$.

(v) Since $M \subseteq M \cup Q$ and $Q \subseteq M \cup Q$, it follows from part (iii) that $\text{PNPS}\alpha\text{-Cl}(M) \subseteq \text{PNPS}\alpha\text{-Cl}(M \cup Q)$ and $\text{PNPS}\alpha\text{-Cl}(Q) \subseteq \text{PNPS}\alpha\text{-Cl}(M \cup Q)$. Hence we obtain $\text{PNPS}\alpha\text{-Cl}(M) \cup \text{PNPS}\alpha\text{-Cl}(Q) \subseteq \text{PNPS}\alpha\text{-Cl}(M \cup Q)$.

(vi) Since $\text{PNPS}\alpha\text{-Cl}(M)$ is a $\text{PNPS}\alpha\text{-closed}$ set, we have by Theorem (3.8) part (ii), $\text{PNPS}\alpha\text{-Cl}(\text{PNPS}\alpha\text{-Cl}(M)) = \text{PNPS}\alpha\text{-Cl}(M)$.

Theorem 3.12. Let $(U, \mathfrak{T}_{\text{PNP}})$ be a PNP-Top-Space and M be PNP-set in U . Then the following statements are true:

- (i) $\text{PNP-Int}(M) \subseteq \text{PNP}\alpha\text{-Int}(M) \subseteq \text{PNPS}\alpha\text{-Int}(M) \subseteq \text{PNPS}\alpha\text{-Cl}(M) \subseteq \text{PNP}\alpha\text{-Cl}(M) \subseteq \text{PNP-Cl}(M)$.
- (ii) $\text{PNP-Int}(\text{PNPS}\alpha\text{-Int}(M)) = \text{PNPS}\alpha\text{-Int}(\text{PNP-Int}(M)) = \text{PNP-Int}(M)$.
- (iii) $\text{PNP}\alpha\text{-Int}(\text{PNPS}\alpha\text{-Int}(M)) = \text{PNPS}\alpha\text{-Int}(\text{PNP}\alpha\text{-Int}(M)) = \text{PNP}\alpha\text{-Int}(M)$.
- (iv) $\text{PNP-Cl}(\text{PNPS}\alpha\text{-Cl}(M)) = \text{PNPS}\alpha\text{-Cl}(\text{PNP-Cl}(M)) = \text{PNP-Cl}(M)$.
- (v) $\text{PNP}\alpha\text{-Cl}(\text{PNPS}\alpha\text{-Cl}(M)) = \text{PNPS}\alpha\text{-Cl}(\text{PNP}\alpha\text{-Cl}(M)) = \text{PNP}\alpha\text{-Cl}(M)$.
- (vi) $\text{PNPS}\alpha\text{-Cl}(M) = M \cup \text{PNP-Int}(\text{PNP-Cl}(\text{PNP-Int}(\text{PNP-Cl}(M))))$.
- (vii) $\text{PNPS}\alpha\text{-Int}(M) = M \cap \text{PNP-Cl}(\text{PNP-Int}(\text{PNP-Cl}(\text{PNP-Int}(M))))$.
- (viii) $\text{PNP-Int}(\text{PNP-Cl}(M)) \subseteq \text{PNPS}\alpha\text{-Int}(\text{PNPS}\alpha\text{-Cl}(M))$.

Proof. We shall prove only (ii), (iii), (iv), (vii) and (viii). The proofs of (i), (v) and (vi) are easy.

(ii) Since $\text{PNP-Int}(M)$ is a PNP-open set. Then $\text{PNP-Int}(M)$ is a $\text{PNPS}\alpha\text{-open}$ set. Hence (by Theorem (3.8)), $\text{PNP-Int}(M) = \text{PNPS}\alpha\text{-Int}(\text{PNP-Int}(M))$. Thus $\text{PNP-Int}(M) = \text{PNPS}\alpha\text{-Int}(\text{PNP-Int}(M))$. ----(*)

Since $\text{PNP-Int}(M) \subseteq \text{PNPS}\alpha\text{-Int}(M)$. This implies $\text{PNP-Int}(\text{PNP-Int}(M)) \subseteq \text{PNP-Int}(\text{PNPS}\alpha\text{-Int}(M))$. Therefore $\text{PNP-Int}(M) \subseteq \text{PNP-Int}(\text{PNPS}\alpha\text{-Int}(M))$. Also $\text{PNPS}\alpha\text{-Int}(M) \subseteq M$ implies that $\text{PNP-Int}(\text{PNPS}\alpha\text{-Int}(M)) \subseteq \text{PNP-Int}(M)$. Hence, $\text{PNP-Int}(M) = \text{PNP-Int}(\text{PNPS}\alpha\text{-Int}(M))$(**). Therefore by (*) and (**), we get $\text{PNP-Int}(\text{PNPS}\alpha\text{-Int}(M)) = \text{PNPS}\alpha\text{-Int}(\text{PNP-Int}(M)) = \text{PNP-Int}(M)$.

(iii) Since $\text{PNP}\alpha\text{-Int}(M)$ is a $\text{PNP}\alpha\text{-open}$ set. Then $\text{PNP}\alpha\text{-Int}(M)$ is a $\text{PNPS}\alpha\text{-open}$ set. Therefore (by Theorem (3.8)), it follows that $\text{PNP}\alpha\text{-Int}(M) = \text{PNPS}\alpha\text{-Int}(\text{PNP}\alpha\text{-Int}(M))$. -----(*). Since $\text{PNP}\alpha\text{-Int}(M) \subseteq \text{PNPS}\alpha\text{-Int}(M)$. This implies $\text{PNP}\alpha\text{-Int}(\text{PNP}\alpha\text{-Int}(M)) \subseteq \text{PNP}\alpha\text{-Int}(\text{PNPS}\alpha\text{-Int}(M))$. Therefore $\text{PNP}\alpha\text{-Int}(M) \subseteq \text{PNP}\alpha\text{-Int}(\text{PNPS}\alpha\text{-Int}(M))$. Also $\text{PNPS}\alpha\text{-Int}(M) \subseteq M$ implies that $\text{PNP}\alpha\text{-Int}(\text{PNPS}\alpha\text{-Int}(M)) \subseteq \text{PNP}\alpha\text{-Int}(M)$. So, $\text{PNP}\alpha\text{-Int}(M) = \text{PNP}\alpha\text{-Int}(\text{PNPS}\alpha\text{-Int}(M))$(**). Hence by (*) and (**), we get $\text{PNP}\alpha\text{-Int}(\text{PNPS}\alpha\text{-Int}(M)) = \text{PNPS}\alpha\text{-Int}(\text{PNP}\alpha\text{-Int}(M)) = \text{PNP}\alpha\text{-Int}(M)$.

(iv) We know that $\text{PNP-Cl}(M)$ is a PNP-closed set, so its is a $\text{PNPS}\alpha\text{-closed}$ set. Hence by Theorem (3.8), we obtain $\text{PNP-Cl}(M) = \text{PNPS}\alpha\text{-Cl}(\text{PNP-Cl}(M))$(1) Since $\text{PNPS}\alpha\text{-Cl}(M) \subseteq \text{PNP-Cl}(M)$. (by part(i)). Then $\text{PNP-Cl}(\text{PNPS}\alpha\text{-Cl}(M)) \subseteq \text{PNP-Cl}(\text{PNP-Cl}(M)) = \text{PNP-Cl}(M)$. This implies that $\text{PNP-Cl}(\text{PNPS}\alpha\text{-Cl}(M)) \subseteq \text{PNP-Cl}(M)$. Since $M \subseteq \text{PNPS}\alpha\text{-Cl}(M) \subseteq \text{PNP-Cl}(\text{PNPS}\alpha\text{-Cl}(M))$, then $M \subseteq \text{PNP-Cl}(\text{PNPS}\alpha\text{-Cl}(M))$. Therefore $\text{PNP-Cl}(M) \subseteq \text{PNP-Cl}(\text{PNP-Cl}(\text{PNPS}\alpha\text{-Cl}(M))) = \text{PNP-Cl}(\text{PNPS}\alpha\text{-Cl}(M))$. This implies $\text{PNP-Cl}(M) \subseteq \text{PNP-Cl}(\text{PNPS}\alpha\text{-Cl}(M))$ and therefore we obtain

$\text{PNP}-Cl(M) = \text{PNP}-Cl(\text{PNPS}\alpha - Cl(M)) \dots\dots\dots(2)$. Now by (1) and (2), it follows that $\text{PNP}-Cl(\text{PNPS}\alpha - Cl(M)) = \text{PNPS}\alpha - Cl(\text{PNP}-Cl(M)) = \text{PNP}-Cl(M)$.

(vii) Since $\text{PNPS}\alpha - Int(M)$ is a $\text{PNPS}\alpha - open$ set. Therefore we obtain $\text{PNPS}\alpha - Int(M) \subseteq \text{PNP}-Cl(\text{PNP}-Int(\text{PNP}-Cl(\text{PNP}-Int(\text{PNPS}\alpha - Int(M)))) = \text{PNP}-Cl(\text{PNP}-Int(\text{PNP}-Cl(\text{PNP}-Int(M))))$ by (ii) $\text{PNPS}\alpha - Int(M) \subseteq \text{PNP}-Cl(\text{PNP}-Int(\text{PNP}-Cl(\text{PNP}-Int(M))))$, also $\text{PNPS}\alpha - Int(M) \subseteq M$. Then we obtain $\text{PNPS}\alpha - Int(M) \subseteq M \cap (\text{PNP}-Cl(\text{PNP}-Int(\text{PNP}-Cl(\text{PNP}-Int(M)))) \dots\dots\dots(1)$ Now we prove that $M \cap (\text{PNP}-Cl(\text{PNP}-Int(\text{PNP}-Cl(\text{PNP}-Int(M))))$ is a $\text{PNPS}\alpha - open$ set contained in M . It is evident that $M \cap (\text{PNP}-Cl(\text{PNP}-Int(\text{PNP}-Cl(\text{PNP}-Int(M)))) \subseteq \text{PNP}-Cl(\text{PNP}-Int(\text{PNP}-Cl(\text{PNP}-Int(M))))$ and also it is clear that $\text{PNP}-Int(M) \subseteq \text{PNP}-Cl(\text{PNP}-Int(M))$. This implies that $\text{PNP}-Int(\text{PNP}-Int(M)) \subseteq \text{PNP}-Int(\text{PNP}-Cl(\text{PNP}-Int(M)))$. This implies that $\text{PNP}-Int(M) \subseteq \text{PNP}-Int(\text{PNP}-Cl(\text{PNP}-Int(M)))$. Then this implies that $\text{PNP}-Cl(\text{PNP}-Int(M)) \subseteq \text{PNP}-Cl(\text{PNP}-Int(\text{PNP}-Cl(\text{PNP}-Int(M))))$ and $\text{PNP}-Int(M) \subseteq \text{PNP}-Cl(\text{PNP}-Int(M))$. This implies that $\text{PNP}-Int(M) \subseteq \text{PNP}-Cl(\text{PNP}-Int(\text{PNP}-Cl(\text{PNP}-Int(M))))$ and $\text{PNP}-Int(M) \subseteq M$. Therefore we get $\text{PNP}-Int(M) \subseteq M \cap (\text{PNP}-Cl(\text{PNP}-Int(\text{PNP}-Cl(\text{PNP}-Int(M)))) \subseteq \text{PNP}-Cl(\text{PNP}-Int(\text{PNP}-Cl(\text{PNP}-Int(M))))$. Hence $M \cap (\text{PNP}-Cl(\text{PNP}-Int(\text{PNP}-Cl(\text{PNP}-Int(M))))$ is a $\text{PNPS}\alpha - open$ set by Theorem (3.8). We also know that $M \cap (\text{PNP}-Cl(\text{PNP}-Int(\text{PNP}-Cl(\text{PNP}-Int(M)))) \subseteq M$. Since $\text{PNPS}\alpha - Int(M)$ is the largest $\text{PNPS}\alpha - open$ set contained in M . Therefore we get $M \cap (\text{PNP}-Cl(\text{PNP}-Int(\text{PNP}-Cl(\text{PNP}-Int(M)))) \subseteq \text{PNPS}\alpha - Int(M)$. $\dots\dots\dots(2)$

Now by (1) and (2). It follows that $\text{PNPS}\alpha - Int(M) = M \cap (\text{PNP}-Cl(\text{PNP}-Int(\text{PNP}-Cl(\text{PNP}-Int(M))))$.

(viii) Since $\text{PNPS}\alpha - Cl(M)$ is a $\text{PNPS}\alpha - closed$ set. Therefore by Corollary (3.4), it follows that $\text{PNP}-Int(\text{PNP}-Cl(\text{PNP}-Int(\text{PNP}-Cl(\text{PNPS}\alpha - Cl(M)))) \subseteq \text{PNPS}\alpha - Cl(M)$. Hence we get $\text{PNP}-Int(\text{PNP}-Cl(M)) \subseteq \text{PNP}-Int(\text{PNP}-Cl(\text{PNP}-Int(\text{PNP}-Cl(M)))) \subseteq \text{PNPS}\alpha - Cl(M)$. (by part (iv)). Therefore it follows that $\text{PNPS}\alpha - Int(\text{PNP}-Int(\text{PNP}-Cl(M))) \subseteq \text{PNPS}\alpha - Int(\text{PNPS}\alpha - Cl(M))$. By part (ii), it follows that $\text{PNP}-Int(\text{PNP}-Cl(M)) \subseteq \text{PNPS}\alpha - Int(\text{PNPS}\alpha - Cl(M))$.

Theorem 3.13. Let $(U, \mathfrak{T}_{\text{PNP}})$ be a $\text{PNP}-Top-Space$ and M be $\text{PNP}-set$ in X . Then the following statements are equivalent:

- (i) M is $\text{PNPS}\alpha - open$ set.
- (ii) $Q \subseteq M \subseteq \text{PNP}-Cl(\text{PNP}-Int(\text{PNP}-Cl(Q)))$, for some $\text{PNP}-open$ set Q .
- (iii) $Q \subseteq M \subseteq \text{PNPS}-Int(\text{PNP}-Cl(Q))$, for some $\text{PNP}-open$ set Q .
- (iv) $M \subseteq \text{PNPS}-Int(\text{PNP}-Cl(\text{PNP}-Int(M)))$.

Proof. (i) $\Rightarrow$ (ii). Let M be a $\text{PNPS}\alpha - open$ set. Then $M \subseteq \text{PNP}-Cl(\text{PNP}-Int(\text{PNP}-Cl(\text{PNP}-Int(M))))$ and $\text{PNP}-Int(M) \subseteq M$. Hence $Q \subseteq M \subseteq \text{PNP}-Cl(\text{PNP}-Int(\text{PNP}-Cl(Q)))$, where $Q = \text{PNP}-Int(M)$.

(ii) $\Rightarrow$ (iii). Suppose $Q \subseteq M \subseteq \text{PNP-Cl}(\text{PNP-Int}(\text{PNP-Cl}(Q)))$, for some PNP-open set Q . But $\text{PNPS-Int}(\text{PNP-Cl}(Q)) = \text{PNP-Cl}(\text{PNP-Int}(\text{PNP-Cl}(Q)))$. (by Lemme (2.17)). Then we have $Q \subseteq M \subseteq \text{PNPS-Int}(\text{PNP-Cl}(Q))$, for some PNP-open set Q .

(iii) $\Rightarrow$ (iv). Suppose that $Q \subseteq M \subseteq \text{PNPS-Int}(\text{PNP-Cl}(Q))$, for some PNP-open set Q . Since Q is a PNP-open set contained in M . Then $Q \subseteq \text{PNP-Int}(M)$ implies that $\text{PNP-Cl}(Q) \subseteq \text{PNP-Cl}(\text{PNP-Int}(M))$. This implies that $\text{PNPS-Int}(\text{PNP-Cl}(Q)) \subseteq \text{PNPS-Int}(\text{PNP-Cl}(\text{PNP-Int}(M)))$. But by hypothesis $M \subseteq \text{PNPS-Int}(\text{PNP-Cl}(Q))$, then $M \subseteq \text{PNPS-Int}(\text{PNP-Cl}(\text{PNP-Int}(M)))$.

(iv) $\Rightarrow$ (i). Let $M \subseteq \text{PNPS-Int}(\text{PNP-Cl}(\text{PNP-Int}(M)))$. But $\text{PNPS-Int}(\text{PNP-Cl}(\text{PNP-Int}(M))) = \text{PNP-Cl}(\text{PNP-Int}(\text{PNP-Cl}(\text{PNP-Int}(M))))$ (by Lemma 2.17). Hence it follows that $M \subseteq \text{PNP-Cl}(\text{PNP-Int}(\text{PNP-Cl}(\text{PNP-Int}(M))))$. This implies that M is $\text{PNPS}\alpha$ -open set.

Corollary 3.14. Let $(U, \mathfrak{T}_{\text{PNP}})$ be a PNP-Top-Space and M be PNP-set in X . Then the following statements are equivalent:

- (i) M is $\text{PNPS}\alpha$ -closed set.
- (ii) $\text{PNP-Int}(\text{PNP-Cl}(\text{PNP-Int}(Q))) \subseteq M \subseteq Q$, for some PNP-closed set Q .
- (iii) $\text{PNPS-Cl}(\text{PNP-Int}(Q)) \subseteq M \subseteq Q$, for some PNP-closed set Q .
- (iv) $\text{PNPS-Cl}(\text{PNP-Int}(\text{PNP-Cl}(M))) \subseteq M$.

Proof. (i) $\Rightarrow$ (ii). Let M be a $\text{PNPS}\alpha$ -closed set in U . Then by Corollary (3.4), we have $\text{PNP-Int}(\text{PNP-Cl}(\text{PNP-Int}(\text{PNP-Cl}(M)))) \subseteq M$. Since $M \subseteq \text{PNP-Cl}(M)$. Therefore we obtain $\text{PNP-Int}(\text{PNP-Cl}(\text{PNP-Int}(\text{PNP-Cl}(M)))) \subseteq M \subseteq \text{PNP-Cl}(M)$. Put $Q = \text{PNP-Cl}(M)$. Then we get $\text{PNP-Int}(\text{PNP-Cl}(\text{PNP-Int}(Q))) \subseteq M \subseteq Q$.

(ii) $\Rightarrow$ (iii). Let $\text{PNP-Int}(\text{PNP-Cl}(\text{PNP-Int}(Q))) \subseteq M \subseteq Q$, for some PNP-closed set Q . By Lemma (2.17), $\text{PNP-Int}(\text{PNP-Cl}(\text{PNP-Int}(Q))) = \text{PNPS-Cl}(\text{PNP-Int}(Q))$. Hence $\text{PNPS-Cl}(\text{PNP-Int}(Q)) \subseteq M \subseteq Q$, for some PNP-closed set Q .

(iii) $\Rightarrow$ (iv). Let $\text{PNPS-Cl}(\text{PNP-Int}(Q)) \subseteq M \subseteq Q$, for some PNP-closed set Q . Since $M \subseteq Q$ (by hypothesis), hence $\text{PNP-Cl}(M) \subseteq Q$. This implies $\text{PNP-Int}(\text{PNP-Cl}(M)) \subseteq \text{PNP-Int}(Q)$. Then this implies $\text{PNPS-Cl}(\text{PNP-Int}(\text{PNP-Cl}(M))) \subseteq \text{PNPS-Cl}(\text{PNP-Int}(Q)) \subseteq M$. Therefore it follows that $\text{PNPS-Cl}(\text{PNP-Int}(\text{PNP-Cl}(M))) \subseteq M$.

(iv) $\Rightarrow$ (i). Let $\text{PNPS-Cl}(\text{PNP-Int}(\text{PNP-Cl}(M))) \subseteq M$. But by Lemma (2.17), we get $\text{PNPS-Cl}(\text{PNP-Int}(\text{PNP-Cl}(M))) = \text{PNP-Int}(\text{PNP-Cl}(\text{PNP-Int}(\text{PNP-Cl}(M))))$. Therefore it follows that $\text{PNP-Int}(\text{PNP-Cl}(\text{PNP-Int}(\text{PNP-Cl}(M)))) \subseteq M$. This shows that M is $\text{PNPS}\alpha$ -closed set in $(U, \mathfrak{T}_{\text{PNP}})$.

4. Pentapartitioned Neutrosophic Pythagorean Semi Alpha –Continuous Mappings

In this section, we introduce the concepts of Pentapartitioned Neutrosophic Pythagorean Semi Alpha-continuous (briefly, PNPS α –continuous) mappings in PNP–Top–Spaces. Also, we study some of the main results depending on PNPS α –open sets.

Definition 4.1. Let $\xi:(U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ be a mapping. Then ξ is called a PNPS α –continuous mapping if $\xi^{-1}(Q)$ is a PNPS α –open set in U for every PNP –open set Q in W .

Theorem 4.2. Every PNP –continuous mapping is a PNPS α –continuous mapping.

Proof. Let $\xi:(U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ be PNP –continuous mapping. Let Q be a PNP –open set in (W, Γ_{PNP}) . Then $\xi^{-1}(Q)$ is PNP –open set in $(U, \mathfrak{T}_{\text{PNP}})$. Since every PNP –open set is PNPS α –open, $\xi^{-1}(Q)$ is PNPS α –open set in $(U, \mathfrak{T}_{\text{PNP}})$. Hence ξ is PNPS α –continuous mapping.

Theorem 4.3. Let $(U, \mathfrak{T}_{\text{PNP}})$, (W, Γ_{PNP}) and (Z, Ψ_{PNP}) be PNP –Top –Spaces. If $\xi:(U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ and $\eta:(W, \Gamma_{\text{PNP}}) \rightarrow (Z, \Psi_{\text{PNP}})$ are PNPS α –continuous mappings, then $\eta \circ \xi:(U, \mathfrak{T}_{\text{PNP}}) \rightarrow (Z, \Psi_{\text{PNP}})$ is a PNPS α –continuous mapping.

Proof. Let Q be a PNP –open set in Z . Since $\eta:(W, \Gamma_{\text{PNP}}) \rightarrow (Z, \Psi_{\text{PNP}})$ is PNP –continuous, $\eta^{-1}(Q)$ is PNP –open set in W . Since ξ is a PNPS α –continuous mapping, $\xi^{-1}[\eta^{-1}(Q)]$ is a PNPS α –open set in U . But $\xi^{-1}[\eta^{-1}(Q)] = (\eta \circ \xi)^{-1}(Q)$. Then $(\eta \circ \xi)^{-1}(Q)$ is a PNPS α –open set in U . Hence, $\eta \circ \xi$ is a PNPS α –continuous mapping.

Theorem 4.4. Let $(U, \mathfrak{T}_{\text{PNP}})$ and (W, Γ_{PNP}) be two PNP –Top –Spaces. Then prove that $\xi:(U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ is PNPS α –continuous if and only if $\xi^{-1}(Q)$ is PNPS α –closed set in U for every PNP –closed set Q in W .

Proof. Let Q be a PNP –closed set in W . Then Q^c is PNP –open set in W . Since ξ is PNPS α –continuous. Therefore $\xi^{-1}(Q^c)$ is a PNPS α –open set in U . Since $\xi^{-1}(Q^c) = [\xi^{-1}(Q)]^c$, $\xi^{-1}(Q)$ is a PNPS α –closed set in U .

Conversely, Let Q be a PNP –open set in W . Then Q^c is PNP –closed set in W . By assumption $\xi^{-1}(Q^c)$ is PNPS α –closed set in U . Since $\xi^{-1}(Q^c) = [\xi^{-1}(Q)]^c$, $\xi^{-1}(Q)$ is PNPS α –open set in U . Hence ξ is PNPS α –continuous.

Theorem 4.5. Let $(U, \mathfrak{T}_{\text{PNP}})$ and (W, Γ_{PNP}) be two PNP –Top –Spaces and $\xi:U \rightarrow W$ be a mapping. Then ξ is a PNPS α –continuous mapping if and only if $\xi(\text{PNPS}\alpha\text{-Cl}(M)) \subseteq \text{PNP-Cl}(\xi(M))$ for every PNP –set M in U .

Proof. Let M be a PNP –set in U and ξ be a PNPS α –continuous mapping. We note that evidently $\xi(M) \subseteq \text{PNPS}\alpha\text{-Cl}[\xi(M)]$. Then we obtain $M \subseteq \xi^{-1}[\xi(M)] \subseteq \xi^{-1}[\text{PNPS}\alpha\text{-Cl}(\xi(M))]$ and $\text{PNPS}\alpha\text{-Cl}(M) \subseteq \text{PNPS}\alpha\text{-Cl}[\xi^{-1}(\text{PNPS}\alpha\text{-Cl}(\xi(M)))]$. Since ξ is a PNPS α –continuous mapping and

$\text{PNPS}\alpha\text{-Cl}[\xi(M)]$ is a $\text{PNPS}\alpha$ -closed set. Thus we have $\text{PNPS}\alpha\text{-Cl}[\xi^{-1}(\text{PNPS}\alpha\text{-Cl}(\xi(M)))] = \xi^{-1}[\text{PNPS}\alpha\text{-Cl}(\xi(M))]$. Hence, we have $\xi[\text{PNPS}\alpha\text{-Cl}(M)] \subseteq \text{PNP}\text{-Cl}[\xi(M)]$.

Conversely, let $\xi[\text{PNPS}\alpha\text{-Cl}(M)] \subseteq \text{PNP}\text{-Cl}[\xi(M)]$, for each PNP -set M in U . Let Q be a PNP -closed set in W . Then $\text{PNPS}\alpha\text{-Cl}[\xi(\xi^{-1}(Q))] \subseteq \text{PNPS}\alpha\text{-Cl}(Q) = Q$. By assumption, $\xi[\text{PNPS}\alpha\text{-Cl}(\xi^{-1}(Q))] \subseteq \text{PNP}\text{-Cl}[\xi(\xi^{-1}(Q))] \subseteq Q$ and hence $\text{PNPS}\alpha\text{-Cl}[\xi^{-1}(Q)] \subseteq \xi^{-1}(Q)$. Since $\xi^{-1}(Q) \subseteq \text{PNPS}\alpha\text{-Cl}[\xi^{-1}(Q)]$. Thus $\text{PNPS}\alpha\text{-Cl}[\xi^{-1}(Q)] = \xi^{-1}(Q)$. This implies that $\xi^{-1}(Q)$ is a $\text{PNPS}\alpha$ -closed set in U . Therefore by Theorem 4.4, ξ is a $\text{PNPS}\alpha$ -continuous mapping.

Theorem 4.6. Let $\xi:(U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ be a mapping. Then ξ is a $\text{PNPS}\alpha$ -continuous mapping if and only if $\text{PNPS}\alpha\text{-Cl}[\xi^{-1}(Q)] \subseteq \xi^{-1}[\text{PNP}\text{-Cl}(Q)]$ for each PNP -set Q in W .

Proof. Suppose that $\xi:(U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ is a $\text{PNPS}\alpha$ -continuous mapping. Take Q be any PNP -set in W . Clearly $Q \subseteq \text{PNP}\text{-Cl}(Q)$ implies $\xi^{-1}(Q) \subseteq \xi^{-1}[\text{PNPS}\alpha\text{-Cl}(Q)]$. Then $\text{PNPS}\alpha\text{-Cl}[\xi^{-1}(Q)] \subseteq \text{PNPS}\alpha\text{-Cl}[\xi^{-1}(\text{PNP}\text{-Cl}(Q))]$. Since $\text{PNP}\text{-Cl}(Q)$ is PNP -closed set in W . So by Theorem 4.4, $\xi^{-1}[\text{PNP}\text{-Cl}(Q)]$ is a $\text{PNPS}\alpha$ -closed set in U . Therefore, $\text{PNPS}\alpha\text{-Cl}[\xi^{-1}(\text{PNP}\text{-Cl}(Q))] = \xi^{-1}[\text{PNP}\text{-Cl}(Q)]$. Hence, we have $\text{PNPS}\alpha\text{-Cl}[\xi^{-1}(Q)] \subseteq \xi^{-1}[\text{PNP}\text{-Cl}(Q)]$.

Conversely, let $\text{PNPS}\alpha\text{-Cl}[\xi^{-1}(Q)] \subseteq \xi^{-1}[\text{PNP}\text{-Cl}(Q)]$ for every PNP -set Q in W . Let F be a PNP -closed set in W . Since every PNP -closed set is $\text{PNPS}\alpha$ -closed set, $\text{PNPS}\alpha\text{-Cl}[\xi^{-1}(F)] \subseteq \xi^{-1}[\text{PNP}\text{-Cl}(F)] = \xi^{-1}(F)$. This implies that $\xi^{-1}(F)$ is a $\text{PNPS}\alpha$ -closed set in U . Thus by Theorem 4.4, ξ is a $\text{PNPS}\alpha$ -continuous mapping.

Theorem 4.7. Let $(U, \mathfrak{T}_{\text{PNP}})$ and (W, Γ_{PNP}) be two PNP -Top-Spaces and $\xi:U \rightarrow W$ be a mapping.. Then ξ is a $\text{PNPS}\alpha$ -continuous mapping if and only if $\xi^{-1}[\text{PNP}\text{-Int}(B)] \subseteq \text{PNPS}\alpha\text{-Int}[\xi^{-1}(B)]$ for every PNP -set B in W .

Proof. Let B be any PNP -set in W and ξ be a $\text{PNPS}\alpha$ -continuous mapping. Then $\text{PNP}\text{-Int}(B) \subseteq B$ implies $\xi^{-1}[\text{PNP}\text{-Int}(B)] \subseteq \xi^{-1}(B)$. Therefore we obtain $\text{PNPS}\alpha\text{-Int}[\xi^{-1}(\text{PNP}\text{-Int}(B))] \subseteq \text{PNPS}\alpha\text{-Int}[\xi^{-1}(B)]$. Since $\text{PNP}\text{-Int}(B)$ is PNP -open set in W and ξ is $\text{PNPS}\alpha$ -continuous Therefore, $\xi^{-1}[\text{PNP}\text{-Int}(B)]$ is $\text{PNPS}\alpha$ -open set in U . So $\text{PNPS}\alpha\text{-Int}[\xi^{-1}(\text{PNP}\text{-Int}(B))] = \xi^{-1}[\text{PNP}\text{-Int}(B)] \subseteq \text{PNPS}\alpha\text{-Int}[\xi^{-1}(B)]$.

Conversely, let $\xi^{-1}[\text{PNPS}\alpha\text{-Int}(B)] \subseteq \text{PNPS}\alpha\text{-Int}[\xi^{-1}(B)]$ for every PNP -set B in W . Let G be any PNP -open set in W and hence $\text{PNP}\text{-Int}(G) = G$. By hypothesis, we have $\xi^{-1}[\text{PNP}\text{-Int}(G)] \subseteq \text{PNPS}\alpha\text{-Int}[\xi^{-1}(G)]$. Therefore, $\xi^{-1}(G) \subseteq \text{PNPS}\alpha\text{-Int}[\xi^{-1}(G)]$. Also $\text{PNPS}\alpha\text{-Int}[\xi^{-1}(G)] \subseteq \xi^{-1}(G)$. Therefore $\text{PNPS}\alpha\text{-Int}[\xi^{-1}(G)] = \xi^{-1}(G)$. This implies that $\xi^{-1}(G)$ is $\text{PNPS}\alpha$ -open set in U . Hence ξ is a $\text{PNPS}\alpha$ -continuous mapping.

Definition 4.8. Let $p_{(\tau_C \mathcal{G}, \lambda, \mathcal{E})}$ be a PNP-point of a PNP-Top-Space $(U, \mathfrak{T}_{\text{PNP}})$. A PNP-set A of U is called PNP-neighbourhood of $p_{(\tau_C \mathcal{G}, \lambda, \mathcal{E})}$ if there exists a PNP-open set B such that $p_{(\tau_C \mathcal{G}, \lambda, \mathcal{E})} \in B \subseteq A$.

Theorem 4.9. Let ξ be a mapping from a PNP-Top-Space $(U, \mathfrak{T}_{\text{PNP}})$ to a PNP-Top-Space (W, Γ_{PNP}) . Then the following assertions are equivalent.

- (i) ξ is PNPS α -continuous.
- (ii) For each PNP-point $p_{(\tau_C \mathcal{G}, \lambda, \mathcal{E})} \in U$ and every PNP-neighbourhood A of $\xi(p_{(\tau_C \mathcal{G}, \lambda, \mathcal{E})})$, there exists a PNPS α -open set B such that $p_{(\tau_C \mathcal{G}, \lambda, \mathcal{E})} \in B \subseteq \xi^{-1}(A)$.
- (iii) For each PNP-point $p_{(\tau_C \mathcal{G}, \lambda, \mathcal{E})} \in U$ and every NN-neighbourhood A of $\xi(p_{(\tau_C \mathcal{G}, \lambda, \mathcal{E})})$, there exists a PNPS α -open set B in U such that $p_{(\tau_C \mathcal{G}, \lambda, \mathcal{E})} \in B$ and $\xi(B) \subseteq A$.

Proof. (i) $\Rightarrow$ (ii): Let $p_{(\tau_C \mathcal{G}, \lambda, \mathcal{E})} \in U$ be a PNP-point in U and let A be a PNP-neighbourhood of $\xi(p_{(\tau_C \mathcal{G}, \lambda, \mathcal{E})})$ in W such that $\xi(p_{(\tau_C \mathcal{G}, \lambda, \mathcal{E})}) \in B \subseteq A$. Since ξ is PNPS α -continuous, we know that $\xi^{-1}(B)$ is a PNPS α -open set in U and $p_{(\tau_C \mathcal{G}, \lambda, \mathcal{E})} \in \xi^{-1}(B) \subseteq \xi^{-1}(A)$. This implies (ii) is true.

(ii) $\Rightarrow$ (iii): Let $p_{(\tau_C \mathcal{G}, \lambda, \mathcal{E})}$ be a PNP-point in U and let A be a PNP-neighbourhood of $\xi(p_{(\tau_C \mathcal{G}, \lambda, \mathcal{E})})$. The condition (ii) implies that there exists a PNPS α -open set B in U such that $p_{(\tau_C \mathcal{G}, \lambda, \mathcal{E})} \in B \subseteq \xi^{-1}(A)$. Thus $p_{(\tau_C \mathcal{G}, \lambda, \mathcal{E})} \in B$ and $\xi(B) \subseteq \xi[\xi^{-1}(A)] \subseteq A$. Hence (iii) is true.

(iii) $\Rightarrow$ (i): Let B be a PNP-open set in W and let $p_{(\tau_C \mathcal{G}, \lambda, \mathcal{E})} \in \xi^{-1}(B)$. Since B is PNP-open set, $\xi(p_{(\tau_C \mathcal{G}, \lambda, \mathcal{E})}) \in B$, and so B is PNP-neighbourhood of $\xi(p_{(\tau_C \mathcal{G}, \lambda, \mathcal{E})})$. It follows from (iii) that there exists a PNPS α -open set A in U such that $p_{(\tau_C \mathcal{G}, \lambda, \mathcal{E})} \in A$ and $\xi(A) \subseteq B$ so that $p_{(\tau_C \mathcal{G}, \lambda, \mathcal{E})} \in A \subseteq \xi^{-1}[\xi(A)] \subseteq \xi^{-1}(B)$. This implies by definition that $\xi^{-1}(B)$ is a PNPS α -open set in U . Therefore, ξ is a PNPS α -continuous mapping.

Definition 4.10. Let $p_{(\tau_C \mathcal{G}, \lambda, \mathcal{E})}$ be a PNP-point of a PNP-Top-Space $(U, \mathfrak{T}_{\text{PNP}})$. A PNP-set A of U is called PNP-neighbourhood of $p_{(\tau_C \mathcal{G}, \lambda, \mathcal{E})}$ if there exists a PNP-open set B such that $p_{(\tau_C \mathcal{G}, \lambda, \mathcal{E})} \in B \subseteq A$.

Theorem 4.11. Let ξ be a mapping from a PNP-Top-Space $(U, \mathfrak{T}_{\text{PNP}})$ to a PNP-Top-Space (W, Γ_{PNP}) . Then the following assertions are equivalent.

- (i) ξ is PNPS α -continuous.
- (ii) For each PNP-point $p_{(\tau_C \mathcal{G}, \lambda, \mathcal{E})} \in U$ and every PNP-neighbourhood A of $\xi(p_{(\tau_C \mathcal{G}, \lambda, \mathcal{E})})$, there exists a PNPS α -open set B such that $p_{(\tau_C \mathcal{G}, \lambda, \mathcal{E})} \in B \subseteq \xi^{-1}(A)$.
- (iii) For each PNP-point $p_{(\tau_C \mathcal{G}, \lambda, \mathcal{E})} \in U$ and every PNP-neighbourhood A of $\xi(p_{(\tau_C \mathcal{G}, \lambda, \mathcal{E})})$, there exists a PNPS α -open set B in U such that $p_{(\tau_C \mathcal{G}, \lambda, \mathcal{E})} \in B$ and $\xi(B) \subseteq A$.

Proof. (i) $\Rightarrow$ (ii): Let $p_{(\tau_C \mathcal{G}, \lambda, \mathcal{E})} \in U$ be a NN-point in U and let A be a NN-neighbourhood of $\xi(p_{(\tau_C \mathcal{G}, \lambda, \mathcal{E})})$. Then there exists a PNP-open set B in W such that $\xi(p_{(\tau_C \mathcal{G}, \lambda, \mathcal{E})}) \in B \subseteq A$. Since ξ is

PNPS α -continuous, we know that $\xi^{-1}(B)$ is a PNPS α -open set in U and $p_{(\tau_C, \beta, \lambda, \xi)} \in \xi^{-1}(B) \subseteq \xi^{-1}(A)$. This implies (ii) is true.

(ii) $\Rightarrow$ (iii): Let $p_{(\tau_C, \beta, \lambda, \xi)}$ be a PNP-point in U and let A be a PNP-neighbourhood of $\xi(p_{(\tau_C, \beta, \lambda, \xi)})$. The condition (ii) implies that there exists a PNPS α -open set B in U such that $p_{(\tau_C, \beta, \lambda, \xi)} \in B \subseteq \xi^{-1}(A)$. Thus $p_{(\tau_C, \beta, \lambda, \xi)} \in B$ and $\xi(B) \subseteq \xi[\xi^{-1}(A)] \subseteq A$. Hence (iii) is true.

(iii) $\Rightarrow$ (i): Let B be a PNP-open set in W and let $p_{(\tau_C, \beta, \lambda, \xi)} \in \xi^{-1}(B)$. Since B is PNP-open set, $\xi(p_{(\tau_C, \beta, \lambda, \xi)}) \in B$, and so B is PNP-neighbourhood of $\xi(p_{(\tau_C, \beta, \lambda, \xi)})$. It follows from (iii) that there exists a PNPS α -open set A in U such that $p_{(\tau_C, \beta, \lambda, \xi)} \in A$ and $\xi(A) \subseteq B$ so that $p_{(\tau_C, \beta, \lambda, \xi)} \in A \subseteq \xi^{-1}[\xi(A)] \subseteq \xi^{-1}(B)$. This implies by definition that $f^{-1}(B)$ is a PNPS α -open set in U . Therefore, ξ is a PNPS α -continuous mapping.

5. Pentapartitioned Neutrosophic Pythagorean Semi Alpha-Irresolute Mappings

In this section, we introduce the concept of Pentapartitioned Neutrosophic Pythagorean Semi Alpha-irresolute (briefly, PNPS α -irresolute) mappings in PNP-Top-Spaces. Also, we discuss the relationship with PNPS α -continuous mappings.

Definition 5.1. Let $(U, \mathfrak{T}_{\text{PNP}})$ and (W, Γ_{PNP}) be two PNP-Top-Spaces. A mapping $\xi: U \rightarrow W$ is called PNPS α -irresolute if the inverse image of every PNPS α -open set in W is PNPS α -open in U .

Theorem 5.2. Let $(U, \mathfrak{T}_{\text{PNP}})$ and (W, Γ_{PNP}) be two PNP-Top-Spaces. Prove that a mapping $\xi: U \rightarrow W$ is PNPS α -irresolute if the inverse image of every PNPS α -closed set in W is PNPS α -closed set in U .

Proof. Let Q be any PNPS α -closed set in W . Then Q^c is PNPS α -open set in W . Since ξ is PNPS α -irresolute, $\xi^{-1}(Q^c)$ is PNPS α -open set in U and $\xi^{-1}(Q^c) = [\xi^{-1}(Q)]^c$ which implies that $\xi^{-1}(Q)$ is PNPS α -closed set in U .

Conversely, Let G be any PNPS α -open set in W . Then G^c is PNPS α -closed set in W . Thus $\xi^{-1}(G^c)$ is PNPS α -closed set in U and $\xi^{-1}(G^c) = [\xi^{-1}(G)]^c$ which implies that $\xi^{-1}(G)$ is PNPS α -open set in U . Hence $\xi: U \rightarrow W$ is PNPS α -irresolute.

Theorem 5.3. Every PNPS α -irresolute mapping is PNPS α -continuous.

Proof. Let $(U, \mathfrak{T}_{\text{PNP}})$ and (W, Γ_{PNP}) be two PNP-Top-Spaces and $\xi: U \rightarrow W$ be a mapping. Let G be a PNP-open set in W . Since every PNP-open set is PNPS α -open, G is PNPS α -open. Since ξ is PNPS α -irresolute, $\xi^{-1}(G)$ is PNPS α -open in U . Therefore ξ is PNPS α -continuous.

Theorem 5.4. Let $(U, \mathfrak{T}_{\text{PNP}})$ and (W, Γ_{PNP}) be two PNP-Top-Spaces and $\xi: U \rightarrow W$ be a mapping. Then ξ is a PNPS α -irresolute mapping if and only if $\xi(\text{PNPS}\alpha\text{-Cl}(M)) \subseteq \text{PNPS}\alpha\text{-Cl}(\xi(M))$ for every PNP-set M in U .

Proof. Let M be a PNP-set in U and ξ be a PNPS α -iresolute mapping. Then evidently $\xi(M) \subseteq \text{PNPS}\alpha - Cl[\xi(M)]$. Then we have $M \subseteq \xi^{-1}[\xi(M)] \subseteq \xi^{-1}[\text{PNPS}\alpha - Cl(\xi(M))]$ and hence $\text{PNPS}\alpha - Cl(M) \subseteq \text{PNPS}\alpha - Cl[\xi^{-1}(\text{PNPS}\alpha - Cl(\xi(M)))]$. Since ξ is a PNPS α -iresolute mapping and $\text{PNPS}\alpha - Cl[\xi(M)]$ is a PNPS α -closed set. Therefore $\text{PNPS}\alpha - Cl[\xi^{-1}(\text{PNPS}\alpha - Cl(\xi(M)))] = \xi^{-1}[\text{PNPS}\alpha - Cl(\xi(M))]$. Thus $\xi[\text{PNPS}\alpha - Cl(M)] \subseteq \text{PNPS}\alpha - Cl[\xi(M)]$.

Conversely, let $\xi[\text{PNPS}\alpha - Cl(M)] \subseteq \text{PNPS}\alpha - Cl[\xi(M)]$, for each PNP-set M in U . Let H be a PNPS α -closed set in W . Then $\text{PNPS}\alpha - Cl(H) = H$. By assumption, $\xi[\text{PNPS}\alpha - Cl(\xi^{-1}(H))] \subseteq \text{PNPS}\alpha - Cl[\xi(\xi^{-1}(H))] \subseteq \text{PNPS}\alpha - Cl(H) = H$ and then $\text{PNPS}\alpha - Cl[\xi^{-1}(H)] \subseteq \xi^{-1}(H)$. Since $\xi^{-1}(H) \subseteq \text{PNPS}\alpha - Cl[\xi^{-1}(H)]$. Thus $\text{PNPS}\alpha - Cl[\xi^{-1}(H)] = \xi^{-1}(H)$. This shows that $\xi^{-1}(H)$ is a PNPS α -closed set in U . Thus by Theorem 5.3, ξ is a PNPS α -iresolute mapping.

Theorem 5.5. Let $(U, \mathfrak{T}_{\text{PNP}})$ and (W, Γ_{PNP}) be two PNP-Top-Spaces and $\xi: U \rightarrow W$ be a mapping. Then ξ is a PNPS α -iresolute mapping if and only if $\text{PNPS}\alpha - Cl[\xi^{-1}(H)] \subseteq \xi^{-1}[\text{PNPS}\alpha - Cl(H)]$ for each PNP-set H in W .

Proof. Let $\xi: U \rightarrow W$ be a PNPS α -iresolute mapping. Let H be any PNP-set set in W . Since $H \subseteq \text{PNPS}\alpha - Cl(H)$ and therefore we get $\xi^{-1}(H) \subseteq \xi^{-1}[\text{PNPS}\alpha - Cl(H)]$. Then we obtain $\text{PNPS}\alpha - Cl[\xi^{-1}(H)] \subseteq \text{PNPS}\alpha - Cl[\xi^{-1}(\text{PNPS}\alpha - Cl(H))]$. Since $\text{PNPS}\alpha - Cl(H)$ is PNPS α -closed set in W and ξ is a PNPS α -iresolute mapping. So by Theorem 5.3, $\xi^{-1}[\text{PNPS}\alpha - Cl(H)]$ is a PNPS α -closed set in U . Therefore $\text{PNPS}\alpha - Cl[\xi^{-1}(\text{PNPS}\alpha - Cl(H))] = \xi^{-1}[\text{PNPS}\alpha - Cl(H)]$. Hence, we have $\text{PNPS}\alpha - Cl[\xi^{-1}(H)] \subseteq \xi^{-1}[\text{PNPS}\alpha - Cl(H)]$.

Conversely, let $\text{PNPS}\alpha - Cl[\xi^{-1}(H)] \subseteq \xi^{-1}(\text{PNPS}\alpha - Cl(H))$ for all PNP-sets H in W . Let Q be a PNPS α -closed set in W . By assumption, $\text{PNPS}\alpha - Cl[\xi^{-1}(Q)] \subseteq \xi^{-1}[\text{PNPS}\alpha - Cl(Q)] = \xi^{-1}(Q)$. This implies that $\xi^{-1}(Q)$ is a PNPS α -closed set in U . Thus by Theorem 5.3, ξ is a PNPS α -iresolute mapping.

Theorem 5.6. Let $(U, \mathfrak{T}_{\text{PNP}})$ and (W, Γ_{PNP}) be two PNP-Top-Spaces and $\xi: U \rightarrow W$ be a bijective mapping. Then ξ is PNPS α -iresolute if and only if $\text{PNPS}\alpha - Int[\xi(M)] \subseteq \xi[\text{PNPS}\alpha - Int(M)]$ for every PNP-set M in U .

Proof. Let M be any PNP-set in U and ξ be a bijective and PNPS α -iresolute mapping. Let $\xi(M) = Q$. Since $\text{PNPS}\alpha - Int(Q) \subseteq Q$ and hence we obtain $\xi^{-1}[\text{PNPS}\alpha - Int(Q)] \subseteq \xi^{-1}(Q)$. Since ξ is an injective mapping, so $\xi^{-1}(Q) = M$. Thus $\xi^{-1}[\text{PNPS}\alpha - Int(Q)] \subseteq M$. Then this implies that $\text{PNPS}\alpha - Int[\xi^{-1}(\text{PNPS}\alpha - Int(Q))] \subseteq \text{PNPS}\alpha - Int(Q)$. Since ξ is PNPS α -iresolute, $\xi^{-1}[\text{PNPS}\alpha - Int(Q)]$ is PNPS α -open set in U and thus $\xi^{-1}[\text{PNPS}\alpha - Int(Q)] \subseteq \text{PNPS}\alpha - Int(M)$. Hence $\xi[\xi^{-1}(\text{PNPS}\alpha - Int(Q))] \subseteq \xi[\text{PNPS}\alpha - Int(M)]$. So $\text{PNPS}\alpha - Int[\xi(M)] \subseteq \xi[\text{PNPS}\alpha - Int(M)]$.

Conversely, let $\text{PNPS}\alpha - Int[\xi(M)] \subseteq \xi[\text{PNPS}\alpha - Int(M)]$ for every PNP-set M in U . Let G be a PNPS α -open set in W . Since ξ is surjective and therefore, by hypothesis, $G = \text{PNPS}\alpha - Int(G)$

$= \text{PNPS}\alpha - \text{Int}[\xi(\xi^{-1}(G))] \subseteq \xi[\text{PNPS}\alpha - \text{Int}(\xi^{-1}(G))]$. It follows that $\xi^{-1}(G) \subseteq \text{PNPS}\alpha - \text{Int}[\xi^{-1}(G)]$. Therefore $\xi^{-1}(G)$ is $\text{PNPS}\alpha$ -open set in U . Hence ξ is a $\text{PNPS}\alpha$ -irresolute mapping.

Theorem 5.7. Let $(U, \mathfrak{T}_{\text{PNP}})$ and (W, Γ_{PNP}) be two PNP -Top-Spaces and $\xi: U \rightarrow W$ be a mapping. Then ξ is a $\text{PNPS}\alpha$ -irresolute mapping if and only if $\xi^{-1}[\text{PNPS}\alpha - \text{Int}(H)] \subseteq \text{PNPS}\alpha - \text{Int}[\xi^{-1}(H)]$ for every PNP -set H in W .

Proof. Let H be any PNP -set in W and ξ be a $\text{PNPS}\alpha$ -irresolute mapping. Then $\text{PNPS}\alpha - \text{Int}(H) \subseteq H$ implies $\xi^{-1}[\text{PNPS}\alpha - \text{Int}(H)] \subseteq \xi^{-1}(H)$. Thus $\text{PNPS}\alpha - \text{Int}[\xi^{-1}(\text{PNPS}\alpha - \text{Int}(H))] \subseteq \text{PNPS}\alpha - \text{Int}[\xi^{-1}(H)]$. Since $\text{PNPS}\alpha - \text{Int}(H)$ is $\text{PNPS}\alpha$ -open set in W and ξ is $\text{PNPS}\alpha$ -irresolute, $\xi^{-1}[\text{PNPS}\alpha - \text{Int}(H)]$ is $\text{PNPS}\alpha$ -open set in U . Thus $\text{PNPS}\alpha - \text{Int}[\xi^{-1}(\text{PNPS}\alpha - \text{Int}(H))] = \xi^{-1}[\text{PNPS}\alpha - \text{Int}(H)]$. Then we obtain $\xi^{-1}[\text{PNPS}\alpha - \text{Int}(H)] \subseteq \text{PNPS}\alpha - \text{Int}[\xi^{-1}(H)]$.

Conversely, let $\xi^{-1}[\text{PNPS}\alpha - \text{Int}(H)] \subseteq \text{PNPS}\alpha - \text{Int}[\xi^{-1}(H)]$ for every PNP -set H in W . Let Q be any $\text{PNPS}\alpha$ -open set in W . Then by assumption, $\xi^{-1}(Q) = \xi^{-1}[\text{PNPS}\alpha - \text{Int}(Q)] \subseteq \text{PNPS}\alpha - \text{Int}[\xi^{-1}(Q)]$ and therefore $\xi^{-1}(Q) \subseteq \text{PNPS}\alpha - \text{Int}[\xi^{-1}(Q)]$. This implies that $\xi^{-1}(Q)$ is $\text{PNPS}\alpha$ -open set in U . Hence ξ is a $\text{PNPS}\alpha$ -irresolute mapping.

Theorem 5.8. Let $(U, \mathfrak{T}_{\text{PNP}})$ and (W, Γ_{PNP}) be two PNP -Top-Spaces and $\xi: U \rightarrow W$ be a bijective mapping and $\text{PNPS}\alpha$ -irresolute mapping. Then prove that $\xi[\text{PNPS}\alpha - \text{Fr}(M)] \subseteq \text{PNPS}\alpha - \text{Fr}[\xi(M)]$ for every PNP -set M in U .

Proof. Let ξ be a bijective and $\text{PNPS}\alpha$ -irresolute mapping. Let M be a PNP -set in U . By definition, $\text{PNPS}\alpha - \text{Fr}(M) = \text{PNPS}\alpha - \text{Cl}(M) \cap \text{PNPS}\alpha - \text{Cl}(M^c)$. By Theorem 5.6, $\text{PNPS}\alpha - \text{Int}[\xi(M)] \subseteq \xi[\text{PNPS}\alpha - \text{Int}(M)]$ and from Theorem 5.4, $\xi[\text{PNPS}\alpha - \text{Cl}(M)] \subseteq \text{PNPS}\alpha - \text{Cl}[\xi(M)]$. Therefore $\xi[\text{PNPS}\alpha - \text{Fr}(M)] = \xi[\text{PNPS}\alpha - \text{Cl}(M)] \cap \xi[\text{PNPS}\alpha - \text{Cl}(M^c)] \subseteq \text{PNPS}\alpha - \text{Cl}[\xi(M)] \cap \text{PNPS}\alpha - \text{Cl}[\xi(M)]^c = \text{PNPS}\alpha - \text{Fr}[\xi(M)]$.

Theorem 5.9. Let $(U, \mathfrak{T}_{\text{PNP}})$ and (W, Γ_{PNP}) be two PNP -Top-Spaces and $\xi: U \rightarrow W$ be a bijective $\text{PNPS}\alpha$ -irresolute mapping. Then we have $\text{PNPS}\alpha - \text{Fr}[\xi^{-1}(M)] \subseteq \xi^{-1}[\text{PNPS}\alpha - \text{Fr}(M)]$ for every PNP -set M in W .

Proof. Let ξ be a bijective and $\text{PNPS}\alpha$ -irresolute mapping. Let M be a NN -set in W . By Theorem 5.5, we have $\text{PNPS}\alpha - \text{Cl}[\xi^{-1}(M)] \subseteq \xi^{-1}[\text{PNPS}\alpha - \text{Cl}(M)]$. Now $\xi^{-1}[\text{PNPS}\alpha - \text{Fr}(M)] = \xi^{-1}[(\text{PNPS}\alpha - \text{Cl}(M)) \cap \text{PNPS}\alpha - \text{Cl}(M^c)] = \xi^{-1}[\text{PNPS}\alpha - \text{Cl}(M)] \cap \xi^{-1}[\text{PNPS}\alpha - \text{Cl}(M^c)] \supseteq \text{PNPS}\alpha - \text{Cl}[\xi^{-1}(M)] \cap \text{PNPS}\alpha - \text{Cl}[\xi^{-1}(M^c)] = \text{PNPS}\alpha - \text{Cl}[\xi^{-1}(M)] \cap \text{PNPS}\alpha - \text{Cl}[(\xi^{-1}(M))^c] = \text{PNPS}\alpha - \text{Fr}[\xi^{-1}(M)]$. Therefore, we obtain $\text{PNPS}\alpha - \text{Fr}[\xi^{-1}(M)] \subseteq \xi^{-1}[\text{PNPS}\alpha - \text{Fr}(M)]$.

Theorem 5.10. If $\xi: (U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ and $\eta: (W, \Gamma_{\text{PNP}}) \rightarrow (Z, \Psi_{\text{PNP}})$ are $\text{PNPS}\alpha$ -irresolute, then the composition function $\eta \circ \xi: (U, \mathfrak{T}_{\text{PNP}}) \rightarrow (Z, \Psi_{\text{PNP}})$ is also $\text{PNPS}\alpha$ -irresolute.

Proof. Let Q be a $\text{PNPS}\alpha$ -open set in Z . Since η is a $\text{PNPS}\alpha$ -irresolute mapping, $\eta^{-1}(Q)$ is $\text{PNPS}\alpha$ -open in W . Since ξ is a $\text{PNPS}\alpha$ -irresolute mapping, $\xi^{-1}[\eta^{-1}(Q)] = (\eta \circ \xi)^{-1}(Q)$ is $\text{PNPS}\alpha$ -open in U . Therefore $\eta \circ \xi$ is $\text{PNPS}\alpha$ -irresolute.

Theorem 5.11. If $\xi: (U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ is $\text{PNPS}\alpha$ -irresolute and $\eta: (W, \Gamma_{\text{PNP}}) \rightarrow (Z, \Psi_{\text{PNP}})$ is $\text{PNPS}\alpha$ -continuous, then the composition function $\eta \circ \xi: (U, \mathfrak{T}_{\text{PNP}}) \rightarrow (Z, \Psi_{\text{PNP}})$ is also a $\text{PNPS}\alpha$ -continuous function.

Proof. Let Q be a PNP -open set in Z . Since η is a $\text{PNPS}\alpha$ -continuous mapping, $\eta^{-1}(Q)$ is $\text{PNPS}\alpha$ -open set in W . Since ξ is a $\text{PNPS}\alpha$ -irresolute mapping, $\xi^{-1}[\eta^{-1}(Q)] = (\eta \circ \xi)^{-1}(Q)$ is $\text{PNPS}\alpha$ -open in U . Therefore $\eta \circ \xi$ is $\text{PNPS}\alpha$ -continuous.

6. Pentapartitioned Neutrosophic Pythagorean Semi Alpha -Closed and Open Mappings

In this section, we introduce Pentapartitioned Neutrosophic Pythagorean Semi Alpha-closed (briefly, $\text{PNPS}\alpha$ -closed) mappings and Pentapartitioned Neutrosophic Pythagorean Semi Alpha-open (briefly, $\text{PNPS}\alpha$ -open) mappings in PNP -Top-Spaces and obtain certain characterizations of these classes of mappings.

Definition 6.1. Let $(U, \mathfrak{T}_{\text{PNP}})$ and (W, Γ_{PNP}) be two PNP -Top-Spaces. A function $\xi: (U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ is said to be $\text{PNPS}\alpha$ -closed if the image of each PNP -closed set in U is $\text{PNPS}\alpha$ -closed in W .

Definition 6.2. Let $(U, \mathfrak{T}_{\text{PNP}})$ and (W, Γ_{PNP}) be two PNP -Top-Spaces. A function $\xi: (U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ is said to be $\text{PNPS}\alpha$ -open if the image of each PNP -open set in U is $\text{PNPS}\alpha$ -open in W .

Theorem 6.3. Prove that a function $\xi: (U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ is $\text{PNPS}\alpha$ -closed if and only if $\text{PNPS}\alpha\text{-Cl}[\xi(A)] \subseteq \xi[\text{PNP}\text{-Cl}(A)]$ for every PNP -set A of U .

Proof. Suppose $\xi: (U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ is a $\text{PNPS}\alpha$ -closed function and A is any PNP -set in U . Then $\text{PNP}\text{-Cl}(A)$ is a PNP -closed set in U . Since ξ is $\text{PNPS}\alpha$ -closed, $\xi[\text{PNP}\text{-Cl}(A)]$ is a $\text{PNPS}\alpha$ -closed set in W . Thus we obtain $\text{PNPS}\alpha\text{-Cl}[\xi(\text{PNP}\text{-Cl}(A))] = \xi[\text{PNP}\text{-Cl}(A)]$. Therefore $\text{PNPS}\alpha\text{-Cl}[\xi(A)] \subseteq \text{PNPS}\alpha\text{-Cl}[\xi(\text{PNP}\text{-Cl}(A))] = \xi(\text{PNP}\text{-Cl}(A))$. Thus we obtain $\text{PNPS}\alpha\text{-Cl}[\xi(A)] \subseteq \xi(\text{PNP}\text{-Cl}(A))$.

Conversely, let A be a PNP -closed set in U . Then $\text{PNP}\text{-Cl}(A) = A$ and so $\xi(A) = \xi[\text{PNP}\text{-Cl}(A)]$. By our assumption $\text{PNPS}\alpha\text{-Cl}[\xi(A)] \subseteq \xi(A)$. But $\xi(A) \subseteq \text{PNPS}\alpha\text{-Cl}[\xi(A)]$. So $\text{PNPS}\alpha\text{-Cl}[\xi(A)] = \xi(A)$ and hence $\xi(A)$ is $\text{PNPS}\alpha$ -closed set in W . Thus ξ is a $\text{PNPS}\alpha$ -closed mapping.

Theorem 6.4. A mapping $\xi: (U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ is $\text{PNPS}\alpha$ -closed if and only if for each PNP -set H of W and for each PNP -open set G of U containing $\xi^{-1}(H)$ there exists a $\text{PNPS}\alpha$ -open set V of W such that $H \subseteq V$ and $\xi^{-1}(V) \subseteq G$.

Proof. Suppose ξ is a $\text{PNPS}\alpha$ -closed mapping. Let H be any PNP -set in W and G be a PNP -open set of U such that $\xi^{-1}(H) \subseteq G$. Then $V = [\xi(G^c)]^c$ is $\text{PNPS}\alpha$ -open in W such that $\xi^{-1}(V) \subseteq G$.

Conversely, let H be a PNP -closed of U . Then $\xi^{-1}[(\xi(H))^c] \subseteq H^c$ and H^c is PNP -open in U . By assumption, there exists a $\text{PNPS}\alpha$ -open set V of W such that $[\xi(H)]^c \subseteq V$ and $\xi^{-1}(V) \subseteq H^c$ and so $H \subseteq [\xi^{-1}(V)]^c$. Hence $V^c \subseteq \xi(H) \subseteq \xi[\xi^{-1}(V)]^c \subseteq V^c$, which implies $\xi(H) = V^c$. Since V^c is $\text{PNPS}\alpha$ -closed, $\xi(H)$ is $\text{PNPS}\alpha$ -closed and ξ is $\text{PNPS}\alpha$ -closed mapping.

Theorem 6.5. Let $\xi: (U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ be a PNP -closed map and $\eta: (W, \Gamma_{\text{PNP}}) \rightarrow (Z, \Psi_{\text{PNP}})$ be a $\text{PNPS}\alpha$ -closed mapping. Then their composition $\eta \circ \xi: (U, \mathfrak{T}_{\text{PNP}}) \rightarrow (Z, \Psi_{\text{PNP}})$ is a $\text{PNPS}\alpha$ -closed mapping.

Proof. Let F be a PNP -closed set in U . Since ξ is PNP -closed, $\xi(F)$ is PNP -closed in W . Since η is $\text{PNPS}\alpha$ -closed, $\eta[\xi(F)] = (\eta \circ \xi)(F)$ is $\text{PNPS}\alpha$ -closed in Z . Hence $\eta \circ \xi$ is a $\text{PNPS}\alpha$ -closed mapping.

Theorem 6.6. Let $\xi: (U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ and $\eta: (W, \Gamma_{\text{PNP}}) \rightarrow (Z, \Psi_{\text{PNP}})$ be two mappings such that their composition $\eta \circ \xi: (U, \mathfrak{T}_{\text{PNP}}) \rightarrow (Z, \Psi_{\text{PNP}})$ is $\text{PNPS}\alpha$ -closed. Then the following statements are true.

- (i) If ξ is PNP -continuous and surjective, then η is $\text{PNPS}\alpha$ -closed.
- (ii) If η is $\text{PNPS}\alpha$ -irresolute and injective, then ξ is $\text{PNPS}\alpha$ -closed.

Proof. (i) Let A be a PNP -closed set of W . Since ξ is PNP -continuous, $\xi^{-1}(A)$ is PNP -closed in U . Since $\eta \circ \xi$ is $\text{PNPS}\alpha$ -closed, $(\eta \circ \xi)(\xi^{-1}(A))$ is $\text{PNPS}\alpha$ -closed in Z . Since ξ is surjective, $(\eta \circ \xi)(\xi^{-1}(A)) = \eta(A)$ is $\text{PNPS}\alpha$ -closed in Z . Hence η is $\text{PNPS}\alpha$ -closed.

(ii) Let B be any PNP -closed set of U . Since $\eta \circ \xi$ is $\text{PNPS}\alpha$ -closed, $(\eta \circ \xi)(B)$ is $\text{PNPS}\alpha$ -closed in Z . Since η is $\text{PNPS}\alpha$ -irresolute, $\eta^{-1}((\eta \circ \xi)(B))$ is $\text{PNPS}\alpha$ -closed in W . Since η is injective, $\eta^{-1}((\eta \circ \xi)(B)) = \eta^{-1}(\eta(\xi(B))) = \xi(B)$ is $\text{PNPS}\alpha$ -closed in W . Hence ξ is $\text{PNPS}\alpha$ -closed.

Theorem 6.7. A function $\xi: (U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ is $\text{PNPS}\alpha$ -open if and only if $\xi[\text{PNP-Int}(M)] \subseteq \text{PNPS}\alpha\text{-Int}[\xi(M)]$, for every PNP -set M of U .

Proof. Suppose $\xi: (U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ is a $\text{PNPS}\alpha$ -open function and M is any PNP -set in U . Then $\text{PNP-Int}(M)$ is a PNP -open set in U . Since ξ is $\text{PNPS}\alpha$ -open, $\xi[\text{PNP-Int}(M)]$ is a $\text{PNPS}\alpha$ -open set. Since $\text{PNPS}\alpha\text{-Int}[\xi(\text{PNP-Int}(M))] \subseteq \text{PNPS}\alpha\text{-Int}[\xi(M)]$, $\xi[\text{PNP-Int}(M)] \subseteq \text{PNPS}\alpha\text{-Int}[\xi(M)]$.

Conversely, $\xi[\text{PNP-Int}(M)] \subseteq \text{PNPS}\alpha\text{-Int}[\xi(M)]$ for every PNP -set M in U . Let G be a PNP -open set in U . Then $\text{PNP-Int}(G) = G$ and by hypothesis, $\xi(G) \subseteq \text{PNPS}\alpha\text{-Int}[\xi(G)]$. But $\text{PNPS}\alpha\text{-Int}[\xi(G)] \subseteq \xi(G)$. Therefore, $\xi(G) = \text{PNPS}\alpha\text{-Int}[\xi(G)]$. Then $\xi(G)$ is $\text{PNPS}\alpha$ -open. Hence ξ is a $\text{PNPS}\alpha$ -open mapping.

Theorem 6.8. Let $\xi:(U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ be a $\text{PNPS}\alpha$ -open function. If for each $p_{(t,c,g,u,f)} \in U$ and for each PNP -neighbourhood M of $p_{(t,c,g,u,f)}$ in U , there exists a $\text{PNPS}\alpha$ -neighborhood H of $\xi(p_{(t,c,g,u,f)})$ in W such that $H \subseteq \xi(M)$.

Proof. Let $\xi:(U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ be a $\text{PNPS}\alpha$ -open function. Let $p_{(t,c,g,u,f)} \in U$ and M be any arbitrary PNP -neighbourhood of $p_{(t,c,g,u,f)}$ in U . Then there exists a PNP -open set Q such that $p_{(t,c,g,u,f)} \in Q \subseteq M$. By Theorem 6.7, $\xi(Q) = \xi[\text{PNP-Int}(Q)] \subseteq \text{PNPS}\alpha\text{-Int}[\xi(Q)]$. But, $\text{PNPS}\alpha\text{-Int}[\xi(Q)] \subseteq \xi(Q)$. Therefore, $\text{PNPS}\alpha\text{-Int}[\xi(Q)] = \xi(Q)$ and hence $\xi(Q)$ is $\text{PNPS}\alpha$ -open in W . Since $p_{(t,c,g,u,f)} \in Q \subseteq M$, $\xi(p_{(t,c,g,u,f)}) \in \xi(Q) \subseteq \xi(M)$ and so the result follows by taking $H = \xi(Q)$.

Theorem 6.9. For any bijective mapping $\xi:(U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ the following statements are equivalent:

- (i) $\xi^{-1}:(U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ is $\text{PNPS}\alpha$ -continuous.
- (ii) ξ is $\text{PNPS}\alpha$ -open.
- (iii) ξ is $\text{PNPS}\alpha$ -closed.

Proof. (i) $\Rightarrow$ (ii): Let M be a PNP -open set in U . By assumption, $(\xi^{-1})^{-1}(M) = \xi(M)$ is $\text{PNPS}\alpha$ -open in W and hence ξ is a $\text{PNPS}\alpha$ -open function.

(ii) $\Rightarrow$ (iii): Let Q be a PNP -closed set of U . Then Q^c is a PNP -open set in U . By assumption $\xi(Q^c)$ is $\text{PNPS}\alpha$ -open in W . But $\xi(Q^c) = [\xi(Q)]^c$. Therefore $\xi(Q)$ is $\text{PNPS}\alpha$ -closed set in W . Hence, ξ is $\text{PNPS}\alpha$ -closed.

(iii) $\Rightarrow$ (i): Let M be a PNP -closed set of U . By assumption, $\xi(M)$ is $\text{PNPS}\alpha$ -closed set in W . But $\xi(M) = (\xi^{-1})^{-1}(M)$ and therefore by Theorem 4.4, $\xi^{-1}:W \rightarrow U$ is $\text{PNPS}\alpha$ -continuous.

7. Pentapartitioned Neutrosophic Pythagorean Semi Alpha-Homeomorphisms

In this section, we define and study the concept of Pentapartitioned Neutrosophic Pythagorean Semi Alpha-homeomorphism (briefly, $\text{PNPS}\alpha$ -homeomorphism) in PNP -Top-Space $(U, \mathfrak{T}_{\text{PNP}})$ and obtain some of its properties and characterizations.

Definition 7.1. A map $\xi:(U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ is said to be PNP -homeomorphism if ξ is one-to-one & onto, PNP -open and PNP -continuous.

Definition 7.2. A map $\xi:(U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ is said to be $\text{PNPS}\alpha$ -homeomorphisms if ξ is one-to-one and onto, $\text{PNPS}\alpha$ -open and $\text{PNPS}\alpha$ -continuous.

Theorem 7.3. Let $\xi:(U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ be a one-to-one onto mapping. Then ξ is $\text{PNPS}\alpha$ -homeomorphism if and only if ξ is $\text{PNPS}\alpha$ -closed and $\text{PNPS}\alpha$ -continuous.

Proof. Let $\xi: (U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ be a $\text{PNPS}\alpha$ -homeomorphism. Then ξ is $\text{PNPS}\alpha$ -continuous. Let M be an arbitrary PNP -closed set in $(U, \mathfrak{T}_{\text{PNP}})$. Then $U - M$ is PNP -open. Since ξ is $\text{PNPS}\alpha$ -open, $\xi(U - M)$ is $\text{PNPS}\alpha$ -open in (W, Γ_{PNP}) . That is, $W - \xi(M)$ is $\text{PNPS}\alpha$ -open in (W, Γ_{PNP}) for every PNP -closed set M in $(U, \mathfrak{T}_{\text{PNP}})$ implies that $\xi(M)$ is $\text{PNPS}\alpha$ -closed in (W, Γ_{PNP}) . Hence $\xi: (U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ is $\text{PNPS}\alpha$ -closed.

Conversely, let ξ be a $\text{PNPS}\alpha$ -closed and $\text{PNPS}\alpha$ -continuous function. Let M be a PNP -open set in $(U, \mathfrak{T}_{\text{PNP}})$. Then $U - M$ is PNP -closed in $(U, \mathfrak{T}_{\text{PNP}})$. Since ξ is $\text{PNPS}\alpha$ -closed, $\xi(U - M)$ is $\text{PNPS}\alpha$ -closed in (W, Γ_{PNP}) . That is, $\xi(U - M) = W - \xi(M)$ is $\text{PNPS}\alpha$ -closed in (W, Γ_{PNP}) . Hence $\xi(M)$ is $\text{PNPS}\alpha$ -open in (W, Γ_{PNP}) for every PNP -open set M in $(U, \mathfrak{T}_{\text{PNP}})$. Thus, ξ is $\text{PNPS}\alpha$ -open and hence $\xi: (U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ is $\text{PNPS}\alpha$ -homeomorphism.

Theorem 7.4. : Let $\xi: (U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ be a bijective and $\text{PNPS}\alpha$ -continuous map. Then the following statements are equivalent.

- (i) ξ is a $\text{PNPS}\alpha$ -open map.
- (ii) ξ is a $\text{PNPS}\alpha$ -homeomorphism.
- (iii) ξ is a $\text{PNPS}\alpha$ -closed map.

Proof. (i) $\Rightarrow$ (ii). By the given hypothesis, the map $\xi: (U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ is bijective $\text{PNPS}\alpha$ -continuous and $\text{PNPS}\alpha$ -open. Hence $\xi: (U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ is $\text{PNPS}\alpha$ -homeomorphism.

(ii) $\Rightarrow$ (iii). Let M be a PNP -closed set in $(U, \mathfrak{T}_{\text{PNP}})$. Then M^c is PNP -open in $(U, \mathfrak{T}_{\text{PNP}})$. By assumption, $\xi(M^c)$ is $\text{PNPS}\alpha$ -open in (W, Γ_{PNP}) . That is $\xi(M^c) = (\xi(M))^c = W - \xi(M)$ is $\text{PNPS}\alpha$ -open in (W, Γ_{PNP}) and hence $\xi(M)$ is $\text{PNPS}\alpha$ -closed in (W, Γ_{PNP}) for every PNP -closed set M in $(U, \mathfrak{T}_{\text{PNP}})$. Hence the function $\xi: (U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ is $\text{PNPS}\alpha$ -closed map.

(iii) $\Rightarrow$ (i). Let G be a PNP -open set in $(U, \mathfrak{T}_{\text{PNP}})$. Then G^c is PNP -closed set in $(U, \mathfrak{T}_{\text{PNP}})$. By the given hypothesis, $\xi(G^c)$ is $\text{PNPS}\alpha$ -closed in (W, Γ_{PNP}) . But $\xi(G^c) = (\xi(G))^c = W - \xi(G)$ is $\text{PNPS}\alpha$ -closed. Therefore $\xi(G)$ is $\text{PNPS}\alpha$ -open in (W, Γ_{PNP}) . for every PNP -open set G in $(U, \mathfrak{T}_{\text{PNP}})$. Hence $\xi: (U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ is $\text{PNPS}\alpha$ -open map.

Theorem 7.5. Every PNP -homeomorphism is $\text{PNPS}\alpha$ -homeomorphism.

Proof. If $\xi: (U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ is PNP -homeomorphism, by definition 7.1, ξ is bijective, PNP -continuous and PNP -open. Then $\xi: (U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ is $\text{PNPS}\alpha$ -continuous and $\text{PNPS}\alpha$ -open. Hence the function $\xi: (U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ is $\text{PNPS}\alpha$ -homeomorphism. Every PNP -continuous function is $\text{PNPS}\alpha$ -continuous and every PNP -open map is $\text{PNPS}\alpha$ -open. Then ξ is bijective, $\text{PNPS}\alpha$ -continuous and $\text{PNPS}\alpha$ -open. Therefore ξ is $\text{PNPS}\alpha$ -homeomorphism.

Theorem 7.6. Let $\xi: (U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ be a $\text{PNP}\alpha$ -homeomorphism. Then prove that ξ is $\text{PNPS}\alpha$ -homeomorphism.

Proof. Let $\xi: (U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ be a PNP α -homeomorphism. Then ξ is bijective, PNP α -continuous and PNP α -open. Let H be a PNP-open set in (W, Γ_{PNP}) . Since ξ is PNP α -continuous, $\xi^{-1}(H)$ is PNP α -open set in $(U, \mathfrak{T}_{\text{PNP}})$. Since every PNP α -open set is PNPS α -open set, $\xi^{-1}(H)$ is PNPS α -open in $(U, \mathfrak{T}_{\text{PNP}})$ which implies that ξ is PNPS α -continuous. Let M be a PNP-open set in $(U, \mathfrak{T}_{\text{PNP}})$. Since ξ is PNP α -open, $\xi(M)$ is PNP α -open set in (W, Γ_{PNP}) . Since every PNP α -open set is PNPS α -open set, $\xi(M)$ is PNPS α -open set in (W, Γ_{PNP}) which implies ξ is PNPS α -open map. Therefore, consequently ξ is PNPS α -homeomorphism.

Theorem 7.7. Let $\xi: (U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ be a bijective and PNPS α -irresolute map. Then the following statements are equivalent.

- (i) ξ is a PNPS α -open map.
- (ii) ξ is a PNPS α -homeomorphism.
- (iii) ξ is a PNPS α -closed map.

Proof. Identical to the proof of Theorem 7.4.

Theorem 7.8. Let $\xi: (U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ be a bijective function. Then prove that ξ is PNPS α -homeomorphism if and only if $\xi[\text{PNPS}\alpha\text{-Cl}(M)] = \text{PNP-Cl}(\xi(M))$ for every subset M of $(U, \mathfrak{T}_{\text{PNP}})$.

Proof. If $\xi: (U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ is PNPS α -homeomorphisms, then ξ is PNPS α -continuous and PNPS α -closed. If $M \subseteq U$. Then $\xi[\text{PNPS}\alpha\text{-Cl}(M)] \subseteq \text{PNP-Cl}(\xi(M))$ since ξ is PNPS α -continuous. Since $\text{PNPS}\alpha\text{-Cl}(A)$ is PNP-closed in $(U, \mathfrak{T}_{\text{PNP}})$ and ξ is PNPS α -closed map, $\xi[\text{PNPS}\alpha\text{-Cl}(M)]$ is PNPS α -closed in (W, Γ_{PNP}) . Also $\text{PNPS}\alpha\text{-Cl}[\xi(\text{PNPS}\alpha\text{-Cl}(M))] = \xi(\text{PNPS}\alpha\text{-Cl}(M))$. Since $M \subseteq \text{PNPS}\alpha\text{-Cl}(M)$, $\xi(M) \subseteq \xi[\text{PNPS}\alpha\text{-Cl}(M)]$ and therefore we obtain $\text{PNP-Cl}(\xi(M)) \subseteq \text{PNP-Cl}[\xi(\text{PNPS}\alpha\text{-Cl}(M))] = \xi(\text{PNPS}\alpha\text{-Cl}(M))$. Thus $\text{PNP-Cl}(\xi(M)) \subseteq \xi(\text{PNPS}\alpha\text{-Cl}(M))$. Hence, $\xi[\text{PNPS}\alpha\text{-Cl}(M)] = \text{PNP-Cl}(\xi(M))$ if ξ is PNPS α -homeomorphism.

Conversely, if $\xi[\text{PNPS}\alpha\text{-Cl}(M)] = \text{PNP-Cl}(\xi(M))$ for every subset M of $(U, \mathfrak{T}_{\text{PNP}})$, then ξ is PNPS α -continuous. If M is PNP-closed in $(U, \mathfrak{T}_{\text{PNP}})$, then M is PNPS α -closed in $(U, \mathfrak{T}_{\text{PNP}})$. Then $\text{PNPS}\alpha\text{-Cl}(M) = M$ which implies $\xi[\text{PNPS}\alpha\text{-Cl}(M)] = \xi(M)$. Hence, by the given hypothesis, it follows that $\text{PNP-Cl}(\xi(M)) = \xi(M)$. Thus $\xi(M)$ is PNP-closed in W and hence PNPS α -closed in W for every PNP-closed set M in U . That is, ξ is a PNPS α -closed map. Therefore $\xi: (U, \mathfrak{T}_{\text{PNP}}) \rightarrow (W, \Gamma_{\text{PNP}})$ is PNPS α -homeomorphism.

Conclusions

In this paper, the author has studied the concepts of Pentapartitioned Neutrosophic Pythagorean Semi- α -open (PNPS α -open) and PNPS α -closed sets. Fundamental definitions are established, and several basic properties and characterizations of these sets are obtained. The relationships between PNPS α -open sets and existing classes of PNP-open, PNP α -open, PNP-semi-open, and PNP-closed sets are investigated, and inclusion relations among these classes are discussed. The author has applied PNPS α -open sets and PNPS α -closed sets and introduced the

notions of new classes of functions called PNPS α -continuous maps, PNPS α -irresolute maps, PNPS α -open maps, PNPS α -closed maps, and PNPS α -homeomorphisms, and has investigated fundamental properties and characterizations of these mappings in PNP-Topological Spaces. In future work, the author intends to continue to introduce and investigate properties and characterizations of the notions of Strongly PNPS α -continuous maps, Perfectly PNPS α -continuous maps, Totally PNPS α -continuous maps, Quasi PNPS α -continuous maps, Almost PNPS α -continuous maps, Somewhat PNPS α -continuous maps, PNPS α -Separation Axioms, PNPS α -Regular space, PNPS α -Normal space, almost PNPS α -Normal space, mildly PNPS α -Normal space, PNPS α -Compact space, PNPS α -Lindelof space, PNPS α -Connected space, PNPS α -Closed graph and strongly PNPS α -Closed graph in PNP-Topological Spaces. In future work, PNPS α -open sets can be applied in an application field of real-life experience.

Scientific Ethics Declaration

* The author declares that the scientific ethical and legal responsibility of this article published in EPSTEM Journal belongs to the author.

Conflict of Interest

* The author declares that he has no conflict of Interest.

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