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Determination of Fracture Parameters of Lime-Pumice Composites Using the Double-K Fracture Model

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Abstract: Experimental studies on concrete beams have demonstrated that there are three distinct phases of crack development in cement-based materials: the initiation of cracks, stable crack growth, and unstable fracture. In light of these phases, the Double-K model necessitates two fracture parameters, specifically the unstable stress intensity factor and the initiation stress intensity factor, to effectively simulate the fracture-dominated failure of concrete structures. Particularly, the initiation fracture toughness is crucial in the assessment of certain structures, including nuclear power plants, reservoirs, and water-resource facilities. Roman mortars are considered the initial type of concrete, made from lime, water, sand, and volcanic tuff, enabling the construction of structures that have lasted for nearly two millennia, with prominent examples such as the Pantheon and the Colosseum. The Pantheon temple remains in use as an exhibition hall in the present day. In this study, beams and square prismatic samples featuring three different initial crack lengths were created to determine the fracture characteristics of lime-pumice mixtures containing various aggregate sizes (4 and 16 mm) and subjected to three-point bending tests at fracture test days (28 and 90). The two-parameter model and the double-K fracture model were employed for simulating the results of the fracture test.

Keywords: Double-K fracture model, Lime, Pumice, Two-parameter model

Introduction

Because of its internal structure and the variations in mechanical qualities it undergoes based on its size, concrete is a non-linear building material. When analyzing concrete using fracture mechanics, its non-linear behavior is equally crucial. For over 50 years, a number of academics proposed non-linear fracture models that used at least two size-independent fracture parameters to simulate cracked concrete structures (Hillerborg et al., 1976; Bazant and Oh, 1983; Jenq & Shah, 1985; Nallathambi & Karihaloo, 1986; Bazant & Kazemi, 1990; Xu & Reinhardt, 1999; Hu & Duan, 2008). The non-Hookean fracture models have used specimens with edge notches, such as beams, compact tension specimens, wedge-splitting specimens, and compact compression specimens, to calculate these size-independent fracture characteristics. Additionally, over the past thirty years, square and cylindrical prismatic specimens with central notches have been extensively used to assess the fracture properties of concrete.

Certain fracture metrics, like fracture toughness or fracture energy, have been extensively employed in the design of quasi-brittle materials, including rocks and concrete, based on fracture mechanics. The phrases "fracture toughness" and "fracture energy" are understood in concrete fracture literature as "unstable fracture toughness" or "unstable fracture energy" in relation to the aforementioned factors. In concrete constructions, however, the initiation fracture toughness a measure of crack initiation is just as significant as the fracture toughness a measure of unstable crack propagation. In particular, the analysis of specific structures, like nuclear power plants, pools, and water-resource structures, depends heavily on the initiation fracture toughness. One of

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the two design parameters of the Double-K fracture model for which RILEM has recently recommended technical standards is the initial fracture toughness (Xu et al., 2021).

Roman mortars, which are constructed of lime, water, sand, and volcanic tuff, are thought to have been the first concrete. They allowed for the construction of buildings that have stood for almost two millennia, including the Colosseum and the Pantheon. Even now, the Pantheon temple is still utilized as an exhibition space. The fracture characteristics of lime-pumice mixtures with different aggregate sizes (4 and 16 mm) were investigated in this study using beams and square prismatic samples with three different initial crack lengths. These samples were then put through three-point bending tests on fracture test days (28 and 90). The fracture test results were simulated using the double-K fracture model and the two-parameter model.

Fracture Mechanics of Cementitious Composites

Building components made from quasi-brittle substances, such as rock and cement-based materials, demonstrate a three-phase behavior leading up to failure. As illustrated in a beam with an initial crack length a_0 , whose load versus crack mouth opening displacement relationship is depicted in Figure 1, these phases consist of: (1) crack initiation, which corresponds to the interval between the initial state and the first cracking load (P_i); (2) stable crack propagation, which corresponds to the interval between the first cracking load and the peak load (P_c); and (3) unstable crack propagation following the peak load. The beam, which behaves nearly in a linearly elastic manner due to material defects until the first cracking load, exhibits non-linear behavior beyond this point due to the development of a fracture process zone (FPZ) behind the free crack (Figure 1c).

A number of non-linear fracture models that utilized at least two size-independent fracture parameters to model cracked concrete members. These approaches generally utilize the fracture toughness parameter (K_{Ic}), which is the fundamental parameter in analysis of the cracked structures (Ince & Bildik, 2021). In Among the various elastic fracture models that are considered equivalent, the two-parameter model (TPM) is capable of predicting solely the unstable crack propagation at the peak load in quasi-brittle structures. This is achieved through the use of two critical parameters that relate to the critical crack length ($a_c = a_0 + \Delta a_c$): the fracture toughness (K_{Ic}^s) and the crack tip opening displacement ($CTOD_c$), which effectively characterizes the blunting at the crack tip, as previously defined:

$$K_{Ic}^s = \sigma_{Nc} \sqrt{\pi a_c} Y(g, l) \quad (1)$$

$$CTOD_c = \frac{\lambda \sigma_{Nc} a_c}{E} V_1(g, l) M(g, l) \quad (2)$$

Where, σ_{Nc} is the nominal strength, E is Young's modulus of the material, and Y , V_1 , and M are dimensionless functions of both the structural geometry (g) and the loading configuration (l). In Eq. (2), the coefficient λ takes the value of π for split-tension specimens with central cracks, 4 for beams, and 1 for semi-circular bending specimens and splitting specimens with edge cracks (Ince, 2025a; Ince et al. 2024; Ince, 2021). The term Y is widely referred to as the geometry factor, while M is obtained from the ratio $COD(a_c)/CMOD_c$, where $CMOD_c$ corresponds to the critical crack mouth opening displacement.

Fracture parameters can be deduced using one of two distinct test techniques: the compliance method introduced by RILEM (Shah, 1990) and the peak load method established by Tang et al. (1996) in this approach. In the first technique, these parameters are determined from the relationship between load and crack mouth opening displacement (P-CMOD) of three-point bending specimens that feature a central edge notch, as illustrated in Figure 1a, employing closed-loop testing apparatus. The critical crack length a_c is calculated from two measurements taken from the P-CMOD curve: the initial compliance (C_i) and the unloading compliance (C_u), which is recorded at approximately 95% of the peak load during the descending phase, as depicted in Figure 1b.

Conversely, the peak-load method eliminates the need for complex closed-loop apparatus, thus providing a more straightforward alternative to the RILEM compliance-based procedure for determining fracture parameters in TPM. Nevertheless, this method still requires a minimum of three specimens to account for the variability inherent in concrete. The test specimens may either have identical geometrical dimensions but differ in their initial notch lengths, or they may maintain consistent notch sizes while varying in overall dimensions. For each specimen, the governing equations of TPM can be expressed as outlined by Jenq & Shah (1985).

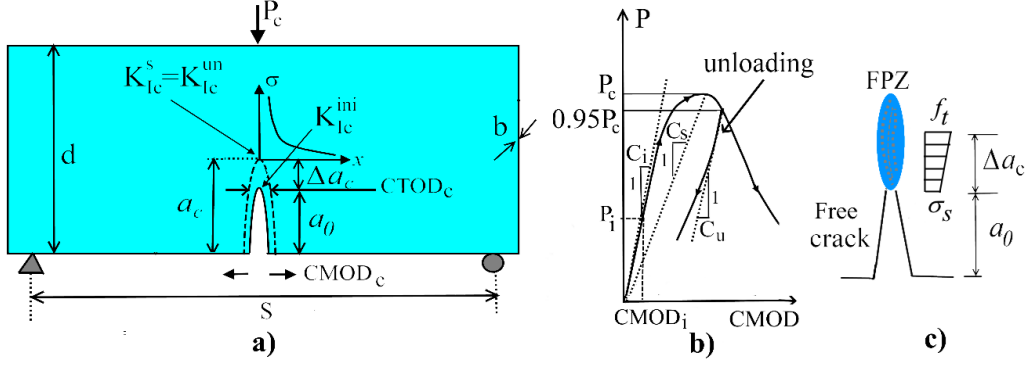


Figure 1. a) A typical concrete beam b) P-CMOD curve c) FPZ formation behind the crack

$$\begin{aligned} K_I^i(\sigma_{Nc}^i, a_c^i) &= K_{Ic}^s, \quad i = 1, 2 \\ CTOD^i(\sigma_{Nc}^i, a_c^i) &= CTOD_c \end{aligned} \quad (3)$$

In which, i refers to the i th sample. Consequently, the parameters associated with fractures can be ascertained by simultaneously resolving four non-linear equations. However, it is crucial to evaluate three or more distinct specimens to ensure statistically reliable results, given that random errors are consistently found in the measured values of σ_{Nc}^i and σ_{Nc}^j . As a result, this system, comprising several equations yet containing merely two unknowns, can solely be addressed through an optimization-based methodology. To facilitate this, Tang et al. (1996) suggested that the K_{Ic}^s and $CTOD_c$ curves relevant to particular notch depths should initially be established for each specimen with a unique a_0 . Subsequently, the sample standard deviation (s) values associated with the identical K_{Ic}^s value are computed utilizing Eq. (4):

$$s(K_{Ic}^s) = \sqrt{\sum_{i=1}^n (CTOD_{cm} - CTOD_{ci})^2 / (n-1)}, \quad i = 1, 2, \dots, n \quad (4)$$

Where, $CTOD_{cm}$ refers to the mean value of $CTOD_{ci}$ values. Consequently, K_{Ic}^s and $CTOD_c$ values corresponding to the minimum value of $s(K_{Ic}^s)$ can be assumed to be the parameters of the model.

Ince (2025b) has recently presented a modified peak load method that uses an optimization algorithm to ascertain K_{Ic}^s and $CTOD_c$. This approach relies on the concurrent resolution of Eq. (3), which acts as a failure criterion for concrete structures, taking into account solely the critical crack extension (Δa_c) at peak load for each specimen. Eren & Ince (2025) and Ince & Eren (2026a) applied this modified approach to lime–pumice composites.

The double-K fracture model simulates a concrete structure including free crack via two-parameter: the initiation fracture toughness K_{Ic}^{ini} according to the initial cracking load and the unstable fracture toughness K_{Ic}^{un} according to the peak load. The parameter K_{Ic}^{un} is computed by using the secant compliance (C_s), as shown in Figure 1b. Therefore, an unloading and reloading procedure is not required, and a closed-loop system is not necessary in this model. In the way proposed by the RILEM TC265-TDK (Xu et al. 2021), the initial cracking load in the computation of can be determined from the turning point between the linear and nonlinear parts of the ascending P-CMOD curve, as shown in Figure 1b. However, K_{Ic}^{ini} can also be computed via the inverse method, which has widely been used since the method was initially suggested by Xu & Reinhardt (1999), which was presented below in detail.

$$K_{Ic}^{ini} = K_{Ic}^{un} - K_{Ic}^c \quad (5)$$

Where, K_{Ic}^c refers to the cohesive fracture toughness, which is determined by utilizing the trapezoidal cohesive stress function at peak load, in which the value of the upper stress is equal to the tensile strength of the materials (f_t). The analysis results according to the compliance method of wedge-splitting tests and beam tests by Xu et al. (2003) indicated that the parameters K_{Ic}^s and $CTOD_c$ that were computed for the TPM were coherent with K_{Ic}^{un} and $CTOD_c$ that were computed for the double-K model. Moreover, tests by Ince (2012) on cubical and cylindrical split-tension specimens revealed that both K_{Ic}^s and K_{Ic}^{un} could also be determined by adopting the peak-load method based on the TPM.

Experimental Program

In this research, the pozzolanic activity tests of pumice material were initially conducted using CL-80-S type lime, in accordance with TS 25 (1975). To achieve this, lime-pumice mortar was poured into cement molds measuring $40 \times 40 \times 160$ mm, which contained three standard prismatic cavities, following the mixing ratios specified in TS 25 (1975). The samples were then covered with a hard mica material, wrapped in stretch film, and allowed to set for one day, after which they were placed in an oven at 55°C for a duration of one week. Four hours prior to the bending test, the samples were taken out of the oven and permitted to cool. Subsequently, they underwent a three-point bending test using a cement testing machine with a capacity of 30 kN and a support span of 100 mm. Six samples, derived from the three bending specimens that were divided in half, were subjected to a compression test in a cement press with a capacity of 40 kN on a loading area of 40×40 mm. Consequently, the flexural-tensile strength of the lime-pumice mortar was determined to be 2.3 MPa, exceeding the threshold of 1.0 MPa, while the compressive strength was found to be 4.4 MPa, surpassing the minimum requirement of 4.0 MPa. It was concluded that the pozzolanic activity of the pumice material is in compliance with TS 25 (1975).

The mixtures containing a maximum aggregate diameter of 4 mm were formulated with a water: lime: pumice: aggregate ratio of 1.53:1.00:1.00:6.00, whereas the mixture with a maximum aggregate diameter of 16 mm was prepared with a water: lime: pumice: aggregate ratio of 1.42:1.00:1.00:6.00. Two types of lime were utilized in the mixtures: CL-80-S lime, which has a specific gravity of 2.08, and CL-90-S lime, with a specific gravity of 2.09. The aggregate was air-dried before the mixing process commenced. The Fuller parabola was employed to ascertain the percentages of the aggregate gradation. Young's modulus of mixtures was determined by utilizing ultrasonic testing techniques. In this study, by using the following formulas, the dynamic the dynamic Young's modulus was determined at the test day for prismatic specimens and cubical specimens, respectively:

$$E_d = 10^3 V^2 \Delta, \quad \text{MPa} \quad (6)$$

$$E_d = 10^3 \frac{V^2 \Delta (1+\nu)(1-2\nu)}{1-\nu}, \quad \text{MPa} \quad (7)$$

In this context, V represents the velocity (km/s) and Δ denotes the unit weight (kg/l). The Poisson ratio (ν) was assumed to be 0.3 for this research, as lime-pumice mixtures are classified as low strength materials (Erdogan, 2003). It is important to note that the dynamic modulus of the material exceeds the static modulus (E_s). The equation: $E_s = 6/7 \times E_d$ was applied for lime-pumice mixtures in this investigation (Lydon, 1984).

In this study, beams, single-notch cube bending (SNCB) specimens, and splitting specimens with edge cracks (SPEC) containing three distinct initial crack lengths were fabricated to determine the fracture parameters of lime-pumice mixtures. All specimens were kept by wrapping in stretch film in the mold until the test day. The specimens were notched with a diamond saw blade of 1.5 mm thickness on the day of the fracture test. For each mixture, cubes with 100 mm were also produced to determine compressive strength. The specimens were notched with a diamond saw blade of 1.5 mm thickness on the day of the fracture test. Then, the compression tests and the bending tests were conducted by employing a digital compression machine with a capacity of 100 kN. The specimens were loaded monotonically until final failure, and care was taken to apply a constant loading rate (Figure 2). Typically, approximately 2 min (± 30 sec) elapsed before the maximum load capacity for each specimen was reached.

Crack patterns observed at the failure of the notched specimens are illustrated in Figure 3. Four of the SNCB specimens exhibited shear fractures rather than bending fractures along the crack line, and these were not presented in Figure 3. The primary reason for this behavior is that SNCB specimens function similarly to deep beams, where the depth of the compression zone exceeds that of the beam subjected to three-point bending loading (Ince & Eren, 2026b).

Table 1 summarizes specimen types, lime types used in the mixtures, the maximum aggregate size (d_{max}), unit weight of the mixtures (Δ), fracture test age (T), compressive strength (f_c), Young's modulus (E_s), and specimen size (d) of the mixtures. In this table, by denoting n_s , the number of specimens in the group, the mean nominal strength values (σ_{Nc}) are also reported according to the grouped relative notch depths ($\alpha_0 = a_0/d$), respectively.

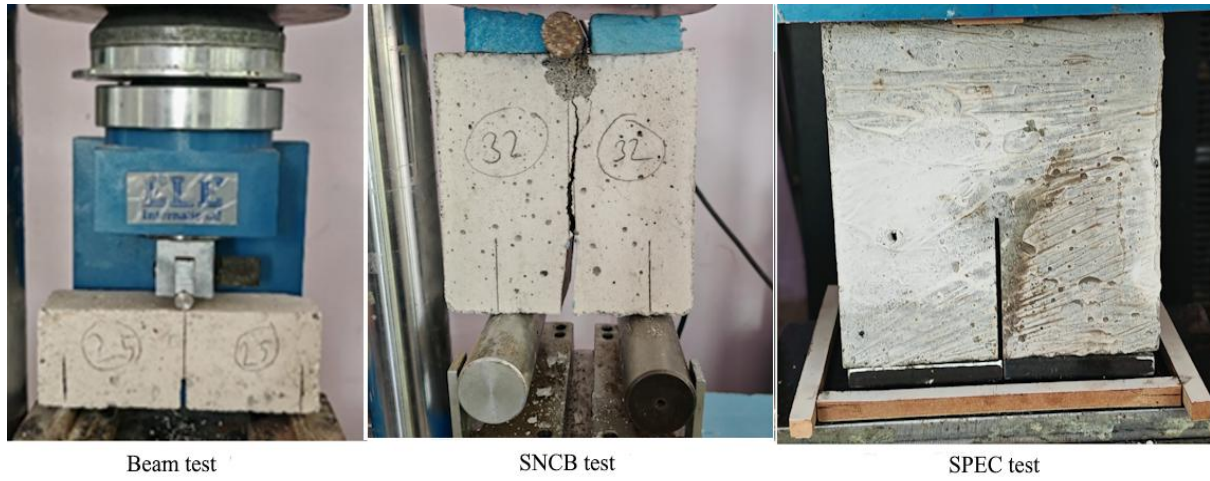


Figure 2. Test setup of the notched specimens

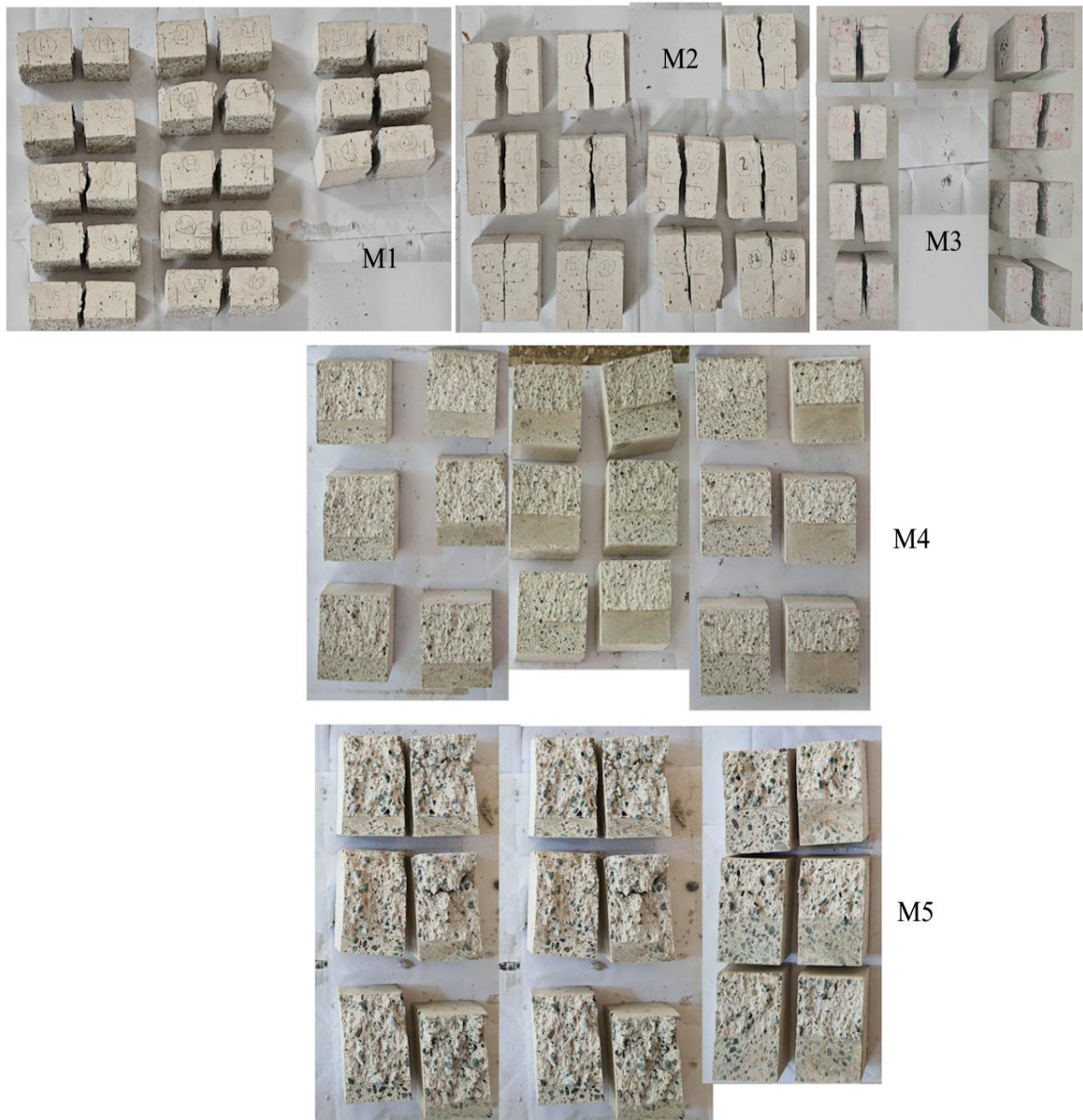


Figure 3. Crack patterns and failure mechanisms of the notched specimens

Table 1. Test results of the notched specimens

Series	Specimen	Lime	d_{max} mm	Δ g/cm ³	T Days	f_c MPa	E_s MPa	d mm	n_s	α_0	σ_{Nc} MPa
M1	Beam	CL-80-S	4	1.93	28	3.79	13384	50	5	0.191	0.821
									5	0.294	0.614
									3	0.449	0.424
M2	SNCB	CL-80-S	4	1.93	28	3.79	13384	150	3	0.316	0.302
									4	0.395	0.268
									4	0.477	0.237
M3	SNCB	CL-90-S	16	2.03	28	3.75	12537	100	4	0.308	0.458
									1	0.399	0.269
									4	0.509	0.303
M4	SPEC	CL-90-S	4	2.01	90	9.82	14216	100	3	0.291	0.611
									3	0.398	0.421
									3	0.48	0.327
M5	SPEC	CL-90-S	16	2.09	90	10.82	20382	150	3	0.188	1.041
									3	0.309	0.886
									3	0.437	0.442

Analysis of Lime-Pumice Composites According to the Double-K Fracture Model

In this study, the experimental results of lime–pumice composites were initially analyzed according to the two-parameter model using the peak-load method developed by Tang et al. (1996). As explained in detail below, this method is based on a specific statistical analysis procedure. Based on Eq. (3), this approach determines the fracture parameters of the material (K_{Ic}^s and $CTOD_c$) using only the peak loads of the specimens. In this method, at least two different groups of specimens are required. Accordingly, the specimen groups should be selected either as specimens having the same dimensions but two different notch lengths or as specimens with the same relative notch length (α_0) but different sizes. However, as previously stated, random errors are always present in the measured nominal strength values (σ_{Nc}). Therefore, in order to obtain statistically valid results, it is recommended that at least three specimen groups be tested.

As shown in Table 1, for the five series, the lime–pumice specimens used in this study were designed as three groups having the same dimensions but three different notch lengths. The applications of the analysis performed according to the peak-load method are presented in Figure 4. First, the K_{Ic}^s - $CTOD_c$ relationships for each group were plotted using Eqs. (1) and (2). Subsequently, the average K_{Ic}^s - $CTOD_c$ relationship for these groups was determined. The statistical procedure of the classical peak-load method is based on the calculation of the sample standard deviation of the groups according to Eq. (4). The following formulas were applied to beams with a span/depth ratio of 2.5 in this study (Yang et al., 1997):

$$Y(\alpha) = \frac{1.83 - 1.65\alpha + 4.76\alpha^2 - 5.3\alpha^3 + 2.51\alpha^4}{\sqrt{\pi}(1+2\alpha)(1-\alpha)^{1.5}} \quad (8)$$

$$V_1(\alpha) = 0.65 - 1.88\alpha + 3.02\alpha^2 - 2.69\alpha^3 + \frac{0.68}{(1-\alpha)^2} \quad (9)$$

$$M(\beta = \alpha_0/\alpha_c, \alpha_c) = \sqrt{(1-\beta)^2 + (1.081 - 1.149\alpha_c)(\beta - \beta^2)} \quad (10)$$

For the SNCB specimens with span/depth=0.6, the LEFM formulas were given by Ince & Eren (2026b):

$$Y(\alpha) = \frac{2.854 - 16.967\alpha + 60.766\alpha^2 - 102.025\alpha^3 + 84.581\alpha^4 - 27.245\alpha^5}{(1+2\alpha)(1-\alpha)^{1.5}} \quad (11)$$

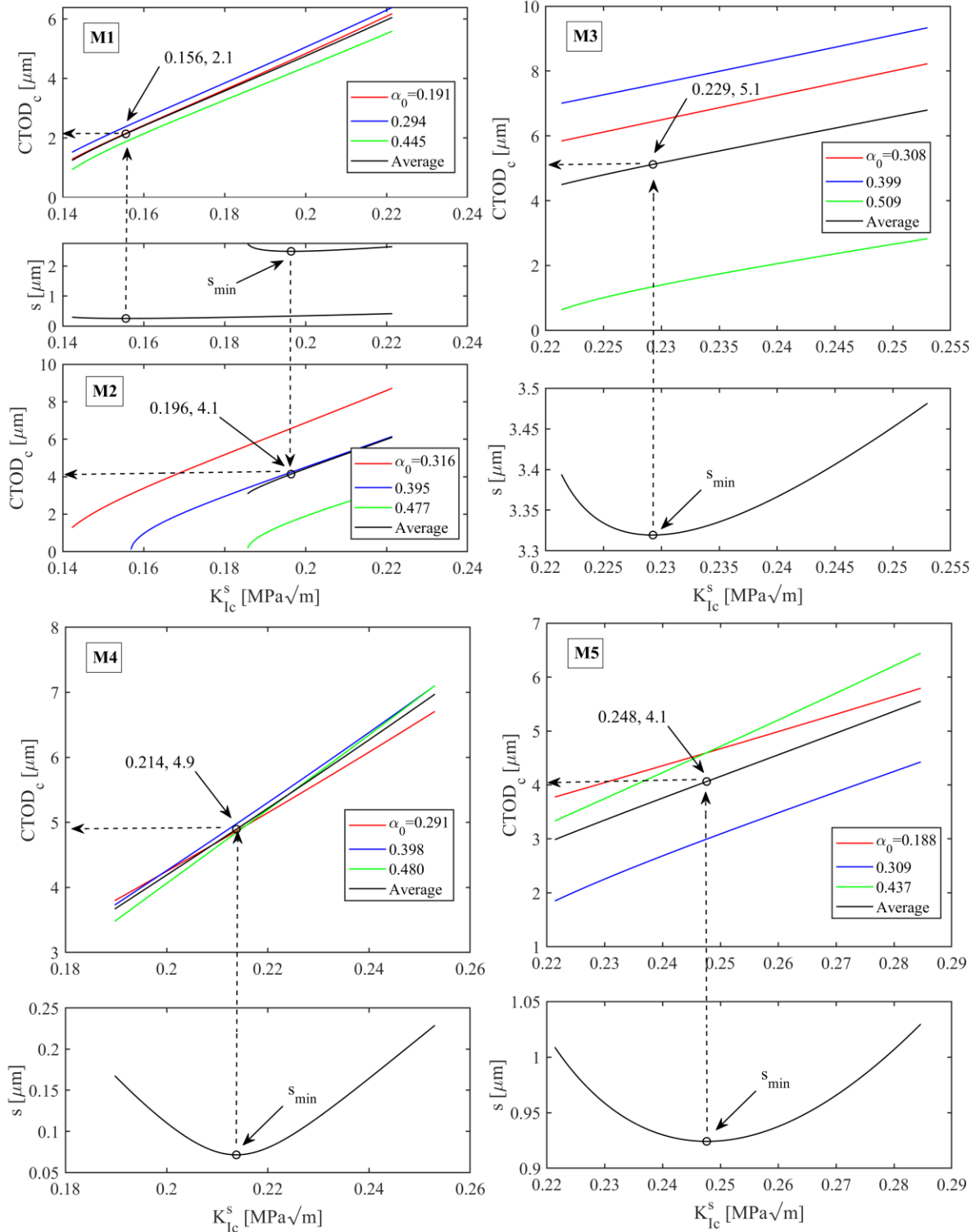


Figure 4. Application of the peak load method to the notched specimens

$$V_1(\alpha) = 8.691 - 27.46\alpha + 5.715\alpha^2 + 74.589\alpha^3 - 101.024\alpha^4 + \frac{4.476}{(1-\alpha)^2} \quad (12)$$

$$M(\beta = \alpha_0/\alpha_c, \alpha_c) = \sqrt{(1-\beta)^2 + (1.007 - 2.747\alpha_c + 2.018\alpha_c^2)(\beta - \beta^2)} \quad (13)$$

For the SPEC specimens subjected to the splitting loading used in Table 2, by denoting γ , the ratio of the width of the distributed load to the specimen depth (γ is 0.1 and 0.2 for the mixtures of M4 and M5, respectively), the LEFM formulas are given by (Ince, 2021):

$$Y(\alpha) = 0.3691 + 0.4492\alpha + 1.3284\alpha^2 + 2.4467\alpha^3 \quad \text{for } \gamma=0.1 \quad (14)$$

$$Y(\alpha) = 0.3341 + 0.0101\alpha + 2.5609\alpha^2 + 0.4067\alpha^3 \quad \text{for } \gamma=0.2 \quad (15)$$

$$V_1(\alpha) = 0.3722 + 2.0219\alpha - 6.6125\alpha^2 + 14.3038\alpha^3 \quad \text{for } \gamma=0.1 \quad (16)$$

$$V_1(\alpha) = 0.3040 + 1.7717\alpha - 5.3749\alpha^2 + 12.2420\alpha^3 \quad \text{for } \gamma=0.2 \quad (17)$$

$$M(\beta = \alpha_0/\alpha_c, \alpha_c) = \sqrt{(1-\beta)^2 + \left(2.16 - 4.02\alpha_c(1-\gamma)^{-0.23} \alpha_c^{3.67} \beta^{1.15}\right)(\beta-\beta^2)} \quad (18)$$

The cohesive fracture toughness of the material (K_{Ic}^c) in Eq. (5) must be computed to determine the imitation fracture toughness (K_{Ic}^{ini}) at first. For this, the following formula proposed by Xu & Reinhardt (1999), which is valid for an infinitely large specimen, was used in this study:

$$K_{Ic}^c = \sqrt{\frac{8\Delta a_c^\infty}{9\pi}} \left[2f_t + \sigma_s (CTOD_c) \right], \quad \Delta a_c^\infty = \frac{\pi}{32} \left(\frac{CTOD_c E_c}{K_{Ic}^s} \right)^2 \quad (19)$$

Where Δa_c^∞ is the crack extension in an infinitely large specimen. f_t and σ_s are the upper and lower limit stress values, respectively, as shown in Figure 1c. In this study, the direct tensile strengths of the mixtures (f_t), which is valid for an infinitely large specimen and proposed by Ince & Fenerli (2022), and the values of σ_s were determined by using Eqs. (20) and (21), as follows:

$$f_t = 0.356 \frac{K_{Ic}^s}{\sqrt{\Delta a_c^\infty}} \quad (20)$$

$$\sigma_s(CTOD_c) = f_t \left(1 - \frac{CTOD_c}{w_0} \right), \quad w_0 = \frac{2G_f}{f_t} \quad (21)$$

In which $G_f (= (K_{Ic}^s)^2 / E_c)$, is the fracture energy of the material. For each series, the computed above fracture properties are summarized in Table 2.

Table 2. Analysis results of the notched specimens

Series	$K_{Ic}^s = K_{Ic}^{un}$ MPa√m	$CTOD_c$ mm	G_f N/m	Δa_c^∞ mm	f_t MPa	σ_s MPa	K_{Ic}^{ini} MPa√m
M1	0.156	0.0021	1.809	3.32	0.961	0.415	0.084
M2	0.196	0.0041	2.881	7.80	0.791	0.342	0.106
M3	0.229	0.0051	4.193	7.70	0.930	0.402	0.124
M4	0.214	0.0049	3.215	10.40	0.746	0.322	0.115
M5	0.248	0.0041	3.008	10.99	0.840	0.363	0.134

Conclusion

The presented work investigated the fracture behavior of lime-pumice mixtures using the peak load method based on the double-K fracture model in fracture mechanics of concrete. The results of this work are detailed below:

The fracture toughness and strength values of the lime-pumice mixtures are approximately one-fifth of those found in normal cementitious mixtures. This reduction may be attributed to the low strength of the lime-pozzolan binder phase. The mechanism at play is that cracks tend to travel around the aggregate, as clearly illustrated in the M4 and M5 series specimens shown in Figure 4. Naturally, this process results in minimal energy absorption.

The initiation and unstable fracture toughness values (K_{Ic}^{ini} and K_{Ic}^{un}) of 90-day mortar are approximately 20% greater than those of 28-day mortar. The fracture toughness values of the lime-pumice mixture with a maximum aggregate size of 16 mm are approximately 30% greater than those of the mixture with a maximum aggregate size of 4 mm after 90 days. Additionally, for the 28-day samples, the fracture toughness values of the mixture with a maximum aggregate size of 16 mm are about 10% greater than those of the mortar.

Recommendations

Further works can come up with more reliable results by scrutinizing various types and sizes of aggregates and specimen types to verify the above findings.

Scientific Ethics Declaration

* The authors declare that the scientific ethical and legal responsibility of this article published in EPSTEM Journal belongs to the authors.

Conflict of Interest

* The authors declare that they have no conflicts of interest

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